جمهورية العراق

وزارة التعليم العالي والبحث العلمي

جامعة بغداد

كلية التربية للعلوم الصرفة/أبن الهيثم

TRI-TOPOLOGICAL SPACES

بحث مقدم الى قسم الرياضيات في كلية التربية أبن الهيثم للعلوم الصرفة كجزء

من متطلبات نيل درجة البكالوريوس تربية في الرياضيات

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الشكر والاهداء(Dedication and Thanks):

الشكر

في مثل هذه اللحظات يتوقف اليراع ليفكر قبل أن يخط الحروف ليجمعها في كلمات ... تتبعثر الأحرف وعبثاً أن يحاول تجميعها في سطور

سطوراً كثيرة تمر في الخيال ولا يبقى لنا في نهاية المطاف إلا قليلاً من الذكريات وصور تجمعنا برفاق كانوا إلى جانبنا............

فواجب علينا شكرهم ووداعهم ونحن نخطو خطوتنا الأولى في غمار الحياة

ونخص بالجزيل الشكر والعرفان إلى كل من أشعل شمعة في دروب عملنا و

وإلى من وقف على المنابر وأعطى من حصيلة فكره لينير دربنا

إلى الأساتذة الكرام في كلية التربية ونتوجه بالشكر الجزيل إلى

ا٠م٠د٠يوسف يعكوب يوسف

الذي تفضل بإشراف على هذا البحث فجزاه الله عنا كل خير فله منا كل التقدي والاحترام..

الإهداء

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

(قل إعملوا فسيرى الله عملكم ورسوله والمؤمنون)

صدق الله العظيم

إلهي لايطيب الليل إلا بشكرك ولايطيب النهار إلى بطاعتك .. ولاتطيب اللحظات إلا بذكرك .. ولا تطيب الآخرة إلا بعفوك .. ولا تطيب الجنة إلا برؤيتك

الله ﷻ

إلى من بلغ الرسالة وأدى الأمانة .. ونصح الأمة .. إلى نبي الرحمة ونور العالمين..

سيدنا محمد ﷺ

إلى من كلله الله بالهيبة والوقار .. إلى من علمني العطاء بدون انتظار .. إلى من أحمل أسمه بكل افتخار .. أرجو من الله أن يمد في عمرك لترى ثماراً قد حان قطافها بعد طول انتظار وستبقى كلماتك نجوم أهتدي بها اليوم وفي الغد وإلى الأبد..

أمي الحبيبة

إلى ملاكي في الحياة .. إلى معنى الحب وإلى معنى الحنان والتفاني .. إلى بسمة الحياة وسر الوجود

إلى من كان دعائها سر نجاحي وحنانها بلسم جراحي إلى أغلى الحبايب

ابي الحبيب

إلى من بها أكبر وعليه أعتمد .. إلى شمعة متقدة تنير ظلمة حياتي

إلى من بوجودها أكتسب قوة ومحبة لا حدود لها..

إلى من عرفت معها معنى الحياة

أخي.............

إلى من أرى التفاؤل بعينه .. والسعادة في ضحكته

إلى شعلة الذكاء والنور

إلى الوجه المفعم بالبراءة ولمحبتك لأزهرت أيامي وتفتحت براعم للغد

أختي.......

إلى الأخوات اللواتي لم تلدهن أمي .. إلى من تحلو بالإخاء وتميزوا بالوفاء والعطاء إلى ينابيع الصدق الصافي إلى من معهم سعدت ، وبرفقتهم في دروب الحياة الحلوة والحزينة سرت إلى من كانوا معي على طريق النجاح والخير

إلى من عرفت كيف أجدهم وعلموني أن لا أضيعهم

المحتويات (CONTENTS):

:(Abstract) الملخص

The topology is a word translated from the English word topology, and the word the topological is divided into two sections of the first section (topo), which belong to the Greek origin to (topos), which means "place" and the second section is (logy). which belong to the origin of the Greek (logos) , which means "study". If we connect the stake holders in the word. we find that the topology is the modern engineering in the study of all the compositions and components of different spaces…

(One of the most famous statements of humor in the topology is):

:*" A topology is a person who cannot tell a coffee cup from a dough nut "*

1.1 introduction:

Q / What is the topology ? 1.1.1:

The topology is a word translated from the English word topology, and the word the topological is divided into two sections of the first section (topo), which belong to the Greek origin to (topos), which means "place" and the second section is (logy). which belong to the origin of the Greek (logos) , which means "study". If we connect the stake holders in the word. we find that the topology is the modern engineering in the study of all the compositions and components of different spaces… (One of the most famous statements of humor in the topology is):

:" *A topology is a person who cannot tell a coffee cup from a dough nut* "

Definition 1.1.2: let X be a nonempty set and of X (i.e $\tau \subseteq IP(X)$) we say τ is a topology on X if satisfy the following condition:

(1) $X, \emptyset \in \tau$

(2) if $u,v \in \tau$ then $u \cap v \in \tau$

The finite intersection of elements from τ again an element of τ

(3) if $u_\alpha \in \tau$; $\alpha \in \Lambda$ then $\cup_{\alpha \in \Lambda} u_\alpha \in \tau$ $\forall \alpha \in \Lambda$

The arbitrary (finite or infinite) union of elements of τ is again an element of τ

we called a pair (X,τ) Topological space

Definition 1.1.3: open set

let (X,τ) be a topological space The subsets of X belonging to τ are called open sets in the space X

i.e , If $A \subseteq X \wedge A \in \tau \rightarrow$ A open set

Definition 1.1.4: closed set

let (X,τ) be a topological space The subsets of X is called closed set in the space X if it is complement $X \backslash A$ is open set we will denoted the family of closed sets by Ŧ

i.e , If $A \subseteq X \wedge A \in$ Ŧ$\rightarrow$ A closed set

Remark 1.1.5: the set in (X,τ) may be:

(1) open & not closed

(2) closed & not open

(3) clopen (closed & open)

(4) not open & not closed

Remarks 1.1.6:

(1) $\tau=\{X,\emptyset\}$ is a topology on X and it's the smallest topology that we can defined on any set X and called [Indescrete topology] And denoted by $(I=\{X,\emptyset\})$

(2) $\tau=IP(X)$ is a topology on X and it's the largest topology that we can defined on any set X and called [Discrete topology] And denoted by $(D=IP(X))$

(3) If τ any topology on X then $I \subseteq \tau \subseteq D$

(4) $\tau=D$ if and only if $\{x\} \in \tau \ \forall \ x \in X$

Theorem 1.1.7:

let (X,τ) be a topological space and Ŧ be a family of closed sets on X then:

(1) X ,Ø ∈ Ŧ

(2) if A,B∈Ŧ then AUB ∈Ŧ ∀A,B∈Ŧ

(3) if A_α ∈Ŧ; $\alpha\in\Lambda$ then $\cap_{\alpha\in\Lambda} A_\alpha$ ∈Ŧ ∀ A_α∈Ŧ

Proof:

(1) Ø∈τ→$Ø^c$∈Ŧ→X∈Ŧ

X∈τ → X^c∈Ŧ →Ø∈Ŧ

(2) let A,B ∈Ŧ → A^c,B^c ∈τ (def of closed)

→$A^c\cap B^c$∈τ (def of top.)

→$(AUB)^c$∈τ (Demorgan)

→AUB ∈Ŧ (def of closed)

A_α ∈Ŧ ∀α∈Λ (3) let

→ A_α^c∈τ ∀α∈Λ

→ $U_{\alpha\in\Lambda} A_\alpha^c$∈τ (Third condition of def of top.)

→ $(\cap_{\alpha\in\Lambda} A_\alpha)^c$∈τ (De morgan's laws)

→ $\cap_{\alpha\in\Lambda} A_\alpha$ ∈Ŧ (def of closed set)

Definition 1.1.8: (subspace topology)

let (X,τ) be a topological space and $w \subseteq k$. Then the topology τ_w is called the subspace or (induced) topology for w and the pair (w,τ_w) is called subspace of (X,τ)

Definition 1.1.9:(Interior points)

let (X,τ) be a topological space and $A\subseteq X$. A point $x\in A$ is called an interior point of A iff there exists an open set $U\cup\tau$ containing X such that $x\in U\subseteq A$. the set of all interior points of A is called the interior of A and is denoted by A^{o} or Int(A)

i.e. $x\in A^{0}\leftrightarrow\exists U\in\tau;\ x\in U\subset A$

or $A^{o} = \cup\{U\in\tau\ ;\ U\subseteq A\}$ this means A^{o} is the lage open set contain in A

Definition 1.1.10: (Derived set)

let (X,τ) be a topological space and $A \subseteq X$, A point $x\in X$ is called a (cluster point) or (limit point) or (a ccumulation point) of A iff every open set containing X contains at least one point of A different from X the set of all cluster point of A is called the derived set of A and is denoted by $A^{/}$

i.e. $x\in A \leftrightarrow \forall U\in\tau\ ;\ x\in U \wedge U\setminus\{x\}\cap A\neq\emptyset$

Definition 1.1.11:(closure of set)

let (X,τ) be a topological space and $A \subseteq X$, the closure of a set A is $A \cup A^{/}$ and is denoted by $\bar{A}$ or cl(A)

i.e. $\bar{A} = A\cup A^{/}$

* $\bar{A} = \cap\{F \subseteq X\ ;\ F^{C} \in \tau \wedge A \subseteq F\}$

$\bar{A}$ is smallest closed set contains A,[0].

2.1 TRI open And TRI closed Sets:

In 1965, Njastad[1],introduced a generalization of open set in topological space called αopen set. A subset A of a topological space is called αopen set if $A \subset$ int cl int A. In the definition of α open set, the same topology is used thrice. In this chapter we use three different topologies and extend this concept to a tritopological space.To denote a topology we use the symbol T for convenience.

Definition 2.1.1:

let X be a non empty set and T_1,T_2 and T_3 be three topologies on X. X together with three topologies is called a tri topological space. it is denoted by (X,T_1,T_2,T_3)

Example 2.1.2:

let $X=\{a,b,c,d\}$, $T_1=\{\emptyset,X\}$, $T_2=P(X)$, $T_3=\{\emptyset,X,\{a\}\}$

then (X,T_1,T_2,T_3) is a tri topological space

Note 2.1.3: Any topological space is a tri topological space . let (X,τ) be a topological space . then (X,T_1,T_2,T_3) is a tri topological space

Note 2.1.4: Any tri topological space is not a topological space . but any tri topological space induces a topological space in many ways . If we take the intersection of all topologies , then we get a topological space .

Example 2.1.5:

$X=\{a,b,c,d\}$, $T_1=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$, $T_2=\{\emptyset,X,\{a\},\{c\},\{a,c\}\}$,

$T_3= \{\emptyset,X,\{a\},\{d\},\{a,d\}\}$

let $T=T_1 \cap T_2 \cap T_3$

then $T=\{X,\emptyset,\{a\}\}$

(X,τ) is a topological space

Definition 2.1.6:

let (X,T_1,T_2,T_3) be a tri topological space . then (X,τ) where $T=T_1 \cap T_2 \cap T_3$ is called the induced topological space

Definition 2.1.7:

let (X,T_1,T_2,T_3) be a tri topological space . let $A \subset X$,A is called a tri open set in X if A is open in the induced topology

i.e. $A \in T_1 \cap T_2 \cap T_3$

Definition 2.1.8:

let (X,T_1,T_2,T_3) be a tri topological space . let $A \subset X$. A is called a tri closed set in X if A is closed in the induced topology

Example2.1.9:

in the example 2.1.5 {a} is tri open and {b,c} is tri closed

Result 2.1.10:

(1) A is tri open iff A is open with respect to each topology

(2) A is tri closed iff A is closed with respect to each topology

(3) A is tri closed iff A^c is tri open

(4) Ø is always tri open

(5) X is always tri open

Theorem 2.1.11:

let (X,T_1,T_2,T_3) be a tri topological space A is tri open iff $A \subset T_1$ int (T_2 int (T_3 int A))

proof:

if A is tri open , then A is open with respect to each topology

Hence $A=T_1$ in A for i=1,2,3

T_1int T_2 int T_3 int A= T_1 int T_2 int A= T_1 int A=A

Hence A $\subset T_1$ int T_2 int T_3 int A

conversely, suppose we have A $\subset$ T_1 int T_2 int T_3 int A

now T_1 int T_2 int T_3 int A $\subset$ T_1 int T_2 int A $\subset$ T_1 int A $\subset$ A

Hence we have $A=T_1$ int T_2 int T_3 int A and this implies $A=T_i$ int A for i=1,2,3 and hence A is tri open

Theorem 2.1.12:

let (X,T_1,T_2,T_3) be a tri topological space A is tri closed iff A $\supset$ T_1 cl (T_2 cl (T_3 cl A))

proof:

A is tri closed $\rightarrow$ A^C is tri open

$\rightarrow$ $A^c \subset T_1$ int T_2 int T_3 int A^c

$\rightarrow$ $A^c \subset$ T_1 int T_2 int(T_3 cl $A)^c$

$\rightarrow$ $A^c \subset$ T_1 int(T_2 cl T_3 cl $A)^c$

$\rightarrow A^c \subset (T_1 \text{ cl } T_2 \text{ cl } T_3 \text{ cl } A)^c$

$\rightarrow A \supset T_1 \text{ cl } T_2 \text{ cl } T_3 \text{ cl } A$

Retracing the above steps, we get the converse

2.2 TRI α OPEN AND TRI α CLOSED SETS:

Definition 2.2.1:

Let (X, T_1, T_2, T_3) be a tri topological space . A subset A of X is called tri αopen inX, if $A \subset T_1$ int T_2 cl T_3 int A. The complement of triα open set is called tri αclosed set

Example 2.2.2:

Let X={a,b,c}, T_1={Ø,X,{a}},T_2={Ø,X,{a},{a,b}},T_3={Ø,X,{a},{a,c}}

Let A={a};

T_1 int T_2 cl T_3 int {a}=T_1 int T_2 cl {a}

=T_1 int X

=X

$A \subset T_1$ int T_2 cl T_3 int A

A={a} is triαopen

Theorem 2.2.3:

A is tri open A is tri αopen

Proof:

A is tri open $A \subset T_1$ int T_2 int T_3 int A

$A \subset T_1$ int T_2 cl T_3int A

Hence A is tri αopen

Result: Converse is not true

Example 2.2.4:

Consider R with usual metric as T_1 and T_3

T_2=Indiscrete Topology

A=[a,b], A is triαopen

but A is not tri open

Theorem 2.2.5:

Arbitrary union of tri α open sets is tri αopen

Proof:

Let $\{A_\alpha / \alpha \in I\}$ be a family of tri αopen sets in X

For each $\alpha \in I$, $A_\alpha \subset T_1$ int T_2 cl T_3 int A_α

Hence $\cup A_\alpha \subset \cup$ (T_1 int T_2 cl T_3 int A_α)

$\subset T_1$ int ($\cup$ T_2 cl T_3 int A_α)

$\subset T_1$ int T_2 cl ($\cup$ T_3 int A_α)

$\subset T_1$ int T_2 cl T_3 int ($\cup A_\alpha$)

Therefore $\therefore \cup A_\alpha$ is tri αopen

Theorem 2.2.6:

Arbitrary intersection of tri αclosed sets is tri αclosed

Proof:

Let $\{B_\alpha / \alpha \in I\}$ be a family of tri αclosed sets in X

Let $A_\alpha = B_\alpha^c$

$\{A_\alpha / \alpha \in I\}$ is a family of tri αopen sets in X

Arbitrary union of tri α open sets is tri αopen. Hence $\cup A_\alpha$ is tri αopen Hence$(\cup A_\alpha)^c$ is tri αclosed

$\cap A_\alpha^c$ is tri αclosed (i.e) $\cap B_\alpha$ is tri αclosed. Hence

arbitrary intersection of tri αclosed sets is tri αclosed

Result2.2.7: Intersection of tri α open sets need not be tri αopen

Example 2.2.8:

X=R , T_1=P(R) , T_2={Ø,R}, T_3= Usual topology in R

A=[a,b] , T_1 int T_2 cl T_3 int [a,b] = T_1 int T_2cl (a,b)

= T_1 int R

$[a,b] \subset T_1$ int T_2 cl T_3 int $[a,b]=R$

Hence A is tri α open. Hence any closed interval is tri α open

Take B=[b,c], B is also tri α open

$A \cap B = [a,b] \cap [b,c] = \{b\}$

T_1 int T_2 cl T_3 int $\{b\}$ = T_1 int T_2 cl $\emptyset$ = T_1 int $\emptyset$ = $\emptyset$

Therefore $A \cap B = \{b\} \not\subset T_1$ int T_2 cl T_3 int $\{b\} = \emptyset$

$A \cap B$ is not tri α open

Theorem: 2.2.9:

In a tri topological space (X,T_1,T_2,T_3) the set of all tri α open sets form a generalized topology.

Proof:-

Proof follows from Result 2.1.10 , Theorem 2.2.3, Theorem 2.2.5 and Result 2.2.7.

Definition 2.2.10:

Let $(X, T_1, T_2, T3)$ be a tri topological space. Let $A \subset X$. An element $x \in A$ is called tri αinterior point of A, if $\exists$ a tri αopen set V such that $x \in V \subset A$.

Example 2.2.11:

Let X = R , T_1 = R with usual Topology, $T_2 = \{\emptyset, R\}$ and $T_3 = P(R)$

Let A = [-1,1] Then 0 is a tri αinterior point of A.

Definition 2.2.12:

The set of all tri αinterior points of A is called the tri αinterior of A and is denoted astri αint A.

Theorem 2.2.13:

Let $A \subset X$ be a tri topological space. Tri αint A is equal to the union of all tri αopensets contained in A.

Proof: $A \subset X$. we want to prove

Tri αint A = $\cup\{ B / B \subset A$, B is tri αopen$\}$

$x \in$ tri αint A $\exists$ a tri αopen set B such that $x \in B \subset A$.

Hence $x \in \cup$ { B / B$\subset$ A, B is tri αopen}

Therefore tri αint A$\subset \cup${B / B$\subset$ A, B is tri αopen}

Suppose $x \in \cup$ {B / B$\subset$ A, B is tri αopen}

$x \in B_0$, B_0 is triαopen and $B_0 \subset A$. Hence $\exists$ a tri αopen set B_0 such that $x \in B_0 \subset A$

Therefore $x \in$ tri αint A. Hence $\cup${ B / B $\subset$ A, B is tri αopen} $\subset$ tri αint A

Hence tri αint A = $\cup$ {B / B $\subset$ A, B is tri αopen}

Note 2.2.14:

1.Tri α int A $\subset$ A.

2. Tri αint A is tri α open.

Theorem 2.2.15:

Tri αint A is the largest tri αopen set contained in A.

Proof: Follows from the Theorem 2.2.13.

Theorem 2.2.16:A is tri αopen iff A = tri αint A.

Proof:

A is tri αopen and A $\subset$ A. Therefore A $\in$ {B / B $\subset$ A, B is tri αopen}

A is in the collection and every other member in the collection is a subset of A and hence the union of this collection is A. Hence $\cup${B / B $\subset$ A, B is tri αopen} = A and hence tri αint A = A.

Conversly since tri αint A is tri αopen,

A = tri αint A implies that A is tri αopen.

Theorem 2.2.17: Tri αint (A$\cup$B) $\supset$ tri αint A $\cup$ tri αint B

Proof:

Tri αint A $\subset$ A and tri αint A is tri αopen.

Tri αint B $\subset$ B and tri αint B is tri αopen.

Union of two tri αopen sets is tri αopen and hence tri αint A ∪ tri αint B is a tri αopen set.Also tri αint A ∪ tri αint B ⊂ AUB.Tri αint A ∪ tri αint B is one tri αopen subset of AUB and tri αint (AUB) is the largest tri αopen subset of AUB.

Hence tri αint A ∪ tri αint B ⊂ tri αint (AUB)

Definition 2.2.18:

Let (X, T_1, T_2, T_3) be a tri topological space and let A⊂X. The intersection of all tri αclosed sets containing A is called the tri αclosure of A and is denoted as tri αcl ATri αcl A = ∩ { B / B⊃A , B is tri αclosed}

Note 2.2.19:

Since intersection of tri αclosed sets is tri αclosed, tri αcl A is a tri αclosed set

Note 2.2.20:

Tri αcl A is the smallest tri αclosed set containing A

Theorem 2.2.21:

A is tri αclosed iff A = tri αcl A

Proof:

Tri αcl A = ∩ { B / B ⊃ A, B is tri αclosed}

If A is a tri αclosed then A is a member of the above collection and each member contains A. Hence their intersection is A. Hence tri αcl A = A Conversely if A = tri αcl A, then A is tri αclosed because tri αcl A is a tri αclosed set

Definition 2.2.22:

Let A ⊂ X, a tri topological space. x ∈ X is called a tri αlimit point of A, if every tri αopen set U containing x, intersects A – {x}.(i.e.) every tri αopen set containing x, contains a point of A other than x

Example 2.2.23:

Let X={a,b,c}, T_1={Ø,{a},{a,b},X}

T_2={Ø,{a},X}, T_3={Ø,{a},{a,c},X}

Tri αopen sets are Ø,X,{a},{a,b},{a,c}

Consider A={a,c}. Then b is a tri αlimit point of A.

Definition 2.2.24:

Let $A \subset X$. The set of all tri αlimit points of A is called the tri αderived set of A and is denoted as tri αD(A).

Theorem 2.2.25:

Tri αcl A = A ∪ tri αD(A)

Proof:

Let $x \in$ tri αcl A. If $x \in A$ then $x \in A \cup$ tri αD(A). If $x \notin A$, then we claim that x isa tri αlimit point of A. Let U be a tri αopen set containing x. Suppose $U \cap A = \emptyset$

Then $A \subset UC$ and UC is tri αclosed and hence tri αcl $A \subset UC$. This implies x ε UC⇐ Hence $U \cap A \neq \emptyset$. Therefore every tri αopen set U containing x intersectsA−{x}.

Hence $x \in$ tri αD(A) and $x \in A \cup$ tri αD(A)

tri αcl $A \subset A \cup$ tri αD(A) Conversely it is clear that $A \subset$tri αcl A.Therefore It is enough to prove tri αD(A) ⊂ tri αcl A.

Let $x \in$ tri αD(A).If $x \in A$ then it is true. So let us take $x \notin A$. Now we have to provethat $x \in$ every tri αclosed set containing A. Suppose not, $x \notin B$ where B is a tri αclosed set containing A. $B \supset A$ Now $x \in BC$, BC is tri αopen and $BC \cap A = \emptyset$

Contradiction to the fact that x is a tri αlimit point of A. Hence $x \in$ every tri αclosedset containing A. Therefore $x \in$ tri αcl A. Hence A ∪ tri αD(A)⊂tri αcl A

Hence tri αcl A = A ∪ tri αD(A).

2.3. TRI α CONVERGENCE OF NETS.

The concept of nets in topological spaces can be extended to tri topological spaces.

Let D be a directed set and $f : D \to X$ be a map. Then f is called a net in X. Also wehave studied the convergence of nets in usual topology. Now we extend the conceptof net convergence to tri topological space.

Definition 2.3.1:

Let X be a tri topological space and let (x_λ) be a net in X. (x_λ) is said to tri αconvergeto a point $x \in X$, if for every tri αopen set U containing x, $\exists\ \lambda_0 \in D$ such that$\lambda \geq \lambda_0 \to x_\lambda \in U$. When (x_λ) tri αconverges to x, we write $(x_\lambda) \to x$.

Theorem 2.3.2:

If (x_λ) is a net in A, $(x_\lambda) \to x$ and $x \notin A$, then x is a tri αlimit point of A.

Proof:

$(x_\lambda)_\lambda \in D$ is a net in A. $(x_\lambda) \to x$ and $x \notin A$. Let U be a tri αopen set containing x. As$(x_\lambda) \to x$, $\exists\ \lambda_0 \in D$ such that $\lambda \geq \lambda_0 \to x_\lambda \in U$In particular $x_{\lambda 0} \in U$. As $x \notin A$, $x_{\lambda 0} \neq x$. Now $x_{\lambda 0} \in U$ and $x_{\lambda 0} \in A$.Therefore $U \cap A \backslash \{x\} \neq \emptyset$. This is true for every tri αopen set containing x. Hence x is a tri αlimit point of A. Hence $x \in$ tri $\alpha D(A)$.

Theorem 2.3.3:

If (x_λ) is a net in A and $(x_\lambda) \to x$, then $x \in$ tri αcl A.

Proof:

(x_λ) is a net in A, $(x_\lambda) \to x$ .For each $\lambda \in D$, $x_\lambda \in A$.

Case 1:

Let $x \in A$

$x \in A$ $x \in$ tri αcl A.

Case 2:

Let $x \notin A$

Now (x_λ) is a net in A, $(x_\lambda) \to x$, $x \notin A$

Hence by Theorem2.3.2 $x \in$ tri $\alpha D(A)$

Hence $x \in$ tri αcl A.

In both cases $x \in$ tri αcl A.

Theorem 2.3.4:

If $x \in$ tri αcl A then $\exists$ a net (x_λ) in A such that $(x_\lambda) \to x$.

Proof:

$x \in$ tri αcl A

Case 1:

Let $x \in A$

Then for any directed set D, consider $f : D \to A$ defined as $f(\lambda) = x$. Then the net $(x_\lambda)$where $x_\lambda = x$ is the constant net converging to x.

Hence $\exists$ a net (x_λ) in A such that $(x_\lambda) \to x$.

Case 2:

Let $x \notin A$ then $x \in$ tri αD(A)

Let U = set of all tri αopen sets containing x,Define an order in U as $B_1 > B_2$ if $B_1 \subset B_2$. With respect to this order U becomes an ordered set. Now we define a map f : $U \to A$as follows. Take any $B \,\varepsilon\, U$ then B is a tri αopen set containing x, sincex $\in$ tri αD(A), every tri αopen set containing x intersects $A - \{x\}$.Hence $B \cap A - \{x\} \neq \emptyset$. Take an element $x_B \in B \cap A - \{x\}$. Now we define

$f(B) = x_B$. Now f is a net whose elements are in A. Therefore f is a net in A.

Claim :$f \to x$

Let B be a tri αopen set containing x.

Consider $B \in U$Now $B_1 > B \to B_1 \subset B \to x_{B1} \in B_1 \subset B \to x_{B1} \in B$. Hence $B_1 > B \to x_{B1} \in B$For every tri αopen set B containing x, $\exists$ Bi such that $x_{Bi} \in B\ \forall\ Bi > B$Hence net $f = (x_B)$ converges to x.Hence $\exists$ a net (x_λ) in A such that $(x_\lambda) \to x$.

Theorem 2.3.5:

$x \in$ tri αcl A iff $\exists$ a net (x_λ) in A such that $(x_\lambda) \to x$.

Proof: Follows from the previous theorems.

2.4. CHARACTERISATIONS.

We get some characterisations when T_2 is discrete topology.

Theorem 2.4.1: Let (X,T_1,T_2,T_3) be a tri topological space where T_2 is discrete topology. Let $A \subset X$.A is tri αopen $\to$ A is T_3 open.

Proof: If A is not T_3 open. Then T_3 int A is a proper subset of A.

Hence T_2 cl T_3 int A is a proper subset of T_2 cl A.Since T_2 is discrete topology, T_2 cl A = A.Hence T_2 cl T_3 int A is a proper subset of A.Therefore T_1 int T_2 cl T_3 int A is a proper subset of A.

Therefore A is not a subset of T_1 int T_2 cl T_3 int A.Hence A is not tri α open. A is not T_3 open $\rightarrow$A is not tri αopen.Hence A is tri αopen $\rightarrow$ A is T_3 open, if T_2 is discrete topology.

Result 2.4.2: Converse is not true.

Example 2.4.3:

X={a,b,c}T_1={Ø,{a},X},T_2=P(X),T_3={Ø,{b},X} , A={b} is T_3 open.

T_1 int T_2 cl T_3 int A = T_1 int T_2 cl A=T_1 int A = Ø.T_1 int T_2 cl T_3 int A= Ø.

Hence A is not a subset of T_1 int T_2 cl T_3 int AA is not tri α open.

A is T_3 open but A is not tri αopen.

Theorem 2.4.4:

Let (X,T_1,T_2,T_3) be a tri topological space where T2 is discrete topology on X.Let A $\subset$ X. Then A is tri αopen $\rightarrow$ A is T_1 open.

Proof:T_3 int A $\subset$ A. T_2 cl T_3 int A $\subset$ T_2 cl A. Since T_2 cl A=A, T_2cl T_3 intA $\subset$ A.T_1 int T_2 cl T_3 int A $\subset$ T_1 int A $\subset$ A.

A is tri αopen. $\rightarrow$ A $\subset$ T_1 int T_2 cl T_3 int A.

T_1 int T_2 cl T_3 int A $\subset$ A $\subset$ T_1 int T_2 cl T_3 int A.

T_1 int T_2 cl T_3 int A = A. T_1 int T_3 int A = A.

This is possible only when T_1 int A = A.

Hence A is T_1 open.

A is tri αopen$\rightarrow$ A is T_1 open.

Result 2.4.5: Converse is not true.

Example 2.4.6:

X={a,b,c}. T_1={Ø,{a},X}, T_2=P(X), T_3={Ø,{b},X} , A={a} is T_1 open.

T_1 int T_2 cl T_3 int A = T_1 int T_2 cl $\emptyset$=T_1 int $\emptyset$= $\emptyset$.

T_1 int T_2 cl T_3 int A = $\emptyset$.

Hence A is not a subset of T_1 int T_2 cl T_3 int A

A is not tri αopen.

A is T_1 open but A is not tri αopen.

Theorem 2.4.7:

When T_2 = P(X), A is tri αopen iff A is T_1 open and T_3 open.

Proof:

Follows from the previous theorems, we have A is tri αopen implies A is T_1 openand T_3open

conversely,If A is T_1 open and T_3 open.

T_1 int T_2 cl T_3 int A = T_1 int T_2 cl A

= T_1 int A

= A

Hence A $\subset$ T_1 int T_2 cl T_3 int A

Hence A is tri αopen.

Theorem 2.4.8:

When T_2 = P(X), A is tri αopen iff A is tri open

Proof: Follows from the previous theorem.

Theorem 2.4.9:

When T_2 = P(X), A is tri αclosed iff A is T_1 closed and T_3 closed

Proof: Follows from the previous theorem.

Theorem 2.4.10: When T_2 = P(X), A is tri αclosed iff A is tri closed

Proof: Follows from the previous theorem.

3.1: Tri α Continuous Functions:

Definition 3.1.1:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces. A function $f: X \to Y$ is called a tri αcontinuous function if $f^{-1}(V)$ is tri αopen in X, for every tri αopen set V in Y.

Example 3.1.2:

Let $X = \{1, 2, 3\}$, $T_1 = \{\phi, \{1\}, X\}$,

$T_2 = \{\phi, \{1\}, \{1,3\}, X\}$, $T_3 = \{\phi, \{1\}, \{1,2\}, X\}$

Let $Y = \{a, b, c\}$, $T_1' = \{\phi, \{a\}, Y\}$

$T_2' = \{\phi, \{a\}, \{a, b\},Y\}$, $T_3' = \{\phi, \{a\}, \{b\},Y\}$

Let $f : X \to Y$ be a function defined as

$f(1) = a$; $f(2) = b$; $f(3) = c$.

Tri αopen sets in (X,T_1,T_2,T_3) are $\phi,\{1\},\{1,2\},\{1,3\},X$.

Tri αopen sets in (Y,T_1',T_2',T_3') are $\phi,\{a\}, \{a,b\},\{a,c\},Y$.

Since $f^{-1}(V)$ is tri αopen in X for every tri αopen set V in Y,

f is tri αcontinuous.

Definition 3.1.3:

Let X and Y be two tri topological spaces. A function $f : X \to Y$ is said to be tri α continuous at a point $a \in X$ if for every tri α open set V containing f(a), $\exists$ a tri α open set U containing a , such that $f(U) \subset V$.

Theorem 3.1.4:

$f: X \to Y$ is tri α continuous iff f is tri α continuous at each point of X.

Proof: Let $f: X \to Y$ be tri α continuous.

Take any $a \in X$. Let V be a tri α open set containing f(a).

$f: X \to Y$ is tri α continuous,

Since $f^{-1}(V)$ is tri α open set containing a. Let $U = f^{-1}(V)$.

Then $f(U) \subset V \rightarrow \exists$ a tri α open set U containing a and $f(U) \subset V$.

Hence f is tri α continuous at a.

Conversely,Suppose f is tri α continuous at each point of X.Let V be a tri α open set of Y. If $f^{-1}(V) = \phi$ then it is tri α open.Take any $a \in f^{-1}(V)$ f is tri α continuous at a.

Hence $\exists$ Ua, tri α open set containing a and $f(Ua) \subset V$.

Let $U = \cup \{U_a / a \in f^{-1}(V)\}$.

Claim: $U = f^{-1}(V)$.

$a \in f^{-1}(V) \rightarrow U_a \subset U \rightarrow a \in U$.

$x \in U \rightarrow x \in U_a$ for some $a \rightarrow f(x) \in V \rightarrow x \in f^{-1}(V)$. Hence $U = f^{-1}(V)$.

Each U_a is tri α open. Hence U is tri α open. $\rightarrow f^{-1}(V)$ is tri α open in X.

Hence f is tri α continuous.

Result 3.1.5:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3')be two tri topological spaces.

Any function $f : X \rightarrow Y$ is tri αcontinuous function if T_3**'** is indiscrete topology.

We know that if T_3**'**is indiscrete Topology $\{\phi, Y\}$ then the only tri αopen sets in Y are ϕ and Y.

$f_{-1}(\phi) = \phi$. $f_{-1}(Y) = X$. ϕ & X are tri αopen in X. Hence $f : X \rightarrow Y$ is tri α continuous function .In this case any function defined from X to Y is tri αcontinuous function.

Theorem 3.1.6:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces.

Then $f: X \rightarrow Y$ is tri αcontinuous function iff $f^{-1}(V)$ is tri αclosed in X whenever V is tri αclosed in Y.

Proof:

Let $f : X \rightarrow Y$ be tri αcontinuous function.

Let V be any tri αclosed in Y.

$\rightarrow V^C$ is tri αopen in Y $\rightarrow f^{-1}(V^c)$ is tri α open in X.

$\rightarrow [f^{-1}(V)]^c$ is tri α open in X.

$\rightarrow f^{-1}(V)$ is tri α closed in X.

Hence $f^{-1}(V)$ is tri αclosed in X whenever V is tri αclosed in Y.

Conversely, suppose $f^{-1}(V)$ is tri αclosed in X whenever V is tri αclosed in Y.V is a tri αopen set in Y.

$\rightarrow V^c$ is tri αclosed in Y.

$\rightarrow f^{-1}(V^c)$ is tri αclosed in X.

$\rightarrow [f^{-1}(V)]c$ is tri α closed in X.

$\rightarrow f^{-1}(V)$ is tri αopen in X.

Hence f is tri αcontinuous.

Theorem 3.1.7:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces. Then,

$f : X \rightarrow Y$ is tri αcontinuous iff f [tri αcl A] $\subseteq$ tri αcl [f(A)] $\forall A \subseteq X$.

Proof: Suppose $f: X \rightarrow Y$ is tri αcontinuous.

Let $A \subseteq X \rightarrow f(A) \subset Y \rightarrow$ tri αcl [f(A)] is tri αclosed set in Y.

$\rightarrow f^{-1}$ [tri αcl {f(A)}] is tri αclosed in X ---- 1

[since f is tri αcontinuous]

tri αcl [f(A)] $\supseteq$ f(A)

$\rightarrow f^{-1}$ [tri αcl {f(A)}] $\supseteq f^{-1}[f(A)] \supseteq$ A----- 2

1 & 2 $\rightarrow f^{-1}$ [tri αcl {f(A)}] is tri αclosed set containing A.

But tri αcl A is the smallest tri αclosed set containing A.

Hence tri αcl A $\subseteq f^{-1}$ [tri αcl {f(A)}]$\rightarrow$ f [tri αcl A] $\subseteq$ tri αcl [f(A)].

Converse: Suppose f [tri αcl A] $\subseteq$ tri αcl [f(A)] $\forall A \subseteq X$

Claim: $f: X \rightarrow Y$ is tri αcontinuous.

It is enough to provef^{-1} (F) is tri αclosed in X for every tri αclosed set F in Y

Let $A = f^{-1}(F) \rightarrow f(A) \subseteq F$

$f[\text{tri } \alpha cl\ A] \subseteq \text{tri } \alpha cl\ [f(A)]$ [Hypothesis]

$\rightarrow f\ [\text{tri } \alpha cl\ f^{-1}(F)] \subseteq \text{tri } \alpha cl\ (F) = F$

$\rightarrow f[\text{tri } \alpha cl\ f^{-1}(F)] \subseteq F$

$\rightarrow \text{tri } \alpha cl\ f^{-1}(F) \subseteq f^{-1}(F)$ --- 1

$f^{-1}(F) \subseteq \text{tri } \alpha cl\ [f^{-1}(F)]$ --- 2

From 1 & 2 $f^{-1}(F) = \text{tri } \alpha cl\ [f^{-1}(F)]$

Hence $f^{-1}(F)$ is tri αclosed in X for every tri αclosed set F in Y.

Result 3.1.8:

Under a tri αcontinuous function the image of a tri αopen set need not be tri αopen.

Example: 3.1.9

Let $X = \{1,2\}$, $T_1 = \{\phi,\{1\},X\}$

$T_2 = \{\phi,\{2\},X\}$, $T_3 = \{\phi,\{1\},X\}$

Let $Y = \{a,b\}$, $T_1' = \{\phi, \{a\},Y\}$

$T_2' = P(Y)$, $T_3' = \{\phi,Y\}$

Define $f(1) = a$ and $f(2) = b$

f is tri αcontinuous. $\{1\}$ is tri αopen in X.

But $f(\{1\}) = \{a\}$ is not tri αopen in Y.

Hence under a tri αcontinuous function image of a tri α open set need not be tri αopen.

Result 3.1.10:

f is tri α continuous but f^{-1} need not be tri α continuous.

Let $g = f^{-1} : Y \rightarrow X$

$g(a) = 1$, $g(b) = 2$

$\{1\}$ is tri α open in X. But $g^{-1}\{1\} = \{a\}$ is not tri α open in Y.

Hence f^{-1} need not be tri α continuous.

Theorem 3.1.11:

X and Y are tri topological spaces. $x_0 \in X$.

$f : X \to Y$ is tri αcontinuous at x_0 iff$(x\lambda) \to x_0 \to (\ f(x\lambda)\ ) \to f(x_0)$ for every net $(x\lambda)$ in X.

Proof:

Let f: $X \to Y$ be tri αcontinuous.

f is tri α continuous at a.

Let $(x_\lambda) \to a$.

Claim: $f(x_\lambda) \to f(a)$.

Let V be a tri α open set containing f(a). Since f is tri α continuous at a,

$\exists$ tri α open set U containing a and $f(U) \subset V$.

Now $(x_\lambda) \to a$.

Hence $\exists\ \lambda_0$ such that $x_\lambda \in U$ for all $\lambda \geq \lambda_0$.

$\to f(x_\lambda) \in V$ for all $\lambda \geq \lambda_0$.

Hence $(f(x_\lambda)) \to f(a)$.

Conversely, suppose $(x_\lambda) \to a \to f(x_\lambda) \to f(a)$.

Let V be a tri α open set containing f(a).

Let A = { U/ U is a tri α open set containing a} order A by set inclusion.

$U_1 \leq U_2$ if $U_2 \subset U_1$.

Now A is a partially ordered set.

Suppose for some U, $f(U) \subset V$ then f is tri α continuous at a.

If not, for every $U \in A$, $f(U) \not\subset V$.

For each $U \in A$, choose $x_U \in U \ni f(x_U) \notin V$.

Now the net $(x_U) \to a$.

$f(x_U) \notin V$ for each U.

Hence the net $(f(x_U))$ does not converge to f(a).

We get contradiction.

Hence $\exists\ U \in A$ such that $f(U) \subset V$.

Hence f is tri α continuous at a.

3.2.Tri α homeomorphisms:

Definition 3.2.1:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces.

A function f: X $\rightarrow$Y is called tri αopen map if f (V) is tri αopen in Y for every tri αopen set V in X.

Example 3.2.2:

In example 3.1.2f is tri αopen map also.

Definition 3.2.3:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces.

Let f: X $\rightarrow$Y be a mapping. f is called tri αclosed map if f(F) is tri α closed in Y for every tri αclosed set F in X.

Example 3.2.4:

The function f defined in the example 3.1.2is a tri αclosed map.

Result 3.2.5:

Let X & Y be two tri topological spaces. Let f: X $\rightarrow$Y be a mapping.

f is tri αcontinuous iff f^{-1} : Y $\rightarrow$X is tri αopen map.

Definition 3.2.6:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces.

Let f: X $\rightarrow$Y be a mapping. f is called a tri αhomeomorphism.

If (i) f is a bijection.

(ii) f is tri αcontinuous.

(iii) f^{-1} is tri αcontinuous.

Example 3.2.7:

The function f defined in the example 3.1.2is

(i) a bijection. (ii) f is tri αcontinuous. (iii) f^{-1} is tri αcontinuous.

Therefore f is a tri αhomeomorphism.

Theorem 3.2.8:

Let (X,T_1,T_2,T_3)be a tri topological space where T_2 is indiscrete topology.

A non empty subset A of X is tri α open iff A has non empty T_3 interior.

Proof:

Let $A \neq \phi$ and T_3int $A \neq \phi$

T_2 cl T_3 int A = X.

T_1 int T_2 cl T_3 int A = X.

$A \subset T_1$ int T_2 cl T_3 int A. Hence A is tri α open.

Conversely,A non empty set A is tri α open.

Suppose T_3 int A = ϕ, then

T_1 int T_2 cl T_3 int A = T_1int T_2 cl ϕ

= T_1 int ϕ = ϕ

A is not a subset of T_1 int T_2 cl T_3 int A.

A is not tri α open.

Hence A has non empty T_3 interior.

3.3 Tri β open and tri β closed sets:

Definition 3.3.1:

Let (X,T_1,T_2,T_3) be a tri topological space. Let $A \subset X$. A is called a tri β open set if $A \subset T_1clT_2intT_3clA$.

Definition 3.3.2:

Let (X,T_1,T_2,T_3) be a tri topological space. Let $A \subset X$. A is called a tri β closed set if AC is tri β open.

Example 3.3.3:

Let X = {a,b,c}, T1={ϕ, {a},X}, T2={ϕ,{b},X}, T3=P(X)

Tri β open sets are ϕ, {b}, {b,c}, X.

Example 3.3.4:

Let X = {a,b,c}, T_1={ϕ,{a},X}, T_2={ϕ,{b},X}, T_3=P(X)

Tri β closed sets are ϕ, {a}, {a,c}, X.

Note 3.3.5:

Always ϕ and X are tri β open sets.

Theorem 3.3.6:

In a tri topological space (X,T_1,T_2,T_3), $A\subset X$.

A is tri open $\rightarrow$ A is tri β open.

Proof:

A is tri open $\rightarrow A \subset T_1intT_2intT_3intA$

$\rightarrow A \subset T_1clT_2intT_3intA$

$\rightarrow A \subset T_1clT_2intT_3clA$

$\rightarrow$ A is tri β open.

Result 3.3.7:

Converse is not true.

Example 3.3.8:

Let $X = \{a,b,c\}$, $T_1 =\{\phi,\{a\},X\}$, $T_2=T_3=P(X)$.

Tri open sets are ϕ, $\{a\}$, X.

Tri β open sets are ϕ, $\{b\}$, $\{b,c\}$, X.

Theorem 3.3.9:

Arbitrary union of tri β open sets is tri β open.

Proof: Let $\{A_\alpha/\alpha\in I\}$ be a family of tri β open sets in X.

For each $\alpha\in I$, $A_\alpha\subset \cup\, T_1clT_2intT_3clA_\alpha$

Now $\cup_{A\alpha}\subset \cup T_1clT_2intT_3clA_\alpha$

$\subset T_1cl\, [\cup\, T_2intT_3clA_\alpha]$

$\subset T_1clT_2int\, [\cup T_3clA_\alpha]$

$\subset T_1clT_2intT_3cl\, [\cup_{A\alpha}]$

Hence $\cup_{A\alpha}$ is tri β open.

Theorem 3.3.10: Arbitrary intersection of tri β closed sets is tri β closed.

Proof: Let $\{B_\alpha/\alpha\in I\}$ be a family of tri β closed sets in X.

Let $A_\alpha = B_\alpha^C$ for each $\alpha\in I$.

$\{A_\alpha/\alpha\in I\}$ is a family of tri β open sets.

Hence $\cup_{A\alpha}$ is tri β open .

Therefore $(\cup_{A\alpha})^C$

is tri β closed.

Hence $\cap A_\alpha^C$ is tri β closed and therefore $\cap B_\alpha$ is tri β closed.

Result 3.3.11:

Intersection of tri β open sets need not be tri β open.

Example 3.3.12:

X=R, $T_1 = T_2 = T_3$ = Usual Topology in R.

A = [a,b] , = $T_1 cl T_2 int$ [a,b]

= T_1 cl (a,b) = [a,b].

A = [a,b] = $T_1 cl T_2 int\ T_3 cl$ [a,b]

[a,b] is tri β open.

Similarly B = [b,c] is tri β open.

$A\cap B = [a,b] \cap [b,c] = \{b\}$

T_1 cl T_2 int T_3 cl {b} = T_1 cl T_2 int {b}

= T_1 cl $\phi = \phi$

Hence {b} is not tri β open.

Hence $A\cap B$ is not tri β open.

Theorem 3.3.13:

Let (X,T_1,T_2,T_3) be a tri topological space. Let $A\subset X$. Then

A is tri open $\rightarrow$ A is tri β open.

Proof:

A is tri open $\rightarrow A \subset T_1 int T_2 int T_3 int A$

$\rightarrow A \subset T_1 cl T_2 int T_3 cl A$

$\rightarrow$ A is tri β open.

Result 3.3.14:

Converse is not true.

Example 3.3.15:

X= {a,b,c}

$T_1 = \{\phi,X\}$, $T_2 = P(X)$, $T_3 = P(X)$

Take A = {a}

$T_1clT_2intT_3clA = T_1clT_2intA = T_1clA = X$

$A \subset T_1clT_2intT_3clA$

Hence A is not tri β open.

A = {a} is not open in T_1

A is not tri open.

Hence A is not tri β open does not imply A is not tri open.

Definition 3.3.16:

Let (X,T_1,T_2,T_3) be a tri topological space. Let $A\subset X$. An element $x\in A$ is called tri β

interior point of A if $\exists$ a tri β open set U such that $x\in U\subset A$.

Example 3.3.17:

X= {a,b,c}, $T_1 = \{\phi,\{b,c\},X\}$, $T_2 = \{\phi,\{a\},X\}$, $T_3 = P(X)$

A = {a,b}, Consider $a\in A$.

Take U = {a}, U is tri β open.

$a\in U\subset A$. Hence a is tri β interior point of A.

Consider $b\in A$. Take U = {b}

$T_1clT_2intT_3cl\, U = T_1clT_2int\{b\}$

$= T_1cl\phi$

$= \phi$

Hence U is not tri β open.

Take U = {a,b}

$T_1clT_2intT_3cl$ U = T_1clT_2int {a,b}

= T_1cl{a}

= {a}

U is not tri β open.

∃ no U with b∈U⊂ A

b is not a tri β interior point of A

Hence tri β interior point of A is a.

Definition 3.3.17:

Let (X,T_1,T_2,T_3) be a tri topological space. Let A⊂ X. The set of all tri β interior point of A is called the tri β interior of A and it is denoted by tri β int A.

Example 3.3.18:

X= {a,b,c}

T_1 = {ϕ,{b,c},X}, T_2 = {ϕ,{a},X}, T_3 = P(X)

A = {a,b}, tri β int A = {a}

Theorem 3.3.19:

Let (X,T_i) be a tri topological space. Let A⊂X. The tri β int A is equal to the union of all tri β open sets contained in A.

Proof:

Let A⊂X.

Let S = union of all tri β open sets contained in A

Claim: tri β int A = S

x∈tri β int A → x∈U⊂ A where U is tri β open → x∈S.

Hence tri β int A ⊂ S.

Now x∈S → x∈U⊂ A for some U, tri β open ® x∈ tri β int A

Hence S ⊂ tri β int A.

Hence tri β int A = S.

Theorem 3.3.20:

Let $A \subset X$. Then tri β int A is the largest tri β open set contained in A.

Proof:

Follows from above theorem.

Definition 3.3.21:

Let X be a tri topological space. Let $A \subset X$. The intersection of all tri β closed sets containing A is called the tri β closure of A and is denoted by tri β clA.

Theorem 3.3.22:

Let X be a tri topological space. $A \subset X$. The tri β clA is the smallest tri β closed set containing A.

Proof: Follows from above definition.

Theorem 3.3.23:

A is tri β closed iff tri β clA = A.

Definition 3.3.24:

Let $A \subset X$. $x \in X$ is called a tri β limit point of A if every tri β open set U containing xintersects $A - \{x\}$.

Definition 3.3.25:

Let $A \subset X$. The set of all tri β limit points of A is called tri β derived set of A and is denoted by tri β D(A).

Theorem 3.3.26:

Tri β clA = $A \cup$ tri β D(A)

Definition 3.3.27:

Let X be a tri topological space. Let (x_λ) be a net in X. (x_λ) is said to tri β converge to a point $x \in X$, if for every tri β open net U containing x, $\exists\ \lambda_0 \in D$ such that $\lambda > \lambda_0 \rightarrow x_\lambda \in U$. We write $(x_\lambda) \rightarrow x$. x is called the tri β limit of the net (x_λ).

Example 3.3.28:

Let X= {a,b,c}

$T_1 = \{\phi, X\}$, $T_2 = \{\phi,\{a\},X\}$, $T_3 = P(X)$

$A_1 = \{a\}$, $T_1clT_2intT_3clA_1 = T_1clT_2int\{a\} = T_1cl\{a\} = X$

A_1 is tri β open.

$A_2 = \{b\}$, $T_1clT_2intT_3clA_2 = T_1clT_2int\ \{b\} = T_1cl\{\phi\} = X$

A_2 is not tri β open.

$A_3 = \{c\}$, $T_1clT_2intT_3clA_3 = T_1clT_2int\ \{c\} = T_1cl\{\phi\} = X$

A_3 is not tri β open.

$A_4 = \{a,b\}$, $T_1clT_2intT_3clA_4 = T_1clT_2int\ \{a,b\} = T_1cl\ \{a\} = X$

A_4 is tri β open.

$A_5 = \{a,c\}$, $T_1clT_2intT_3clA_5 = T_1clT_2int\ \{a,c\} = T_1cl\ \{a\} = X$

A_5 is tri β open.

$A_6 = \{b,c\}$, $T_1clT_2intT_3clA_6 = T_1clT_2int\ \{b,c\} = T_1cl\ \phi = \phi$

A_6 is not tri β open.

$A_7 = \phi$ and A_7 is tri β open.

$A_8 = X$ and A_8 is tri β open.

tri β open sets are ϕ, {a},{a,b},{a,c},X.

$D = P(X) - \{\phi\}$. D is a directed set with reverse inclusion $A \leq B$ if $B \subset A$

Define $f : D \rightarrow X$

f{a} = a ; f{a,b} = a; f{c} = a; f{a,c} = c

f{b} = a ; f{b,c} = b; f{a,b,c} =b

Claim: the net $f \rightarrow a$.

Take {a}, tri β open set containing a. $\lambda_0 = \{a,b\}$

Take {a,b}, $\lambda_0 = \{a,b\}$

Take {a,c}, $\lambda_0 = \{c\}$

This net f tri β converges to a.

Result 3.3.29:

Tri β limit of a net need not be unique.

Example 3.3.30:

Let X= {a,b,c}

T1 = {ϕ,X}, T2 = {ϕ,{a},X}, T3 = P(X)

tri β open sets are ϕ, {a},{a,b},{a,c},X.

D = P(X) - {ϕ}.

Define f : D $\rightarrow$x as

{a} $\rightarrow$a {a,b} $\rightarrow$a {a,b,c} $\rightarrow$a

{b} $\rightarrow$b {b,c} $\rightarrow$b

{c} $\rightarrow$c {a,c} $\rightarrow$c

f $\rightarrow$a ,f $\rightarrow$b and f $\rightarrow$c

This net f tri β converges to every point of X.

Theorem 3.3.31:

If (x_λ) be a net in A. $(x_\lambda) \rightarrow x$ then $x \in$tri β clA

Proof:

(x_λ) is a net in A.

$(x_\lambda) \in A$ for every $\lambda \in D$.

Hence $(x_\lambda) \rightarrow x$

Case 1:

Let $x \in A$

Then $x \in$tri β cl A

Case 2:

Let $x \notin A$

Let U be any tri β open set containing x.

Since $(x_\lambda) \rightarrow x$, $\exists\ \lambda_0 \in$ D such that $\lambda \geq \lambda_0 \rightarrow x_\lambda \in U$.

In particular $x_{\lambda 0} \in U$. Also $x_{\lambda 0} \neq x$.

Therefore $U \cap A - \{x\} \neq \phi$. This is true for each tri β open set containing x.

Hence $x \in$ tri β D(A).

Hence $x \in$ tri β clA.

3.4. Tri β continuous functions:

Similar to tri α functions we define tri β functions also.

Definition 3.4.1:

Let (X,T_1,T_2,T_3) and (Y,T_1',T_2',T_3') be two tri topological spaces. Let $f : X \rightarrow Y$ is called tri β continuous if $f^{-1}(V)$ is tri β open set in X for every tri β open set V in Y.

Definition 3.4.2:

Let X and Y be two tri topological spaces. A function f from X to Y is said to be tri β continuous at a point a ε X if for every tri β open set V containing f(a), ∃ a tri β open set U containing a such that $f(U) \subset V$. We can easily prove the following theorems.

i) f is tri β continuous iff f is tri β continuous at each point $x \in X$.

ii) f is tri β continuous iff $f^{-1}(V)$ is tri β closed set in X for every tri β closed set V in Y

iii) Any function defined from X to Y is tri β continuous if T_2' is indiscretetopology.Similarly we can define tri β open map, tri β closed map and tri β homeomorphismalso.

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