Republic of Iraq
Ministry of Higher Education and Scientific Research
University of Baghdad
College of Education for Pure Sciences Ibn Al Haitham
Department of Mathematics

The Fuzzy Sets

A project submitted to the department of Mathematics College of Education for Pure Sciences Ibn Al Haitham in partial fulfitment for the requirement of the Bachelor of Education Degree in Mathematics

By the student

(Zeenah Khalid Mahmmod)

Supervised by

(Ass. Prof. Hatam. Y. Khalaf)

2018-2019

أؤيد بأن المشروع المعد من قبل الطالبة زينه خالد محمود

قد تم بأشرافي في كلية التربية للعلوم الصرفة ابن الهيثم/ قسم علوم الرياضيات

توقيع المشرف:

اسم المشرف:

التاريخ:

الإهداء

إلى من شرفني بحمل أسمه ... أبي...

إلى نبع الحنان ... أمي ...

إلى رفاق دربي ... أخوتي و أخواتي ...

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Abstract:

In our research we presenfed the concept of fuzzy sets, fuzzy subgroups and fuzzy ideals, And showed how to build mathematical exercises between the two concepts presented and was borroved some of the sources that provided us with many of the basies of this subject.

Introduction:

This research conststs of three chapter. The first chapter contains three parts, the first section deals with the explanation of the concept of the fuzzy sets and fuzzy subsets. The second section explains the processes on these sets. The third section shows the multiplication by $(\cdot)$ (Direct product), between two fuzzy sets.

The second chapter talks about the fuzzy groups , fuzzy subgroups and consists of two section. The first section explains the fuzzy subgroups and the second section speaks of the normal subgroups.

Chapter three and the later consists of two parts. The first section explains the partial episodes of the fuzzy ring $(R, +,\cdot)$ and the second section addresses the foggy ideals.

CHAPTER 1

FUZZYSETS

Introduction:

Now, we shall start by the concepts of fuzzy sets and the basic definition with some explain fuzzy sets, and we shall give some important definition and properties with operations on fuzzy sets, also we shall give the binary operation on fuzzy set. This research into three subsections.

Now, we must firstly begin to explain "Fuzzy". Zadeh introduced the fundamental concept of a fuzzy set in 1965.

Zadeh's original definition of fuzzy set "**A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership(characteristic) function which assigns to each object a grade of membership ranging between zero and one".**

The classes of objects encountered in the real physical world do not have precisely defined criteria of membership. **For example,** the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the class of all numbers, which are much greater than.

Clearly, the "class of beautiful women", or "the class of tall men", do not constitute classes or sets in the usual mathematical sense of these terms.

1.1. Fuzzy Set on a Set R

We shall introduce some definitions and concepts related to fuzzy sets of R.

Now, in order to put fuzzy set in mathematical frame, we give the basic definition of fuzzy set.

Definition 1.1.1: Let R be a non-empty set and I will denote the closed unit interval [0,1] of the real line (real numbers).

Let $I^R = \{A: R \to I | A\ is\ a\ mapping\}$ be a collection of all mappings from R into I. A member of I^R is called a fuzzy subset of R or a fuzzy set in R.

So A is a fuzzy subset of R if and only if $A: R \to I$ is a mapping and A is a characteristic function which associates with each point x in R a real number $A(x)$ in the interval [0,1] with the value of A at x representing the grade of membership of x in A.

From this definition, we recall that the nearer value of $A(x)$ to unity the higher grade of membership of x in A.

The following example explain this definition:

Example 1.1.2: Let R be the real line R and let A be a fuzzy subset of numbers which are much greater than 1.

Then one can give a precise characterization of A by specifying:

$$A(x) = \begin{cases} \dfrac{x-1}{x} & if\ x > 1 \\ 0 & if\ x \leq 1 \end{cases}$$

Remark1.1.3: If we want to know the difference between fuzzy sets and ordinary sets, we observe that when A is a set in the ordinary sense of the term, so its membership function can take only two values 0 and 1 with a characteristic function:

$$A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases}$$

Then $A(x) \in \{0,1\}$ for all $x \in R$ While if A is a fuzzy set in a set R then

$A(x) \in [0,1]$for all $x \in R$.Thus the ordinary sets become a special case of fuzzy sets.

Next, we shall give some definitions and concepts related to fuzzy subsets of R.

Definition 1.1.4: Let $x_t: R \to I$ be a fuzzy subset of R, where $x \in R$ and $t \in [0,1]$ defined by: for all $y \in R$

$$x_t(y) = \begin{cases} t & if\ x = y \\ 0 & if\ x \neq y \end{cases}$$

Then x_t is called a fuzzy singleton.

Definition 1.1.5: Let A and B be two fuzzy subsets of R. Then:

(i) $A = B\ if\ and\ only\ if\ A(x) = B(x)\ for\ all\ x \in R.$

(ii) $A \subseteq B\ if\ and\ only\ if\ A(x) \leq B(x)\ for\ all\ x \in R.$

If $A \subseteq B$ and there exists $x \in R$ such that $A(x) < B(x)$,

By part (ii), we can deduce that $x_t \subseteq A$ if and only if $x_t(y) \leq A(y)$ for all $y \in R\ if\ t > 0, then\ A(x) \geq t.$

Now, we shall introduce a definition of level subsets, which is a set between fuzzy set and ordinary set

Definition 1.1.6: Let A be a fuzzy subset of a non-empty set R, for $t \in [0,1]$. Then the set $A_t = \{x \in R | A(x) \geq t\}$ is called a level subset of R with respect to A.

Note that is a subset of R in the ordinary sense.

Operations on Fuzzy Subset of R 1.2.

We can give the definitions of intersection and union of fuzzy subsets of R.

Definition 1.2.1: Let A and B be two fuzzy sets in R. Then :

$(i)\ (A \cap B) = min\{A(x), B(x)\}, for\ all\ x \in R.$

$(ii)\ (A \cup B) = max\{A(x), B(x)\}, for\ all\ x \in R.$

Notice that $A \cap B, A \cup B$ are fuzzy sets in R.

If we generalize this definition by a collection of fuzzy sets, then:

$$\left(\underset{i\in}{\cap} A^i\right)(x) = inf\left\{A^i(x) | i \in \ \right\}, for\ all\ x \in R.$$

$$\left(\underset{i\in}{\cup} A^i\right)(x) = sup\left\{A^i(x) | i \in \ \right\}, for\ all\ x \in R.$$

Which are also fuzzy sets in R.

The following properties of level subset hold, for each

$(i)\ \ (A \cap B)_t = A_t \cap B_{t.}$

$(ii)\ \ (A \cup B)_t = A_t \cup B_{t.}$

$(iii)\ \ A = B\ if\ and\ only\ if\ A_t = B_t.$

1.3.Direct Prodct of Fuzzy Subset

We will recall the definitions of algebraic structure of fuzzy sets.

Definition 1.3.1: Let " $\cdot$ " be a binary operation in R, and $A, B \in I^R$.Then, for each $x \in R$ we define:

$$A \cdot B(x) = \begin{cases} sup\left\{min\{A(y), B(z)\}\right\} & if\ x = yz \qquad y, z \in R \\ 0 & otherwise \end{cases}$$

It is clear that $A \cdot B$ is a fuzzy subset of R. And it we take a collection $\left\{A^i(x) | i \in \ \right\}$ of fuzzy subsexts, then for each $x \in R$:

$$\left(\prod_{i\in} A^i\right)(x) = \begin{cases} sup\left\{inf\{A^i\left(x^i\right)\}\right\} & if\ x = \prod_{i\in} x^i\ , \qquad x^i \in R \\ 0 & otherwise \end{cases}$$

Proposition 1.3.2: Let " · " be a binary operation in R, and $x_t \cdot y_s$ be two fuzzy singletons, where $y, z \in R\ and\ t, s \in [0,1]. Then\ x_t \cdot y_s = (xy)_r\ where\ r = min\{s, t\}$.

CHAPTER 2

FUZZY SUBGROUPS OF A GROUP

This section consists of the concepts of the fuzzy groups, which was coined by Rosenfeld. Also, this section is divided into three subsections. In subsection one, we introduce the basic concept of fuzzy group. In subsection two, we give the definition of fuzzy normal subgroup. In subsection three, we will introduce the concept of fuzzy subgroup of a commutative group.

2.1. Fuzzy Subgroups of a Group $(\boldsymbol{R},.)$

We shall introduce the definition of fuzzy subgroup of a group and the definition of level subgroup of a group.

Definition 2.1.1: Let R be a non-empty set closed under the binary operation " $\cdot$ " ,(i.e., R be a groupoid), $A \in I^R, A \neq \emptyset, where\ \emptyset\ is\ the\ empty\ fuzzy\ set$ defined by $\emptyset(x) = 0, for\ all\ x \in R. Then, (A,\cdot)$ is called closed (i.e., A is a fuzzy subgroupoid) if and only if $A \cdot A \subseteq A$.

The following proposition gives an equivalent definition to fuzzy closed subset.

Proposition 2.1.2: Let $A \in I^R, A \neq \emptyset$. The following statements are equivalent:

$(i)\ A\ is\ a\ fuzzy\ subgroupoid\ (i.e., (A,\cdot)\ is\ closed)$.

$(ii)\ For\ any\ x_t, y_s \subseteq A. Then\ x_t \cdot y_s \subseteq A\ for\ all\ x, y \in R.$

$(iii)\ A(xy) \geq min\{A(x), A(y)\}, for\ all\ x, y \in R.$

Now, we are ready to define a fuzzy subgroup of a group.

Definition 2.1.3 : Let $(R,\cdot)$ be a group and $A \in I^R$ such that $A \neq \emptyset$. Then A is called a fuzzy subgroup of R if and only if for each $x, y \in R$,

1. $A(xy) \geq min\{A(x), A(y)\}$.
2. $A(x) = A(x^{-1})$.

If we suppose (e) be the identity element of R, then:

$A(e) = A(xx^{-1}) \geq min\{A(x), A(x^{-1})\} = A(x). Hence, A(e) \geq A(x)$ *for each* $x \in R. So\ A(x)$ *is the nearer value of* $A(x)$*to unity and* may be equal to unity.

Now, when $(R,\cdot)$ is a group, we define $A \cdot B$ as follows:

Definition 2.1.4: Let A,B be two fuzzy subsets of a group R. Then for each $x \in R$ we define:

$$A \cdot B(x) = sup\{min\{A(y), B(z)| \ x = yz \qquad y, z \in R\}\}$$

is a fuzzy subset of R. Also, let $\{A^i(x)|i \in \ \}$ be a collection of fuzzy subsets. Then, for each $x \in R$, we have:

$$A \cdot B(x) = sup\left\{inf\left\{A^i(x^i)|x = \prod_{i\in} x^i \ , \qquad x^i \in R\right\}\right\}$$

is a fuzzy subset of R.

In section one, we spoke about level sets or level subsets and mentioned that a level set lies between fuzzy set and ordinary set. The same meaning is carried on level subgroup.

Definition 2.1.5: Let R be a group and A be a fuzzy subgroup of R. Then the subgroup $A_t(where\ t \in [0,1]\ and\ A(e) \geq t)$ are called level subgroups of A.

All the properties of level subsets in the previous section are valid in the level subgroups. Furthermore, many properties valid in ordinary groups are also valid in level subgroups.

2.2.Fuzzy Normal Subgroup of a Group$(\boldsymbol{R},\cdot)$

We will recall the concept of fuzzy normal subgroup of a group.

Definition 2.2.1: Let A and B be any two fuzzy subgroups of $(R,\cdot)$ such that $B \subseteq A$ Then B is said to be fuzzy normal in A if and only if $x_t.B = B.x_t \; for\ each\ x_t \subseteq A$.

2.3.Fuzzy Subgroup on a Commutative Group$(\boldsymbol{R},+)$

We will give the concept of fuzzy subgroup on a commutative group

Definition 2.3.1: Let $(\mathrm{R},+)$ be a commutative group and $A \in I^R\ such\ that\ A \neq \emptyset$. Then A is called a fuzzy subgroup of R if and only if for each $x, y \in R$.

Now, the following remark give some conditions of a group which can translate into the collection I^R and to a fuzzy subgroup A such as associative, commutative, identity and inverse.

Remark 2.3.2: Let $(\mathrm{R},+)$ be a commutative group. Then the conditions of associative and commutative hold in $I^R, i.e., for\ each\ A, B, C \in I^R$,

(i) $A + (B + C) = (A + B) + C$.

(ii) $A + B = B + A$.

Also, the following conditions hold in a fuzzy subgroup A of R, for each $t \in [o, 1]$:

(i) $Associative: For\ x_t, y_t, z_t \subseteq A, we\ have\ x_t + (y_t + z_t) = (x_t + y_t) + z_{t.}$

(ii) $Commutative: For\ x_t, y_t \subseteq A, we\ have\ x_t + y_t = y_t + x_t$.

(iii) $Identity: Let\ (0)$ be $the\ identity\ of\ R. Then\ 0_t\ is\ the\ identity$

$of\ A\ and\ \ x_t + 0_t = (x + 0)_t = 0_t + x_t = x_t, for\ each\ x_t \subseteq A$.

(iv) $Inverse: Let\ (-x)\ be\ the\ inverse\ element\ of\ x\ in\ R. Then\ (-x)$

$$is\ inverse\ of\ af\ fuzzy\ singleton\ in\ A\ and\ x_t + (-x)_t = \big(x + (-x)\big)_t = 0_t$$
$$= (-x)_t + x_t.$$

Finally, we will give the concept of fuzzy subset-X a group R in the following proposition

Proposition2.3.3: Let X be a fuzzy subset of R and A be a fuzzy subgroup of R. Define the fuzzy subset $-X$ of R by $\forall\, z \in R, (-X)(z) = X(-z). If\ X \subseteq A, then -$ $\boldsymbol{X} \subseteq \boldsymbol{A}$.

Also, for any fuzzy singleton x_t of R, it follows easily that $-(x_t) = (-x)_t$.

CHAPTER 3

FUZZY SUBRINGS AND FUZZY IDEALS OF ARING $(\mathbf{R},+,\cdot)$

This section consists of the concepts of the fuzzy rings and fuzzy ideals, which were coined by Liu, who found many basic properties in ring theory, carried over an fuzzy ring. So in subsection one, we give the concepts of the fuzzy subring and in subsection two, we introduce the concepts related to fuzzy ideals.

Fuzzy Subrings of a ring $(\boldsymbol{R},+,\cdot)$

We will introduce the definitions about fuzzy subring of a ring and fuzzy subring of a fuzzy ring. Also, we give some properties about fuzzy subring.

From this definition, we can obtain the following proposition which appeared in many references like without proof

Proposition3.1.1: Let $(\mathrm{R},+,\cdot)$ be a ring and x_t, y_s be two fuzzy singletons, where $x, y \in R$ *and* $t, s \in [0,1]$. *Then* $x_t + y_s = (x+y)_r$ and $x_t \cdot y_s = (xy)_r$, $\mathrm{r} = \min\{t, s\}$.

proof

For each $\mathrm{z} \in \mathrm{R}$,

$$(x_t + y_s)(z) = sup\{min\{x_t(u), y_s(v) | u, v \in R, \quad z = u + v, \quad z \in R\}\}$$

$$= min\{t, s\} \quad \text{for } \mathrm{x} = \mathrm{u}, \mathrm{y} = \mathrm{v}\ , \mathrm{z} = \mathrm{x} + \mathrm{y}, \mathrm{z} \in \mathrm{R}$$

$$= r \quad for\ z = x + y \ldots\ldots\ldots\ldots..(1)$$

On the other hand,

$$(x_t + y_s)(z) = r \quad if\ z = x + y \ldots\ldots\ldots\ldots\ldots..(2)$$

From (1) and (2), we have $x_t + y_s = (x+y)_r$, *where* $\mathrm{r} = \min\{t, s\}$.

For each $z \in R$

$$(x_t \cdot y_s)(z) = \begin{cases} sup\{min\{x_t(u), y_s(v)\}\} & if\ u, v \in R,\ z = uv,\ z \in R \\ 0 & otherwise \end{cases}$$

$$= \begin{cases} min\{t,s\} & if\ x = u, y = v, \quad z = uv, \quad z \in R \\ 0 & otherwise \end{cases}$$

$$= \begin{cases} r & z = xy \\ 0 & otherwise \end{cases} \dots\dots\dots\dots\dots\dots\dots\dots. (1)$$

On the other hand,

$$(xy)_r(z) = \begin{cases} r & z = xy \\ 0 & otherwise \end{cases} \dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

From (1) and (2), we have $x_t \cdot y_s = (xy)_r, where$ r $= \min\{t, s\}$.

Definition3.1.2: Let R be a ring with respect to two binary operations "+","." and $A \in I^R, A \neq \emptyset. A$ will be called a fuzzy subring of R, if A is a fuzzy subgroup for the binary operation "+" in I^Rinduced by "+" in R, and A is a fuzzy subgroupoid for the binary operation "." in I^Rinduced by "+" in R.

If A is a fuzzy subring of R, then $A(0) \geq A(x)\ for\ all\ x \in R$.

The following proposition gives an equivalent definition to fuzzy subring of a ring.

Proposition3.1.3: Let R be a ring, $A \in I^R, A \neq \emptyset$. Then A is a fuzzy subring of R if and only if, for all $x, y \in R$,

(i) $A(x - y) \geq min\{A(x), A(y)\}$

(ii) $A(xy) \geq min\{A(x), A(y)\}$

Now, we are ready to define a fuzzy subring of a fuzzy ring A.

Definition3.1.4: A fuzzy subring of a fuzzy ring A is a fuzzy subring B of R satisfying:

B$(x) \leq A(x)\ for\ all\ x \in R$.

In section two, we spoke about level subgroups and mentioned that a level subgroup between fuzzy group and ordinary group. The same meaning is carried on level subring and give the concept about level subring in the following proposition:

Proposition3.1.5: Let A be a fuzzy subset of a ring R. Then the level subset $A_t\big(where\ 0 \le t \le A(0)\big)$ is a subring of R if and only if A is a fuzzy subring of R.

$proof$: $(\Longrightarrow)$

Let A_t be a subring of R, we must prove that A is a fuzzy subring of R.

Let A_t be a subring of R, $\forall t \in [0, A(0)]. Then\ 0 \in A_t. Hence, A(0) \ge t, \forall t \in [0, A(0)]$,

Let $x, y \in R. Let\ A(x) = t, A(y) = s, for\ some\ t, s \in [0, A(0)]$. Without loss of generality, we may assume that $s \ge t. Then\ A(y) = s \ge t, hence\ x, y \in A_t. Since\ A_t\ is\ a\ subring\ of\ R, then\ x - y \in A_t.\ \ So\ A(x - y) \ge t = min\{A(x), A(y)\}. Since\ x, y \in A_t\ and\ A_t\ is\ subring\ of\ R, then\ xy \in A_t. So\ A(xy) \ge t = min\{A(x), A(y)\}$. Therefore, A is a fuzzy subring of R.

$(\Longleftarrow)$

Let A be a subring of R, we must prove that A_t is a fuzzy subring of R.

Let

$0 \le t \le A(0). Then\ 0 \in A_t. Thus\ A_t \ne \emptyset\ (\emptyset\ is\ empty\ set).\ \ Let\ x, y \in A_t. Then\ A(x) \ge t, A(y) \ge t. Since\ A(x - y) \ge\ min\{A(x), A(y)\} \ge t, then\ x - y \in A_t. Let\ x, y \in A_t. Since\ \ A(xy) \ge min\{A(x), A(y)\} \ge t, then\ xy \in A_t$. Therefore, A_t is a fuzzy subring of R

Definition3.1.6: Let A be a fuzzy subring of a ring R. Then the subring where $t \in [0,1]$ are called level subrings of A.

All the properties of level subset in section one are valid in the level subrings. Furthermore, many properties valid in ordinary rings are also valid in level subrings.

Now, we give another subrings of R related to fuzzy subrings in the following definition:

Definition3.1.7: Let R be a ring with identity 1 and A be a fuzzy subring of R. Then we define the following:

(i) $supp(A) = \{x \in R | A(x) > 0\}$ $is\ caled\ the\ support\ of\ A$.

(ii) $A_* = \{x \in R | A(x) = 0\}$.

(iii) $A_\# = \{x \in R | A(x) > A(1)\}$.

It is easily show that $supp(A), A_* and\ A_\#$ are subrings of R.

The following remark give some conditions of a ring, which can translate into the collection I^R and to a fuzzy subring A such as associative, commutative, identity and inverse.

Remark 3.1.8: Let $(R, +, \cdot)$ be a commutative ring with identity 1. Then the conditions of associative and commutative hold in $I^R, i.e., for\ each\ A, B, C \in R$.

(i) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$.

(ii) $A \cdot B = B \cdot A$.

Also, the following conditions hold in a fuzzy subgroup A of R, for each $t \in [o, 1]$:

(i) $Associative: For\ x_t, y_t, z_t \subseteq A, we\ have\ x_t \cdot (y_t \cdot z_t) = (x_t \cdot y_t) \cdot z_{t.}$

(ii) $Commutative: For\ x_t, y_t \subseteq A, we\ have\ \ x_t \cdot y_t = y_t \cdot x_t$.

(iii) $Identity: Let$ (1) be $the\ identity\ of\ R. Then\ 1_t\ is\ the\ identity$

$of\ A\ and\ \ x_t \cdot 1_t = (x \cdot 1)_t = 1_t \cdot x_t = x_t, for\ each\ x_t \subseteq A$.

(iv) $Inverse: Let\ (x^{-1})\ be\ the\ inverse\ element\ of\ x\ in\ R. Then\ (x^{-1})$

$$is\ inverse\ of\ af\ fuzzy\ singleton\ in\ A\ and\ x_t \cdot (x^{-1})_t = (x \cdot x^{-1})_t = 1_t$$
$$= (x^{-1})_t \cdot x_t.$$

Now, we give the definition of a fuzzy subfield of a field.

Definition3.1.9: A fuzzy subset A of a field F is a fuzzy subfield of F if:

$(i)\ A(1) = 1$

$(ii)\ A(x - y) \geq min\{A(x), A(y)\} \quad for\ each\ x, y \in F$

$(iii)\ A(xy^{-1}) \geq min\{A(x), A(y)\} \quad for\ each\ x, y \in F, y \neq 0$

Let A be a fuzzy subfield of F. $If\ x \in F, x \neq 0. Then\ A(0) = A(1) \geq A(x) = A(-x) = A(x^{-1})$.

Finally, we give the concept of fuzzy subset X^{-1} of a field F in the following proposition.

Proposition3.1.10: Let X be a fuzzy subset of F, A be a fuzzy subfield of F and $X \subseteq A$. Define the fuzzy subset X^{-1} of F be for each $z \in F, (X^{-1})(z) = X(z^{-1}), then\ X^{-1} \subseteq A$ and for any singleton $x_t$of F, $(x_t)^{-1} = (x^{-1})_t$.

$proof$:

$$(X^{-1})(z) = X(z^{-1}) \leq A(z^{-1}) = A(z). Hence\ X^{-1} \subseteq A$$

$$(x_t)^{-1}: F \to I, for\ each\ z \in F\ and\ (x_t)^{-1}(z) = (x_t)(z^{-1}).$$

$$(x_t)^{-1}(z) = \begin{cases} t & if\ x = z^{-1} \\ 0 & if\ x \neq z^{-1} \end{cases}$$

$$= \begin{cases} t & if\ x^{-1} = z \\ 0 & if\ x^{-1} \neq z \end{cases}$$

$$= (x^{-1})_t(z)$$

$Hence\ \ (x_t)^{-1}(z) = (x_t)(z^{-1}) = (x^{-1})_t(z)$

$Therefore, (x_t)^{-1} = (x^{-1})_t$

3.2.Fuzzy Ideals of a Ring $(\boldsymbol{R}, +,\cdot)$

We shall give the concept related to fuzzy ideals of a ring R and fuzzy ideals of fuzzy ring and introduce the concepts to special types of fuzzy ideals such as prime, completely prime, radical, maximal and primary fuzzy ideal.

Now, we are ready to introduce the concept of fuzzy ideal.

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