جامعة بغداد

كلية التربية للعلوم الصرفة / ابن الهيثم

التفاضل والتكامل

قسم الرياضيات

المرحلة الاولى

الفصل السابع

اساتذة المادة

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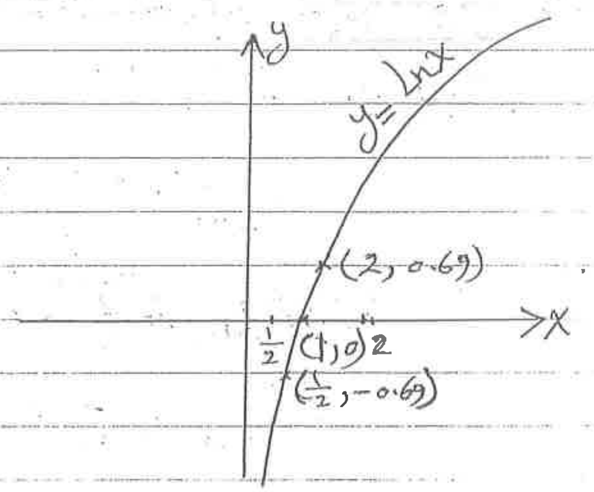
(Natural Logarithm):

$y = \ln x \quad ; \quad x > 0$

$\ln : (0, \infty) \longrightarrow \mathbb{R}$



$\ln e = 1$
$\ln 1 = 0$
$\ln 2 = 0.69$
$\ln 10 = 2.3$

$y = \ln(u(x))$ تعميم

حيث $u(x)$ دالة موجبة لـ x

مثال:

$y = \ln(2x)$
$y = \ln(\cos x)$
$y = \ln\left(\frac{x}{e}\right)$

Properties of Natural Logarithm

1- $\ln(x \cdot y) = \ln x + \ln y \quad ; \quad x > 0, \; y > 0$
2- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
3- $\ln\left(\frac{1}{x}\right) = -\ln x$
4- $\ln x^a = a \ln x$

Examples:- find the value of : ((if $\ln 2 = 0.69$))

$\ln 16 = \ln 2^4 = 4 \ln 2 = 4(0.69) = 2.76$
$\ln \sqrt{2} = \ln 2^{\frac{1}{2}} = \frac{1}{2} \ln 2 = (0.5)(0.69) = 0.345$
$\ln 8 = \ln 2^3 = 3 \ln 2 = 3(0.69) = 2.07$
$\ln \frac{1}{2} = -\ln 2 = -0.69$

Examples :- Prove that :-

1- $2 \ln\left(\cos\frac{\theta}{2}\right) = \ln \frac{1+\cos\theta}{2}$

الطرف الأيمن $= \ln \frac{1+\cos\theta}{2}$

$\boxed{\cos(2\theta) = 2\cos^2\theta - 1}$

$\cos(\theta) = 2\cos^2\left(\frac{\theta}{2}\right) - 1$

$= \ln \frac{\cancel{1} + 2\cos^2\left(\frac{\theta}{2}\right) \cancel{-1}}{2}$

$= \ln \frac{\cancel{2}\cos^2\left(\frac{\theta}{2}\right)}{\cancel{2}}$

$= \ln \cos^2\left(\frac{\theta}{2}\right)$

$= 2 \ln \cos\left(\frac{\theta}{2}\right)$

= الطرف الأيسر

2- $\ln\left(x+\sqrt{x^2-1}\right) = -\ln\left(x-\sqrt{x^2-1}\right)$

الطرف الأيسر $= \ln\left(x+\sqrt{x^2-1}\right)$

$= \ln \frac{\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)}{\left(x-\sqrt{x^2-1}\right)}$

$= \ln \frac{x^2-(x^2-1)}{x-\sqrt{x^2-1}}$

$= \ln \frac{\cancel{x^2}-\cancel{x^2}+1}{x-\sqrt{x^2-1}}$

$= \ln \frac{1}{x-\sqrt{x^2-1}}$

$= -\ln\left(x-\sqrt{x^2-1}\right) =$ الطرف الأيمن

3- $2 \ln \sin\theta = \ln(1-\cos\theta) + \ln(1+\cos\theta)$

الطرف الأيمن $= \ln(1-\cos\theta) + \ln(1+\cos\theta)$

$= \ln\left((1-\cos\theta)(1+\cos\theta)\right)$

$= \ln(1-\cos^2\theta)$

$= \ln \sin^2\theta$

$= 2 \ln \sin\theta =$ الطرف الأيسر

The derivative of Natural logarithm

$$\frac{d \ln u}{dx} = \frac{d \ln u}{du} \cdot \frac{du}{dx}$$ حسب قانون السلسلة

$$= \frac{1}{u} \cdot \frac{du}{dx}$$

$$\boxed{\frac{d \ln u}{dx} = \frac{\frac{du}{dx}}{u}}$$

The Limit of Natural Logarithm

$$\lim_{x \to \infty} \ln x = +\infty$$

$$\lim_{x \to 0^+} \ln x = -\infty$$

Ex :- Find the limits if exists :-

1- $\lim_{x \to 1} (x - \ln x) = 1 - \ln 1 = 1 - 0 = 1$

2- $\lim_{x \to 1} \cos(\ln x) = \cos(\ln 1) = \cos(0) = 1$

3- $\lim_{x \to 1} \ln x^{(x+1)} = \lim_{x \to 1} \{(x+1) \cdot \ln x\}$

$= \lim_{x \to 1} (x+1) \cdot \lim_{x \to 1} \ln x$

$= (1+1) \cdot \ln 1 = 2(0) = 0$

4- $\lim_{x \to 0} \frac{\ln(x+1)}{x}$ $\qquad \frac{\ln(1)}{0} = \frac{0}{0}$

نستخدم قاعدة لوبيتال

$= \lim_{x \to 0} \frac{\frac{1}{x+1}}{1} = \lim_{x \to 0} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1$

Ex: find y' :-

1- $y = \ln(x^2 + 2x)$

$y' = \frac{2x+2}{x^2+2x}$

2- $y = \ln(\tan x + \sec x)$

$y' = \frac{\sec^2 x + \sec x \cdot \tan x}{\tan x + \sec x} = \frac{\sec x\,\cancel{(\sec x + \tan x)}}{\cancel{(\tan x + \sec x)}}$

$y' = \sec x$

3- $y = (\ln x)^3$

$y' = 3(\ln x)^2 \cdot \frac{1}{x}$

4- $y = \ln(\cos x)$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

5- $y = \ln(\ln x)$

$$y' = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x} \cdot \frac{1}{\ln x} = \frac{1}{x \cdot \ln x}$$

6- $y = \ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1} x}{x}$

$$y' = \frac{1}{x} - \frac{1}{\cancel{2}} \frac{\cancel{2}x}{1+x^2} - \frac{x \cdot \frac{1}{1+x^2} - \tan^{-1}x \cdot 1}{x^2}$$

$$= \frac{1}{x} - \frac{x}{1+x^2} - \frac{\frac{x}{1+x^2} - \tan^{-1}x}{x^2}$$

7- $y = x\left[\sin(\ln x) + \cos(\ln x)\right]$

$$y' = x\left[\cos(\ln x) \cdot \frac{1}{x} - \sin(\ln x) \cdot \frac{1}{x}\right] + \left[\sin(\ln x) + \cos(\ln x)\right] \cdot 1$$

$$= \cos(\ln x) - \cancel{\sin(\ln x)} + \cancel{\sin(\ln x)} + \cos(\ln x)$$

$$= 2\cos(\ln x)$$

8- $y = x \cdot \sec^{-1} x - \ln\left(x + \sqrt{x^2-1}\right)$; $x > 1$

$$y' = x \cdot \frac{1}{|x|\sqrt{x^2-1}} + \sec^{-1}x \cdot 1 - \frac{1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2-1}}$$

$$= \frac{\cancel{x}}{\cancel{x}\sqrt{x^2-1}} + \sec^{-1}x + \frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}$$

$$= \frac{1}{\sqrt{x^2-1}} + \sec^{-1}x + \frac{\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}$$

$$= \frac{1}{\sqrt{x^2-1}} + \sec^{-1}x + \frac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}} \cdot \frac{1}{\cancel{x+\sqrt{x^2-1}}}$$

$$= \frac{1}{\sqrt{x^2-1}} + \sec^{-1}x + \frac{1}{\sqrt{x^2-1}}$$

$$= \frac{2}{\sqrt{x^2-1}} + \sec^{-1}x$$

9- $y = x\ln(a^2+x^2) - 2x + 2a\tan^{-1}\frac{x}{a}$

Constant = a

$$y' = x\cdot\frac{2x}{a^2+x^2} + \ln(a^2+x^2)\cdot 1 - 2 + 2\cancel{a}\cdot\frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{\cancel{a}}$$

$$= \frac{2x^2}{a^2+x^2} + \ln(a^2+x^2) - 2 + \frac{2}{\frac{a^2+x^2}{a^2}}$$

$$= \frac{2x^2}{a^2+x^2} + \ln(a^2+x^2) - 2 + \frac{2a^2}{a^2+x^2}$$

$$= \frac{2x^2+2a^2}{a^2+x^2} + \ln(a^2+x^2) - 2$$

$$= \frac{2\cancel{(x^2+a^2)}}{\cancel{(a^2+x^2)}} + \ln(a^2+x^2) - 2 = \cancel{2} + \ln(a^2+x^2) - \cancel{2}$$

$$= \ln(a^2+x^2)$$

Exercises :- Find y' :- (H.W.)

1- $y = \ln\left(x\sqrt{x^2+1}\right)$

2- $y = \ln\left(3x\sqrt{x+2}\right)$

3- $y = x\ln x - x$

4- $y = x^3 \ln(2x)$

5- $y = \frac{1}{2}\ln\frac{1+x}{1-x}$

6- $y = \frac{1}{3}\ln\frac{x^3}{1+x^3}$

7- $y = \ln\frac{x}{2+3x}$

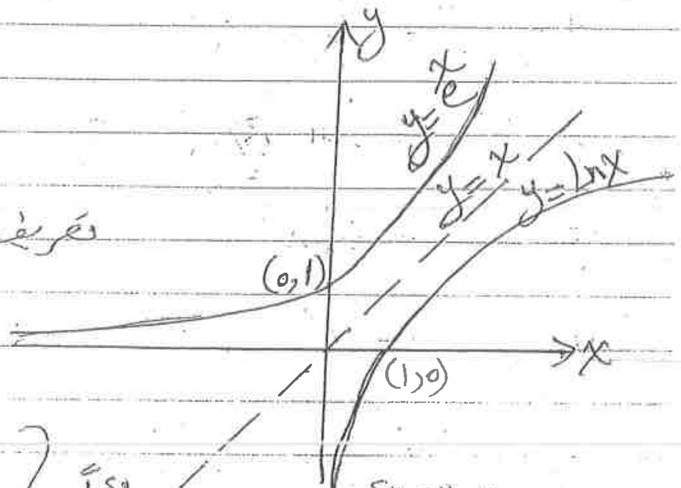
8- $y = \ln(x^2+4) - x\tan^{-1}\frac{x}{2}$

9- $y = x(\ln x)^3$

(The exponential function)

$y = e^{x}$

$e^{x} : \mathbb{R} \longrightarrow (0, \infty)$



$\boxed{e^{x} = Ln^{-1}(x)}$ تعريف:

$e = 2.718$

$$\left.\begin{aligned} e^{0} &= (2.7)^{0} = 1 > 0 \\ e^{2} &= (2.7)^{2} = 7.29 > 0 \\ e^{-1} &= (2.7)^{-1} = \frac{1}{2.7} > 0 \end{aligned}\right\}$$

الدالة الأسية موجبة دائماً ولا تساوي صفر

$e^{x} = exp(x) = EXP(x)$ رمزها:

$y = e^{u(x)}$ تعميم:

$y = e^{\sin x}$ أمثلة:

$y = e^{\tan^{-1} x}$

$y = e^{\sqrt{x}}$

$y = e^{3x}$

$y = e^{x^2}$

$y = \frac{1}{e^{x}}$

Relation between exponential fun. and Natural Logarithm fun.

$$\boxed{y = e^{x} \Longleftrightarrow x = \ln y}$$

Properties of exponential function.

1- $e^{0} = 1$

2- $e^{x_1} \cdot e^{x_2} = e^{x_1 + x_2}$

3- $\dfrac{e^{x_1}}{e^{x_2}} = e^{x_1 - x_2}$

4- $(e^{x})^{r} = e^{rx} \quad \forall r \in \mathbb{R}$

5- $e^{-x} = \dfrac{1}{e^{x}}$

6- $e^{\ln x} = x = \ln e^{x}$

Ex: find the value of the following :-

1- $\ln(e^{-x^2}) = -x^2$

2- $\ln(e^{\frac{1}{x}}) = \dfrac{1}{x}$

3- $e^{\ln \frac{1}{x}} = \dfrac{1}{x}$

4- $e^{2\ln x} = e^{\ln x^2} = x^2$

5- $\exp(\ln x - 2\ln y)$

$= \exp(\ln x - \ln y^2) = \exp\left(\ln \dfrac{x}{y^2}\right) = \dfrac{x}{y^2}$

6- $e^{x + \ln x} = e^{x} \cdot e^{\ln x}$

$= e^{x} \cdot x$

$= x e^{x}$

The Limit of exponential function :-

$$\lim_{x \to +\infty} e^{x} = +\infty$$

$$\lim_{x \to -\infty} e^{x} = 0$$

The derivative of exponential function :-

$y = e^{u}$ حيث $u=f(x)$ دالة قابلة للاشتقاق

$$\boxed{\frac{d e^{u}}{dx} = e^{u} \cdot \frac{du}{dx}}$$

$e = 2.718$

Ex : Find the derivative of :-

1- $y = e^{\tan^{-1} x}$

$$y' = e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

2- $y = \ln \frac{e^{x}}{1+e^{x}}$

$$y' = \frac{1}{\frac{e^{x}}{1+e^{x}}} \cdot \frac{(1+e^{x}) \cdot e^{x} \cdot 1 - e^{x} \cdot e^{x} \cdot 1}{(1+e^{x})^{2}}$$

$$= \frac{1}{e^{x}} \cdot \frac{e^{x} + \cancel{e^{2x}} - \cancel{e^{2x}}}{(1+e^{x})}$$

$$= \frac{1}{\cancel{e^{x}}} \cdot \frac{\cancel{e^{x}}}{(1+e^{x})}$$

$$= \frac{1}{(1+e^{x})}$$

3- $y=\frac{1}{2}\left(e^{x}-e^{-x}\right)$

$y'=\frac{1}{2}\left(e^{x}\cdot 1-e^{-x}\cdot -1\right)$

$=\frac{1}{2}\left(e^{x}+e^{-x}\right)$

4- $y=e^{(x^2+ax+b)}$; a, b Constants

$y'=e^{(x^2+ax+b)}\cdot(2x+a)$

5- $y=e^{3}$

$y'=0$

6- $y=e^{x^3}$

$y'=e^{x^3}\cdot 3x^2$

7- $y=\sec^{-1}\left(e^{2x}\right)$

$$y'=\frac{1}{\left|e^{2x}\right|\sqrt{\left(e^{2x}\right)^2-1}}\cdot e^{2x}\cdot 2$$

$$=\frac{2\cancel{e^{2x}}}{\cancel{e^{2x}}\sqrt{e^{4x}-1}}$$

$$=\frac{2}{\sqrt{e^{4x}-1}}$$

16- $\tan y = e^{x} + \ln x$:

$$\sec^2 y \cdot y' = e^{x} \cdot 1 + \frac{1}{x}$$

$$y' = \frac{e^{x} + \frac{1}{x}}{\sec^2 y}$$

Exercises :- Find y' :- (H.W.)

1- $y = x^2 \cdot e^{x}$

2- $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

3- $y = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

4- $y = e^{\sin\sqrt{x}}$

5- $y = (1+2x) \cdot e^{-2x}$

6- $y = (9x^2 - 6x + 2)\, e^{3x}$

7- $y = \frac{ax-1}{a^2} \cdot e^{ax}$ $a =$ constant

8- $y = e^{-x^2}$

9- $y = x^2 \cdot e^{-x^2}$

10- $y = e^{x} \cdot \ln x$

11- $y = e^{\frac{1}{x}}$

12- $y = e^{\sin 2x}$

13- $y = e^{(\sqrt{x} + \tan x)}$

14- $y = e^{(\ln x)^2}$

15- $y = \ln e^{x}$

16- $y = e^{x}\sqrt{x^2+5}$

17- $y = \tan^{-1}(e^{x})$

18- $y = e^{\tan^{-1}(\cos 3x)}$

19- $y = e^{(x - e^{x})}$

20- $y = \ln(e^{2x} + e^{x} - 1)$

21- $y = \sin e^{x+\frac{1}{x}}$

General logarithm:-

$$\log_a x = \frac{\ln x}{\ln a} \; ; \; x>0,\; a>0,\; a\neq 1$$

$$\boxed{\log_a u = \frac{\ln u}{\ln a} \quad u>0,\; a>0,\; a\neq 1}$$

تعميم ،

Properties:

1- $\log_a (x \cdot y) = \log_a x + \log_a y$

2- $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3- $\log_a x^y = y \cdot \log_a x$

4- $\log_a a = 1 \longleftrightarrow \ln e = 1$

5- $\log_a 1 = 0 \longleftrightarrow \ln 1 = 0$

Examples:-

$$\log_2 16 = \frac{\ln 16}{\ln 2} = \frac{\ln 2^4}{\ln 2} = \frac{4 \cancel{\ln 2}}{\cancel{\ln 2}} = 4$$

$$\log_{\frac{1}{7}} 49 = \frac{\ln 49}{\ln \frac{1}{7}} = \frac{\ln 7^2}{-\ln 7} = \frac{2 \cancel{\ln 7}}{-\cancel{\ln 7}} = -2$$

$$\log_{10} 10 = 1 :$$

$$\log_{10} 100 = \log_{10} 10^2 = 2\underbrace{\log_{10} 10}_{=1} = 2(1) = 2$$

$$\log_{10} \frac{1}{1000} = \log_{10} 10^{-3} = -3\underbrace{\log_{10} 10}_{=1} = -3(1) = -3$$

The derivative of general logarithm function :-

$$\log_a u = \frac{\ln u}{\ln a}$$ حسب التعريف

$$\frac{d}{dx}\log_a u = \frac{d}{dx}\left(\frac{\ln u}{\ln a}\right)$$

$$= \frac{1}{\ln a} \cdot \frac{d}{dx}\ln u$$

$$= \frac{1}{\ln a} \cdot \frac{\frac{du}{dx}}{u}$$

$$\boxed{\frac{d \log_a u}{dx} = \frac{du/dx}{u \cdot \ln a} \qquad u>0,\ a>0,\ a\neq 1}$$

Ex :- find y' ?

1- $y = \log_2 (x^2+3x)$

$$y' = \frac{2x+3}{(x^2+3x)\cdot \ln 2}$$

2- $y = \log_7(\tan x + \sin x)$

$$y' = \frac{\sec^2 x + \cos x}{(\tan x + \sin x)\cdot \ln 7}$$

3- $y = \ln x \cdot \log_{10} x$

$$y' = \ln x \cdot \frac{1}{x \cdot \ln 10} + \log_{10} x \cdot \frac{1}{x}$$

4- $y = \log_a \sin^{-1} x + \frac{x}{e^x}$ $\quad a = \text{Constant}$

$$y' = \frac{\frac{1}{\sqrt{1-x^2}}}{\sin^{-1}x \cdot \ln a} + \frac{e^x \cdot 1 - x \cdot e^x \cdot 1}{e^{2x}}$$

واجب ، جد y' :

1- $y = \log_4 \sin x$

2- $y = \log_a(e^x + \sin x)$

The General Exponential function :-

$$x = e^{\ln x}$$

$$a = e^{\ln a}$$

$$a^u = \left(e^{\ln a}\right)^u$$

$$\boxed{a^u = e^{u \ln a}}$$

حيث u عدد حقيقي

$-\infty < u < \infty$

والأساس $a > 0$.

Properties of general exponential function :-

1. $a^1 = a \quad (a>0)$:-
2. $a^0 = 1$
3. $a^u \cdot a^v = a^{u+v}$
4. $\left(a^{m/n}\right)^n = a^m$
5. $(a \cdot b)^u = a^u \cdot b^u \quad$ (حيث $a>0 , b>0$)

The derivative of general exponential function :-

$a^u = e^{u\ln a}$ حسب التعريف

$$\frac{d\,a^u}{dx} = \frac{d}{dx}\left(e^{u\ln a}\right)$$

$$\frac{d\,a^u}{dx} = e^{u\ln a} \cdot \ln a \cdot \frac{du}{dx}$$

$$\boxed{\frac{d\,a^u}{dx} = a^u \cdot \ln a \cdot \frac{du}{dx}}$$

Ex : find y' :-

1. $y = 2^{(x^2+\sec x)}$

$$y' = 2^{(x^2+\sec x)} \cdot \ln 2 \,(2x + \sec x \cdot \tan x)$$

2. $y = 4^{\sin^{-1}x}$

$$y' = 4^{\sin^{-1}x} \cdot \ln 4 \cdot \frac{1}{\sqrt{1-x^2}}$$

3- $y = x^{\pi} \cdot \pi^{x}$

$y' = x^{\pi} \cdot \pi^{x} \cdot \ln\pi \cdot 1 + \pi^{x} \cdot \pi x^{\pi-1} \cdot 1$

Exercises : find the derivative

1- $y = 5^{x^2+x-1}$

2- $y = 6^{\sin x + \ln x + 3}$

3- $y = 2^{\sec x}$

4- $y = 3^{\tan x}$

5- $y = \ln\left(\frac{x^4}{1+x^3}\right) + 7^{x^{2/3}}$

6- $S = 2^{-t^2}$

Ex :-

1- Find x if $3^{x} = 2^{x+1}$

$\ln 3^{x} = \ln 2^{x+1}$ بأخذ Ln الطرفين

$x \ln 3 = (x+1) \ln 2$

$x \ln 3 = x \ln 2 + \ln 2$

$x \ln 3 - x \ln 2 = \ln 2$

$(\ln 3 - \ln 2) x = \ln 2$

$x = \frac{\ln 2}{\ln 3 - \ln 2} = \frac{\ln 2}{\ln \frac{3}{2}} = \frac{\ln 2}{\ln 1.5}$

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$$2-\quad 3^{\log_3 7} + 2^{\log_2 5} = 5^{\log_5 x} \quad \text{Find } x$$

$$e^{\log_3 7 \cdot \ln 3} + e^{\log_2 5 \cdot \ln 2} = e^{\log_5 x \cdot \ln 5}$$

$$e^{\frac{\ln 7}{\ln 3} \cdot \ln 3} + e^{\frac{\ln 5}{\ln 2} \cdot \ln 2} = e^{\frac{\ln x}{\ln 5} \cdot \ln 5}$$

$$e^{\ln 7} + e^{\ln 5} = e^{\ln x}$$

$$7 + 5 = x$$

$$x = 12$$

* Relation between general exponential function and general Logarithm function.

$$\boxed{\log_a y = x \Longleftrightarrow y = a^x \; ; \; a > 0 \; , \; a \neq 1}$$

Note:-

$$\log_{10} y = x \Longleftrightarrow y = 10^x$$

$$* \; \log_a y = \frac{\ln y}{\ln a} = x$$

$$\left\{ \begin{aligned} \ln y &= x \ln a \\ e^{\ln y} &= e^{\ln a^x} \\ y &= a^x \end{aligned} \right.$$

$$\log_e y = \ln y = x \Longleftrightarrow y = e^x$$

$$\log_2 32 = 5 \Longleftrightarrow 32 = 2^5$$

الدالة الأسية واللوغارتمية هي دوال عكسية لبعض البعض ولكن بنفس الأساس.

(Logarithmic differentiation)

* إذا كانت الأسس والأساسات متغيرات

Examples: find the derivative:

$\boxed{1-}$ $y = x^{x^2}$; $x > 0$ لأن

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \cdot \ln x$$

$$\frac{y'}{y} = \cancel{x^2} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x$$

$$y' = y\left(x + 2x \ln x\right)$$

$$y' = x^{x^2}\left(x + 2x \ln x\right)$$

2- $y = (x^2+1)^{\ln x}$

$$\ln y = \ln (x^2+1)^{\ln x}$$

$$\ln y = \ln x \cdot \ln(x^2+1)$$

$$\frac{y'}{y} = \ln x \cdot \frac{2x}{x^2+1} + \ln(x^2+1) \cdot \frac{1}{x}$$

$$y' = y\left(\frac{2x \ln x}{x^2+1} + \frac{\ln(x^2+1)}{x}\right)$$

$$y' = (x^2+1)^{\ln x}\left(\frac{2x \ln x}{x^2+1} + \frac{\ln(x^2+1)}{x}\right)$$

3- $y=(\sin x)^{\tan x}$; $\sin x>0$

$\ln y=\ln(\sin x)^{\tan x}$

$\ln y=\tan x\cdot\ln(\sin x)$

$\frac{y'}{y}=\tan x\cdot\frac{\cos x}{\sin x}+\ln(\sin x)\cdot\sec^2 x$

$y'=y\left(\frac{\cancel{\sin x}}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{\cancel{\sin x}}+\sec^2 x\cdot\ln(\sin x)\right)$

$y'=(\sin x)^{\tan x}\left(1+\sec^2 x\cdot\ln(\sin x)\right)$

$\boxed{4-}$ $y=\sqrt[5]{\frac{(x+2)^3(x+1)^2}{(x-4)^2(x+3)}}$

$\ln y=\ln\left(\frac{(x+2)^3(x+1)^2}{(x-4)^2(x+3)}\right)^{\frac{1}{5}}$

$\ln y=\frac{1}{5}\ln\left(\frac{(x+2)^3(x+1)^2}{(x-4)^2(x+3)}\right)$

$\ln y=\frac{1}{5}\left\{\ln\left((x+2)^3(x+1)^2\right)-\ln\left((x-4)^2(x+3)\right)\right\}$

$\ln y=\frac{1}{5}\left\{\left(\ln(x+2)^3+\ln(x+1)^2\right)-\left(\ln(x-4)^2+\ln(x+3)\right)\right\}$

$\ln y=\frac{1}{5}\left\{3\ln(x+2)+2\ln(x+1)-2\ln(x-4)-\ln(x+3)\right\}$

$\frac{y'}{y}=\frac{1}{5}\left\{\frac{3}{x+2}+\frac{2}{x+1}-\frac{2}{x-4}-\frac{1}{x+3}\right\}$

$$\acute{y} = y\left\{\frac{3}{5(x+2)} + \frac{2}{5(x+1)} - \frac{2}{5(x-4)} - \frac{1}{5(x+3)}\right\}$$

$$\acute{y} = \sqrt[5]{\frac{(x+2)^3 (x+1)^2}{(x-4)^2 (x+3)}}\left\{\frac{3}{5(x+2)} + \frac{2}{5(x+1)} - \frac{2}{5(x-4)} - \frac{1}{5(x+3)}\right\}$$

5- $y = x^{\cos x} + x^{\sin^{-1} x}$

Let $t = x^{\cos x}$, $z = x^{\sin^{-1} x}$

$y = t + z$

$\acute{y} = \acute{t} + \acute{z}$

$t = x^{\cos x}$

$\ln t = \ln x^{\cos x}$

$\ln t = \cos x \cdot \ln x$

$\frac{\acute{t}}{t} = \cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x$

$\acute{t} = t\left(\frac{\cos x}{x} - \ln x \sin x\right)$

$\acute{t} = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right)$

$z = x^{\sin^{-1} x}$

$\ln z = \ln x^{\sin^{-1} x}$

$$\ln z = \sin^{-1}x \, \ln x$$

$$\frac{z'}{z} = \sin^{-1}x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$z' = z\left(\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$z' = x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$y' = t' + z'$$

$$y' = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right) + x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{1-x^2}}\right)$$

Exercises : Find y' ?

1- $y = x^{x}$; $x > 0$

2- $y = e^{\sqrt{x^2+e^x}}$

3- $y = x^{(\sqrt{5}+\ln x)}$

4- $y = x^{\ln x}$

Ex :- Find y' :-

$$y = 5^{\tan^{-1}x} + 3\,\text{csch}(e^{\sqrt{x}}) - \ln\left(\cos^{-1}(x^2 e^x)\right) + \log_2 \sin x + \tanh^{-1}(\cos 2x)$$

$$y' = 5^{\tan^{-1}x} \cdot \ln 5 \cdot \frac{1}{1+x^2} + 3 \cdot -\text{csch}(e^{\sqrt{x}}) \cdot \coth(e^{\sqrt{x}}) \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$- \frac{\frac{-1}{\sqrt{1-(x^2e^x)^2}} \cdot (x^2 e^x + e^x \cdot 2x)}{\cos^{-1}(x^2 e^x)} + \frac{\cos x}{\sin x \ln 2} + \frac{1}{1-\cos^2(2x)} \cdot -\sin(2x) \cdot 2$$