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On g-closed set

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Abstract.

In this paper we will review the concept of g-closed set which was introduced for the

first time by Levine [4]. Some topological properties by using the g-closed set are

studied . So some example were discussed.

Introduction.

The study of open sets and some of their properties and the relationships between

them is one of the most important topics on which the science of topology.

In this paper we will offer other type of near open set, which is called g-open set.

Some of its properties are studied.

On g-closed set

1. On g-closed set

Definition 2.1 [1]. Asubset A of a topological space (X,τ) is called g-closed set if $cl(A)$ is a subset of U whenever A is a subset of U and U is open.

The complement of a g-closed set is called g-open set. The family of all g-closed sets in a space (X,τ) is denoted by $gC(X)$ and the family of all g-open sets in a space (X,τ) is denoted by $gO(X)$

Example2.2. consider the topological space (X,τ) where $X = \{1,2,3\}$

$$and\ \ \tau = \{X, \emptyset, \{1\}\}$$

$$Then\ \ gC(X) = \{X, \emptyset, \{2\}, \{3\}\{1,2\}\{1,3\}\{2,3\}\}.$$

$$and\ \ gO(X) = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}.$$

Remark 2.3.

(i) Every closed set in a topological space (X,τ) is g-closed

(ii) Every open set in a topological space (X,τ) is g-open

$proof:$

(i) $Let\ A\ be\ a\ closed\ set\ in\ a\ space\ (X,\tau), and\ \ A \subseteq U, U\ is\ open$

$set\ in\ X\ so\ A\ is\ closed, then\ cl(A) = A \subseteq U, So\ A\ is\ g-closed\ set.$

(ii) $Let\ A\ be\ an\ open\ set\ in\ \ (X,\tau), then\ (X - A)\ is\ closed\ set. So,$

$by(i)$

$, X - A \ is \ g - closed \ set. Implies, X - (X - A) = A \ is \ g - open \ set.$

In **Example 2.2.** we can see that the converse of the meaning inRemark2.3 need not by true in general

Proposition 2.4. The union of any family of g-closed sets is g-closed set.

$proof: Let \ \{A_\alpha : \alpha \in \Lambda\}$be a family of g-closed subsets of a space (X, τ) and let $\cup \{A_\alpha : \alpha \in \Lambda\} \subseteq U \ whenever \ U \ is \ open \ set \ in \ X.$

$Implice, A_\alpha \subseteq U \ for \ all \ \alpha \in \Lambda.$ Therefore,$\cup \{A_\alpha : \alpha \in \Lambda\} \subseteq U$.
$That \ is \ lead \ to, cl(\cup \{A_\alpha : \alpha \in \Lambda\}) \subseteq U.$

Hence, $\cup \{A_\alpha : \alpha \in \Lambda\}$ is g-closed set.

Corollary 2.5. The intersection of any family of g-open sets is g-open.

$prof: Let \ \{A_\alpha : \alpha \in \Lambda\}$ be a family of g-open sets, then $\{X - A_\alpha : \alpha \in \Lambda\}$ is a family of g-closed sets. So $\cup \{X - A_\alpha : \alpha \in \Lambda\}$ is g-closed set (by Proposition 2.4). But $\cup \{X - A_\alpha : \alpha \in \Lambda\} = X - \cap \{A_\alpha : \alpha \in \Lambda\}$. Implies to $\cap \{A_\alpha : \alpha \in \Lambda\}$ is g-open set.

Remark 2.6:

(i) If A and B are two g-open subsets of a topological space $(X, \tau), A \cup B$ need not be g-open set.

In Example 2.2. $If \ \ A = \{2\} \ \ , B = \{3\} \ are \ g - open \ but \ A \cup B \ not$

$g - open$

(ii) If A and B are two g-closed subsets of a topological space $(X,\tau), A\cap B$ need not be g-closed set..

In Example 2.2. $If\ \ A=\{1,2\}\,, B=\{1,3\}\ are\ g-closed\ but\ A\cap B$

$not\ g-open$

Definition2.7. Let (X,τ) be a topological space, A be a subset of X. Then:

The g-interior of A , simply g-int(A), is the union of all g-open sets contained in A.

The g-closure of A , simply g-cL(A), is the intersection of all g-closed sets containing A.

Remark 2.8.

1. g-int(A) need not to be g- open set.

2. g-cL(A) need not to be g-closed set.

See **Example 2.2.**

Proposition 2.9. Let A be a subset of a topological space (X,τ), then:

$(i)\ int(A)\subseteq g-int(A)$.

$(ii)\ g-cl(A)\subseteq cl(A)$.

$Prrof$: $(i)\ Let\ x\in int(A), imples\ x\in\cup\{u\in\tau : x\in u\subseteq A\}$. Since every open set is g-open, then $x\in\cup\{u\in\ gO(X) : x\in u\subseteq A\}. Hence\ x\in g-int(A)$.

$(ii)\ Let\ x\in g-cl(A), imples\ x\in\cap\{B\in gC(X) : x\in A\subseteq B\}$. Since every closed set is g-closed, then

$$x\in\cap\{B\in gC(X) : x\in B\wedge A\subseteq B\}$$

$$\subseteq \{F : F \text{ is closed} \land A \subseteq F\}$$

$Implies\ x \in cl(A).$

The inverse meaning in Proposition 2.9, may be untrue. For example:

See Example 2.2. Let $A = \{1,3\}. Then\ int(A) = \{1\}\ but\ g - int(A) = A.$

$$And\ cl(A) = X\ but\ \ g - cl(A) = A.$$

Proposition 2.10. Let (X, τ) be a topological space, A and B are two subsets of X. Then:

$(i)\ g - int(X) = X \quad and \quad g - cl(X) = X$

$(ii)\ g - int(\emptyset) = \emptyset \quad and \quad g - cl(\emptyset) = \emptyset$

$(iii)\ g - int(A) \subseteq A \subseteq\ g - cl(A)$

$(iv) If\ A \subseteq B, then\ g - int(A) \subseteq\ g - int(B)\ and$

$g - cl(A) \subseteq\ g - cl(B)$

$(v)\ g - int(A) \cup g - int(B) \subseteq g - int(A \cup B)$

$(vi)\ g - cl(A \cap B) \subseteq g - int(A) \cap g - int(B)$

2. On g-compact space.

In this section we will introduce new type of compactness by using the concept of g-open set which is called g-compactness. Some properties, remarks and examples will be given. Also, the relationships between this compactness and g-compactness space are discussed.

Definition3.1. Let (X, τ) be a topological space, $A \subseteq X$, a family W of subsets of X is said to be a g-open cover of A if and only if W covers A and W is a subfamily of $gO(X)$.

Definition 3.2. A topological space (X, τ) is said to be a "g-compact space" if and only if every g-open cover of X has a finite subcover.

Proposition3.3. Every g-compact space is compact.

$proof$: Follows from every open set is g-open.

Remark 3.4. The convers of proposition 3.3, may by untrue, For example The topological space (N, τ_I) is compact space which is not g-compact. Since $\{\{n\}: n \in N\}$ is a g-open cover of N which has no finite subcover

Proposition 3.5. A g-closed subset of a g-compact space is g-compact.

$proof$: Let (X, τ) be a g-compact space, and let A be a g-closed subset of X. Let $w = \{u_i: i \in \Lambda\} \cup \{X - A\}$ be a g-open cover of X. Since (X, τ) is a g-compact space, w has a finite subcover $\{u_1, u_2, \dots, u_n\} \cup \{X - A\}$ Cover X. As $Since\ X - A$ covers no part of A, a family numbers of

$U_1, U_2, \ldots, U_n$, have the property that $A \subseteq U_1 \cup U_2 \cup \ldots \cup U_n$. Hence, A is g-compact.

Corollary 3.6. A closed subset of a g-compact space is g-compact.

$proof$: Since every closed set is g-closed, then Proposition 3.5. is applicable.

Proposition 3.7. If A and B are g-compact subsets of a topological space (X, τ) , then $A \cup B$ is g-compact.

Remark 3.8. If A and B are g-compact subsets of a topological space (X, τ), then $A \cap B$ need not by g-compact. For example:

Let N be the set of all natural numbers, Let $X = N \cup \{0, -1\}$ *and Let*

$$\tau = P(N) \cup \{H \subseteq X| - 1{,}0 \in H \text{ and } X - H \text{ finite}\}.$$

Then both $A = N \cup \{0\}$ *and* $B = N \cup \{-1\}$ are g-compact subsets of X, but $A \cap B$ is not g-compact.

Definition 3.9. A function $f: (X, \tau) \to (Y, \delta)$ is called g-continuous if and only if the inverse image of any closed subset of Y is a g-closed subset of X.

Proposition 3.10. The g-continuous image of a g-compact space is compact.

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