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Mathematical Modeling to Evaluation the Soil Contamination by Heavy Metals in Diyala Governorate

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سورة البقرة

بسم الله الرحمن الرحيم

قالوا سبحنك لاعلم لنا الا ما علمتنا انك انت العليم الحكيم **٣٢**

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Researcher

Abstract

In this thesis a modified approach to a numerical method based on interpolation theory for functions of two variables is proposed. We develop a multivariate divided difference method. The explicit formulae that connects the classical divided difference interpolation coefficients for one variable with multivariate interpolation coefficients are established.

To illustrate the accuracy and efficiency of the suggested method, we used this method to estimate the rate of contaminated soil by heavy metals; that is, evaluate the concentration of heavy metals in soil of Diyala governorate.

In particular, interpolation methods are extensively applied to model different phenomena where experimental data must be used in computer studies where expressions of those data are required.

The extending of divided difference method in two dimensions are proposed then applied to estimate the effect increasing the contamination in displaced people camp in Diyala government.

List of Symbols and Abbreviations

Symbol	**Definition**
C_l	value of concentration of heavy metals getting from laboratory inspecting
C_s	value of concentration of heavy metals getting from suggested method
Cd	Cadmium
Co	Cobalt
Cr	Chromium
Cu	Copper
e_{HO}	Higher order formula of global error
e_{RE}	Richardson extrapolation of global error
Fe	Iron
GIS	Geographic information system
HM	Heavy metal
Ni	Nickel
Pb	Lead
PH	Potential of hydrogen
PPm	Measure units of heavy metal concentration

"pythINSIM"	Maximum defect error
"pythTDINSIM"	Maximum defect error in two dimensions
"pythMDINSIM"	Maximum defect error in multivariate dimensions
S.M.C	Standard Universal for concentration of heavy metal
XRF	X-Ray Fluorescence
Zn	Zinc

Contents

Introduction

A particular and important aspect in numerical methods is the approximation of the different values. Operation designated as interpolation, which is employed in most of the branches of the science, such as: engineering (Oanta, 2001, [14]), economics (Oanta, 2007, [15]), etc.

The thesis emphasizes the importance of the numerical analysis in applications, being provided a systematic presentation of the methods and techniques of numerical analysis and interpolation of the functions. Basically, there are many types of approximating functions. Thus, any analytical expression may be expressed as an approximating function. The most common approximating functions are: polynomials, trigonometric and exponential functions. Special attention is dedicated to polynomials which are the oldest and simplest methods of approximation.

Interpolation theory for “functions of one variable has a long and distinguished history, dating back to Newton's interpolation formula and the classical calculus of finite differences. Standard numerical approximations to derivatives and many numerical integration methods for differential equations are based on the finite difference.” [4],[8],[19] The problems of interpolation and approximate the functions of several independent variables are important but the methods are less well developed than in the case of functions of a single variable. An immediate indication of the difficulties inherent in the higher dimensional case can be seen in the lack of uniqueness in the general interpolation problem. In many problems in engineering and science, the data consist of sets of discrete points, being required approximated functions which must have the following properties:

• The approximated function should be easy to determine;

• It should be easy to evaluate;

• It should be easy to differentiate;

• It should be easy implemented.

It can be noticed that polynomials satisfy all the proceeding properties, moreover, polynomials have many important properties such as continuity and orthogonally.

Contamination of soil by heavy metals refers to some significant heavy metals of biological toxicity, including zinc (Zn), copper (Cu), nickel (Ni), cadmium (Cd), lead (Pb), chromium (Cr), and arsenic (As), etc. With the development of the global economy, both type and content of heavy metals in the soil caused by human activities have gradually increased in recent years, which have resulted in serious environment deterioration.

In recent years, there are many studies about this problem (see e.g., [5], [10], [13], [20], [21], [37], [39], [41]). However these studies used statistical methods for processing the data. Many researchers studied and designed mathematical models which describe the contamination in soil for different categories and properties of soil, for more details see [3], [12], [22] and the references therein.

The employ of interpolation method in contamination of soil by heavy metals are firstly introduced by Tawfiq, et al. [25-33].

In this thesis consist divided difference method and its modification is used to process the data in order to determine the rate of contamination in soil.

The organization of this thesis is as follows:

Chapter one comprises of some mathematical concepts which will be utilized. In chapter two, the divided difference formula in two dimensions is introduced and developed then generalized in multi-dimensions, which can be used to estimate the concentration of heavy metal in soil. Chapter three illustrates the applications of the suggested method in several cities in Diyala governorate to estimate the concentration of heavy metals in the soil and to determine the accuracy of the suggested method by comparing them with laboratory results. Chapter four talks about error estimation and stability. Chapter five contains the conclusion and future work. In appendix, a general thought regarding contamination of the environment had been presented, particularly soil contamination by heavy metals that is required in the plan of the suggested method.All algorithms in this thesis have been implemented in MATLAB version 2014a

Chapter One

Preliminaries

1.1. Introduction

This chapter gives a brief prologue to some important idea which represents a background that is used throughout the thesis in various places. These topics may be familiar in many cases; however, a limited discussion is provided here in an attempt to make the thesis self-contained.

This chapter consists some of definitions, hypotheses, axioms and theorems which we need in this thesis.

1.2. “Interpolation

Many times description of a problem is given as discrete points such as $(x_0, y_0), (x_1, y_1), \dots \dots, (x_{n-1}, y_{n-1}), (x_n, y_n)$.The question is, how then dose one find the value of *y* at any other value of *x* ?

Well, a continuous function *f(x)* may be used to represent the n+1 data values with *f(x)* passing through *n*+1 points . Then one can find the value of *y* at any other values of *x*. This is called ***interpolation.***” [7] and [17]

Of course, if we required estimating the consort of the value outside ambit the given data, this case is said to be ***extrapolation.***

So, what kind of functions *f(x)* one should choose?

A polynomial is a common choice for an interpolating function because the polynomials are easy to evaluate, differentiate, integrate, orthogonal, and relative to other choices such as a trigonometric, exponential and so on series.

The idea of polynomial interpolation involves finding a polynomial of order *n* that passes through the *n*+1given points. [4] and [8]

That is, we must find a real interpolation function F, which has to satisfy the following conditions:

$F(x_i) = y_i$, i = 0,..., n. The theoretical base of polynomial interpolation is the Weierstrass theorem (existence theorem). This theorem shows that any continuous function can be approximated with accuracy on an interval, using a polynomial function. The interpolation polynomial of function is unique for any given interval. That is, there exist only one polynomial interpolate the given data but there are many formula can be represent these data such as Lagrange, Newton, divided difference, Hermite, spline, Birkhoff, trigonometric and rational interpolation (for more details see [11] and [24].

Theorem 1.1 (Weierstrass Approximation Theorem) [4]

Let f (x) be a continuous function on [a, b], then for any $\varepsilon > 0$, there exist an integer n and polynomial $P_n(x)$ of degree n, such that:

$$| f(x) - P_n(x) | < \varepsilon, \quad \forall x \in [a, b] \qquad (1.1)$$

Theorem 1.2 (Uniqueness Theorem)

There is exactly one polynomial passing through given points $(x_i , f(x_i))$.

Proof

Let $P_n(x)$ and $Q_n(x)$ are interpolation polynomial of degree n, for the unknown function $f(x)$ at the given points x_i , i = 0,1,…, n.

Let $R_n(x) = P_n(x) - Q_n(x)$, hence $R_n(x)$ is a polynomial of degree n.

So, $R_n(x_i) = P_n(x_i) - Q_n(x_i) = 0 \quad \forall\ i= 0,1,\ldots,n.$

Then $R_n(x)$ has (n+1) roots this is contradiction!

So, $P_n(x) = Q_n(x)$.

1.3. Importance of interpolation

Interpolation is the most important topic in numerical analysis and its application. It is the heart and nucleus of classical numerical analysis, for two main reasons. The first reason due to our constant need to search for the value of a function of data scheduled in most arithmetical problems and be of high accuracy even if that data is limited.

In those problems and discussions are not scheduled astronomical we find a value for the function at one or more points not included in the spreadsheet, we must find the function by use the methods of interpolation. As for the second reason, the importance of interpolation is due to the fact that most classical numerical methods in various sectors have been derived from the methods of interpolation.

The numerical methods used in the derivations, the integration of the ordinary differential equations, the quadratic equations, and other segments of classical numerical analysis have been developed and derived directly from the methods of interpolation. Although, other methods used in modern numerical analysis do not depend heavily on methods of interpolation; this does not conflict with the great role and great benefit of interpolation and methods of interpolation.

1.4. Interpolation methods

There are many interpolation methods that are used to find representation for a set of data, say spline, Lagrange, nearest, and so on. All of these methods require that the data x_i be monotonic either always decreasing or always increasing. Each method works with non-uniformed spaced data. Now, we demonstrate some interpolation methods

1.4.1Linear Interpolation [23]

Linear interpolation is the simplest method of getting values at position in between the data points. The points are simply joined by straight line segments. Each segment (bounded by two data points) can be interpolated independently.

1.4.2 Cubic Interpolation [38]

Cubic interpolation is the simple method that offers true continuity between the segments. As such it requires more than just the two endpoints of the segment but also the two points on either side of them. A common solution is the two extra points at the start and end of the sequence, the new points are created so that they have slope equal to the slope of the start or end segment.

1.4.3 Hermite Interpolation [38] and [40]

Hermite interpolation like cubic interpolation but it can achieve a higher degree of continuity. In addition it has nice tension and biasing controls. Tension can be used to tighten up the curvature at known points. The bias is used to twist the curve about the known points.[38] and [40]

1.4.4 The Nearest Neighbor Interpolation [6]

The nearest neighbor interpolation assigns the value of the nearest point to each grid node. This method is useful when data already evenly spaced, but need to be converted to a surfer grid file. Alternatively, in cases where the data are close to being on a grid, with only a few missing values, this method is effective for filling in the holes in the data.

1.4.5 The Data Metrics Interpolation [6]

The collection of data metrics interpolation creates grids of information about the data on a node -by-node basis. The data metrics methods are not in general weight average interpolators of z-value.

There are many interpolation methods but in this thesis, we suggest divided difference method and it is extended in two or more variables to solve our suggested problem.

1.5. Divided Difference Method

Let f (x) be a continuous function given at the distinct point x_i, i= 0,1,2,…,n, define the zeros divided difference of the function f with respect to x_i denoted $f[x_i]$ by

$f[x_i] = f(x_i)$, The first divided difference of the function f with respect to x_i and x_{i+1} denoted $f[x_0, x_1]$ and is defined as [4]:

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

The second divided difference denoted $f[x_i, x_{i+1}, x_{i+2}]$ and is defined as:

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

Similarly the nth divided difference relative to x_0, x_1 ,…., x_{n-1}, x_n, is given by

$$f[x_0, x_1, \ldots\ldots, x_{n-1}, x_n] = \frac{f[x_1, x_2, \ldots\ldots, x_n] - f[x_0, x_1, \ldots., x_{n-1}]}{x_n - x_0}$$

The value of $f[x_0, x_1, \dots \dots, x_n]$ is independent of the order number $x_0, \dots \dots, x_n$. That is, if n= 0, 1, 2, then

$$f[x_o, x_1, x_2] = f[x_1, x_0, x_2] = f[x_2, x_1, x_0]$$

Then a polynomial of degree n in x can be construct to interpolate through x_i, i = 0, 1, …, n; as

$$\mathrm{P}_n(x) = \sum_{i=0}^{n} f[x_0, x_1, \dots, x_i]\,(x - x_o) \dots (x - x_{i-1}), \qquad (1.2)$$

Let $w_{i-1}(x) = \prod_{k=0}^{i-1}(x - x_k)$, so rewrite equation (1.2) as follow

$$\mathrm{P}_n(x) = \sum_{i=0}^{n} f[x_0, x_1, \dots, x_i]\; w_{i-1}(x). \qquad (1.3)$$

where $w_{-1}(x) = 1$

The following theorem describes the relation between nth divided difference formula of the function and its derivative.

Theorem 1.3 [18]

If $f^{(n)}(x)$ is continuous function in $[a, b]$ and $x_0, x_1, \dots, x_n$ are in [a, b] then

$$f[x_0, x_1, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}, \qquad (1.4)$$

where; min $(x_0, x_1, \dots, x_n) \le \xi \le$ max $(x_0, x_1, \dots x_n)$

A particular case of theorem 1.3, is the following corollary.

Corollary 1.1

If $f^{(n)}\ (x)$ is continuous function in a neighborhood of x then

$$f\left[\underbrace{x, x, \ldots, x}_{n+1 terms}\right] = \frac{f^{(n)}\ (x)}{n!} \qquad (1.5)$$

1.6. Speed, Memory, and Smoothness Considerations[9]

When choosing an interpolation method, keep in mind that some require more memory or longer computation time than others. However, you may need to trade off these resources to achieve the desired smoothness in the result.

- Nearest neighbor interpolation is the fastest method. However, it provides the worst results in terms of smoothness.
- Linear interpolation uses more memory than the nearest neighbor method, and requires slightly more execution time. Unlike nearest neighbor interpolation its results are continuous, but the slope changes at the vertex points.
- Cubic interpolation requires more memory and execution time than either the nearest neighbor or linear methods. However, both the interpolated data and its derivative are continuous.
- Cubic spline interpolation has the longest relative execution time, although it requires less memory than cubic interpolation. It produces the smoothest results of all the interpolation methods. You may obtain

unexpected results, however, if your input data is non-uniform and some points are much closer together than others.

The relative performance of each method holds true even for interpolation of two-dimensional or multidimensional data.

Chapter Two

High Dimensional Interpolation

2.1. Introduction

This chapter introduces the concept of interpolation theory for functions of two variables. The divided difference formula in two dimensions is introduced and developed then generalized in multidimensional. The explicit formulae that connect the classical finite difference interpolation coefficients for one variable curve with multivariate interpolation coefficients are established.

2.2. Interpolation in Two Dimensions

Given distinct points $(x_0, y_0), (x_1, y_1), \ldots, (x_m, y_n)$ in the (x, y)- plane, we have to find a polynomial passing through these points by divided difference formula.

Consider the distinct points (x_i, y_j) as a rectangular grid where x_i , $i= 0, 1, 2, \ldots, m$ and y_j , $j= 0, 1, 2, \ldots, n$, then a polynomial of degree m in x and n in y can be constructed to interpolate through (x_i, y_j), by extending one dimension formula equation (1.3), this is, given by.

$$P_{m,n}(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} w_{i-1}(x) w_{j-1}(y) f[x_0, x_1, \dots, x_i, y_0, y_1, \dots, y_j] \quad ,(2.1)$$

where; $w_{i-1}(x) = \prod_{k=0}^{i-1}(x - x_k)$,

$w_{j-1}(y) = \prod_{k=0}^{j-1}(y - y_k)$,

$w_{-1}(x) = 1$ and $w_{-1}(y) = 1$.

2.3. High Dimensional Interpolation

The problems of polynomial interpolation for functions of several independent variables are important but the methods are less well developed than in the case of functions of a one variable. Causes of the difficulties inherent in the higher dimensional case can be seen in the lack of uniqueness in the general interpolation problem.

That is, we ask the following question.

If $x_1, x_2, \dots, x_m$ are m distinct points, say in the x, y-plane, then is there a unique polynomial of specified degree which attains specified values, say $f(x_i)$, at these points? Clearly the answer, in general, must be no since if all of the points (x_i, $f(x_i)$) lie on a straight line in x, y, z-space, then there are infinitely many planes (i.e., linear polynomials) and perhaps higher degree polynomials of the from $z = h(x, y)$ containing this line.

We shall not dwell on these aspects of interpolation in higher dimensions but shall how to construct appropriate polynomials when the points of interpolation are specially chosen. It will also be found that in these special cases the interpolation polynomials are unique. For simplicity, we concentrate on functions of two variables; however, extension to more dimensions offers no difficulty.

Another representation of the interpolation polynomial can be obtained by using divided difference formulae, with $m + 1$ distinct points x_i we have.

$$f(x, y) = \sum_{i=0}^{m} w_{i-1}(x)\, f[x_0, x_1, ..., x_i; y] + w_m(x) f[x_0, ..., x_m, x; y] \quad (2.2)$$

where;

$$w_{-1}(x) \equiv 1;\ w\omega_i(x) = w_{i-1}(x)\,(x - x_i), i = 0,1, ...,$$

The divided differences of a function of several variables are formed by keeping all but one variable fixed and taking the indicated differences with respect to the free variable. Hence, $f[x_0, x_1, ..., x_i; y]$as a function of the independent variable y has the divided difference representation, using the $n+1$ $points\ of$ y_j , such:

$$f[x_0, x_1, \ldots, x_i; y] = \sum_{j=0}^{n} w_{j-1}(y) f[x_0, x_1, \ldots, x_i; y_0, y_1, \ldots, y_j] +$$
$$w_n(y)\, f[x_0, \ldots, x_i; y_0, \ldots, y_n, y] \qquad (2.3)$$

We use equation (2.3) for $i = 0, 1, \ldots, m$ in equation (2.2) to get

$$f(x, y) = Q(x, y) + R(x, y);$$

where

$$Q(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{n} \omega_{i-1}(x)\, w_{j-1}(y) f[x_0, \ldots, x_i; y_0, \ldots y_j], \qquad (2.4)$$

and

$$R(x, y) = w_n(y) \sum_{i=0}^{m} w_{i-1}(x) f[x_0, \ldots, x_i; y_0, \ldots, y_n, y] + w_m(x) f[x_0, \ldots, x_m, x; y] \qquad (2.5a)$$

It is clear that $R(x_i, y_j) = 0$ at the $(m+1)(n+1)$ points. To simplify this expression we again use the divided difference formula and the $m+1$ $points$ x_i to write:

$$f[x; y_0, \ldots, y_n, y] = \sum_{i=0}^{m} w_{i-1}(x) f[x_0, \ldots, x_i; y_0, \ldots, y_n, y] + w_m(x) f[x_0, \ldots, x_m, x; y_0, \ldots, y_n, y].$$

If we multiply this identity by $w_n(y)$ and subtract the result from (2.5a) we finally obtain;

$$R(x,y)=w_m(x)f[x_0,\ldots,x_m,x;y]+w_n(y)f[x;y_0,\ldots,y_n,y]-w_m(x)w_n(y)$$

$f[x_0,\ldots,x_m,x;y_0,\ldots,y_n,y]$. (2.5b)

If $f(x,y)$ has continuous partial derivatives of orders $m+1$ *and* $n+1$, respectively in x *and* y and the appropriate mixed derivative of order $m+n+2$ then the error becomes

$R(x,y) =$

$$\frac{w_m(x)}{(m+1)!}\frac{\partial^{m+1}f(\xi,y)}{\partial x^{m+1}}+\frac{w_n(y)}{(n+1)!}\frac{\partial^{n+1}f(x,\eta)}{\partial y^{n+1}}-\frac{w_m(x)w_n(y)}{(m+1)!(n+1)!}\frac{\partial^{m+n+2}f(\xi',\eta')}{\partial x^{m+1}\,\partial y^{n+1}}. \quad (2.6)$$

We obtain, upon replacing m *by* n *in* (2.2) *and* n *by* $n-i$, in (2.3)

$$f(x,y) = P_n(x,y)+R_n(x,y);$$

where

$$P_n(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{n-i} w_{i-1}(x)w_{j-1}(y)f\left[x_0,\ldots,x_i;y_0,\ldots,y_j\right]. \quad (2.7)$$

and

$$R_n(x,y) = \sum_{i=0}^{n} w_{i-1}(x)w_{n-i}(y)f[x_0,\ldots,x_i;y_0,\ldots,y_{n-i},y] + w_n(x)f[x_0,\ldots,x;y];$$

$$= \sum_{i=0}^{n+1}\frac{w_{i-1}(x)w_{n-i}(y)}{i!(n-i+1)}\left(\frac{\partial}{\partial x}\right)^{i}\left(\frac{\partial}{\partial y}\right)^{n-i+1} f(\xi_{i,}\eta_i). \quad (2.8)$$

The polynomial (2.7) has degree at most n, in this case the points distributed as a trigonometric grid. But the points in equation (2.4) distributed as a rectangular grid. So choosing the formula of polynomial depending on the distributed of given point.

To approximate the partial derivatives of functions of several independent variables, we could proceed for functions of one variable, it is easy to use the Taylor expansion method. If no mixed derivatives occur and the points to be used are on a coordinate line in the direction of differentiation than the one-dimensional analysis is valid.

2.4. Modify Multivariate Interpolation

We can deduce yet another representation of the divided difference, when several multiplicities occur.

Definition 2.1

Let $f^{(n)}(x)$ is continuous function in [a, b], $y_0, y_1, \dots, y_n$ are in [a, b], and x is distinct from any y_i then

$$f[\mathrm{x}, \mathrm{y}_{0,} \mathrm{y}_{1,\dots,} \mathrm{y}_\mathrm{n}] = \frac{f[\mathrm{x}, \mathrm{y}_1, \dots, \mathrm{y}_\mathrm{n}] - f[\mathrm{y}_0, \mathrm{y}_1, \dots, \mathrm{y}_\mathrm{n}]}{\mathrm{x} - \mathrm{y}_0}. \qquad (2.9)$$

The following definition gives the unique continuous extension of the definition of divided difference

Definition 2.2

Let $\{x_i\}, \{y_j\}$ in $[a, b]; x_i \neq y_j, for$ $0 \leq i \leq p, 0 \leq j \leq q; f^{(m)}(x)$ is continuous in [a, b] ; $0 \leq p, q \leq m$, then

$$f[x_0, \dots, x_p, y_0 \dots, y_q] = g[x_0, \dots, x_p] = h[y_0, \dots, y_q] , \qquad (2.10)$$

where

$$g(x) \equiv f[x, y_{0,\dots,} y_q] \;\; , \;\; h(y) \equiv f[x_0, \dots, x_p, y].$$

The conclusions of the above definitions that follow, concern continuity properties and representations for divided differences that are easily established when no multiplicities occur among the arguments. When multiplicities do occur, the definitions establish the same representations under the hypothesis of minimal differentiability of $f(x)$.

The following theorem describes the relation between multi divided difference formula of the function and its derivative.

Theorem 2.1

If f (x) has a continuous derivative of order m in [a, b]; and

$x_0, \ldots, x_p, y_0, \ldots, y_q,\ \ z_0, \ldots, z_r\ are\ in\ [a, b];\ x_i \neq y_j, x_i \neq z_k\ ,\ y_j \neq z_k,$

for all $i, j, k, 0 \leq p, q, r \leq m$; then:

$$f[x_0, \ldots, x_p, y_0, \ldots, y_q, z_0, \ldots, z_r] = \frac{1}{p!q!r!} \frac{\partial^p}{\partial x^p} \frac{\partial^q}{\partial y^q} \frac{\partial^r}{\partial z^r} f[x, y, z]|_{(\xi,\eta,\zeta)} \quad (2.11)$$

where; $\min(x_0, \ldots, x_p) \leq \xi \leq \max(x_0, \ldots, x_p)$;

$\min(y_0, \ldots, y_q) \leq \eta \leq \max(y_0, \ldots, y_q)$;

$\min(z_0, \ldots, z_r) \leq \zeta \leq \max(z_0, \ldots, z_r)$.

Proof

Let $$g(x) \equiv f[x, y_0, \ldots, y_q, z_0, \ldots, z_r] \quad (2.12)$$

$$h(y) \equiv f[x, y, z_0, \ldots, z_r]$$

By Definition (2.2) and theorem (1.3), appropriately generalized for sets of variables $\{x_i\},\{y_j\},\{z_k\}$ we have

$$f[x_0, \ldots, x_p, y_0, \ldots, y_q, z_0, \ldots, z_r] = g[x_0, \ldots, x_p] \quad (2.13)$$

$$g[x_0, \ldots, x_p] = \frac{1}{p!} \frac{\partial^p}{\partial x^p} g(x)|_{x=\xi} \quad (2.14)$$

$$g(x) = h[y_0, \ldots, y_q] = \frac{1}{q!} \frac{\partial^q}{\partial y^q} h(y)|_{y=\eta} \quad (2.15)$$

$$h(y) = k[z_0, \dots, z_r] = \frac{1}{r!}\frac{\partial^r}{\partial z^r} k(z)\Big|_{z=\zeta} \qquad (2.16)$$

The conclusion (2.11) follows from (2.13), (2.14), (2.15), and (2.16). $\blacksquare$

A special case is contained in the following corollary.

Corollary 2.1

If $f^{(m)}(x)$ is continuous in [a, b], x, y, z are distinct points in [a, b]; $0 \le p, q, r \le m$; then

$$f\left[\underbrace{x, \dots, x}_{p+1}, \underbrace{y, \dots, y}_{q+1}, \underbrace{z, \dots, z}_{r+1}\right] = \frac{1}{p!q!r!}\frac{\partial^p}{\partial x^p}\frac{\partial^q}{\partial y^q}\frac{\partial^r}{\partial z^r} f[x, y, z] \qquad (2.17)$$

2.5. Suggested Other Modification

There is an alternative method for generating approximations that has as it basis the divided difference at $x_0, x_1, \dots, x_n$, this alternative method uses the connection between the n^{th} divided difference and the n^{th} derivative of f (see Chapter one theorem(1.3) and corollary (1.1)).

Suppose that the distinct numbers $x_0, x_1, \dots, x_n$ are given together with the values of f and f' at these numbers, define a new sequence: $z_0, z_1, \dots, z_{2n+1}$ by $z_{2i} = z_{2i+1} = x_i$, for each *i*= 0,1,…, n;

and construct the divided difference table in the form of table (2.1) that uses $z_0, z_1, \dots, z_{2n+1}$.

Since $z_{2i} = z_{2i+1} = x_i$, for each i we cannot define $f[z_{2i}, z_{2i+1}]$ by divided difference formula, however, if we assume based on (theorem 1.3, in chapter one) that the reasonable substitution in this situation is $f[z_{2i}, z_{2i+1}] = f'(z_{2i}) = f'(x_i)$,we can use the entries: $f'(x_0), f'(x_1), \dots$, $f'(x_n)$, in place of undefined first divided differences:

$$f[z_0, z_1], f[z_2, z_3], \dots, f[z_{2n}, z_{2n+1}].$$

The remaining divided differences are produced as usual and the appropriate divided differences are employed in divided difference formula. Table (2.1) illustrates the entries that are used for the first three divided difference columns when determining by suggested manner x_0, x_1 and x_2 . The remaining entries are generated in the same manner as in the Table (2.1). The modified polynomial is then given by

$$P_{2n+1}(x) = f[z_0] + \sum_{i=1}^{2n+1} f[z_0, z_1, \dots, z_i](x - z_0)(x - z_1) \dots (x - z_{i-1}). \quad (2.18)$$

Then equation (2.18) can be generalized in two variables formula as follows

$$P_{2m+1,2n+1}(x, y) = f[z_0, v_0] + \sum_{i=1}^{2m+1} \sum_{j=1}^{2n+1} f[z_0, z_1, \dots, z_i;\ v_0, v_1, \dots, v_j]\,(x - z_0) \dots (x - z_{i-1})\,(y - v_0) \dots \left(y - v_{j-1}\right) \quad (2.19)$$

For simplicity,

$$P_{2m+1,2n+1}(x,y) = \sum_{i=0}^{2m+1}\sum_{j=0}^{2n+1} f\left[z_0,\dots,z_i,v_0,\dots,v_j\right] w_{i-1}(z)w_{j-1}(v) \quad (2.20)$$

for all $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$,

where; $z_{2i} = z_{2i+1} = x_i$ and $v_{2j} = v_{2j+1} = y_j$.

Now, from theorem (2.1) we can generalize the suggested modification in two variables and construct table (2.2) as the same manner of table (2.1) but in two variables.

Table 2.1: Suggested modification of divided difference

z	*F(z)*	First divided difference	Second divided difference
Z_0	$f[z_0] = f(x_0)$		
	z		f
		f	
Z_2	$f[z_2] = f(x_1)$		f
Z_3	$f[z_3] = f(x_1)$		f
		f	
Z_4	$f[z_4] = f(x_2)$		f
Z_5	$f[z_5] = f(x_2)$		

Table 2.2: Suggested modification of divided difference in two variables

(x_i, y_j)	First divided difference	Second divided difference
		f
		f
		f

2.6. Suggested Practical Two Dimensions Interpolation

Let the function $f(x,t)$ be defined at the points P_1, P_2, P_3 and P_4 which represent the vertices of a rectangular region (rectangular grid). Let P be any point inside this region. Through *P* draw horizontal and vertical lines intercepting the sides of the region at points *A*, *B*, *C* and *D*. Define the numbers α, β, γ and δ as illustrated in Figure (2.1):

$$\alpha = \frac{AP}{AB}, \beta = \frac{BP}{AB}, \gamma = \frac{CP}{CD}, \delta = \frac{DP}{CD} \tag{2.21}$$

Apply interpolation rule (divided difference) for sides P_1P_2 and P_3P_4 then for line segment *CD*, we can obtain the following formula:

$$L(P) = \delta\beta\, f(P_1) + \alpha\delta\, f(P_2) + \beta\gamma\, f(P_3) + \alpha\gamma\, f(P_4) \tag{2.22}$$

by using two variable vertices to estimate another value which lie among them. It can be expressed as an extension of linear interpolation for interpolating functions of two variables on a regular two dimensions grid, with a key idea of performing linear interpolation in one direction, then again in the other direction. The result is an interpolation which is not linear, although each step is linear. It should be noted that it doesn't make any difference whether we start interpolating with the values of any of the two dimensions followed by the values of the other one; the solution will be the same.

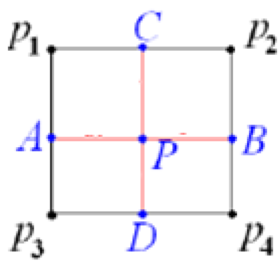


Figure 2.1: Practical 2-Dimensions Interpolation

To understand the topic more clearly, assume that we have the value of the concentration for a specific metal at four given positions $P_{00}=(x_0,t_0)$, $P_{01}=(x_0,t_1)$, $P_{10}=(x_1,t_0)$, and $P_{11}=(x_1,t_1)$, where x represent the distance and t represent the time, then the interpolate function from these positions $C=f(x,t)$ represents the concentration of heavy metals for any position in this region which illustrated in figure (2.2), the idea is using linear interpolation in one direction, and repeat using linear interpolation in the other direction.

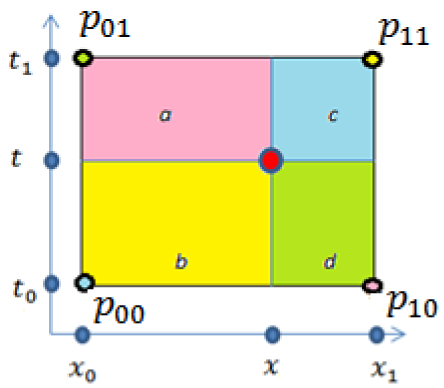


Figure 2.2: The Region of 2-Dimensional Interpolation

Practically, as in figure (2.3), we need to find the value of the function f(x, t) at the red spot which equals to the sum of the product of each coloured spot by the area of the same colour rectangle, divided by the area of all four rectangles.

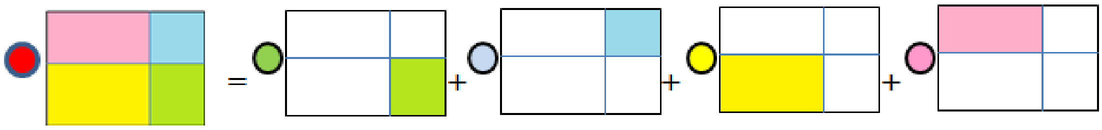


Figure 2.3: Practical representation of 2-D Interpolation

2.7. Suggested Practical Multivariate Interpolation

We suggest a practical method of multivariate interpolation on a 3-dimensional regular grid. It approximates the value of an intermediate point (x, y, z) within the local axial rectangular prism linearly, using data as used in finite on the net points and all the mesh elements are tetrahedral (3D simplexes).

Suggested interpolation is the extension of linear interpolation, which operates in spaces with dimension D=1, and bilinear interpolation, which operates with dimension D=2, to dimension D=3. The order of accuracy is 1 for all these interpolation schemes. There are several ways to arrive at this interpolation; it is a tensor product of 3 linear interpolation operators.

2.7.1. Implementation of Suggested Method

On a periodic and cubic net, let , x_d, y_d and z_d be the differences between each of , x, y, z and the smaller coordinate related, that is:

$$x_d = (x - x_0)/(x_1 - x_0)$$

$$y_d = (y - y_0)/(y_1 - y_0)$$

$$z_d = (z - z_0)/(z_1 - z_0)$$

where x_0 indicates the net point below x, and x_1 indicates the net point above x and similarly for y_0, y_1, z_0, and z_1. figure (2.4) illustrates the distribution of the points.

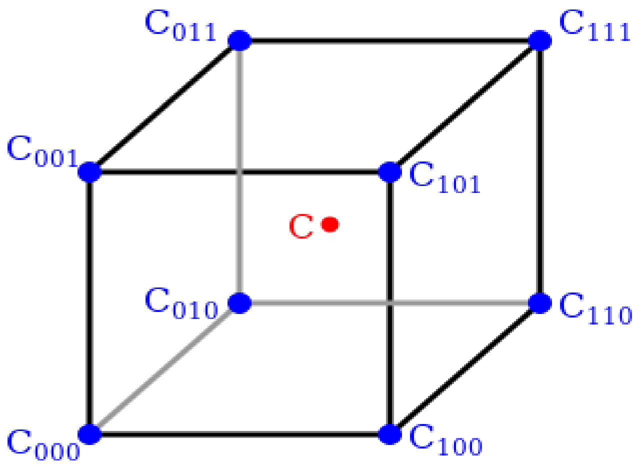


Figure 2.4: Eight corner points on a cube surrounding the interpolation point C

First we interpolate along x (imagine we are pushing the front face of the cube to the back), giving:

$$c_{00} = V[x_0, y_0, z_0](1 - x_d) + V[x_1, y_0, z_0]x_d$$
$$c_{01} = V[x_0, y_0, z_1](1 - x_d) + V[x_1, y_0, z_1]x_d$$
$$c_{10} = V[x_0, y_1, z_0](1 - x_d) + V[x_1, y_1, z_0]x_d$$
$$c_{11} = V[x_0, y_1, z_1](1 - x_d) + V[x_1, y_1, z_1]x_d$$

where $V[x_0, y_0, z_0]$ means that the function value of (x_0, y_0, z_0). Then we interpolate these values along y, as we were pushing the top edge to the bottom), giving:

$$c_0 = c_{00}(1 - y_d) + c_{10}y_d$$
$$c_1 = c_{01}(1 - y_d) + c_{11}y_d$$

Finally, we interpolate these values along z (walking through a line):

$$c = c_0(1 - z_d) + c_1 z_d$$

This gives us a predicted value for the point will be illustrated in figure (2.5).

The result of interpolation is independent of the order of the interpolation steps along the three axes: any other order, for instance along x, then along y, and finally along z, produces the same value.

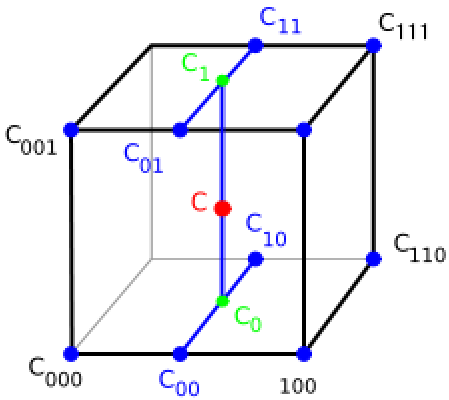


Figure 2.5: Suggested 3D interpolation

The above operations can be visualized as follows: First we find the eight corners of a cube that surround our point of interest. These corners have the values C_{000}, C_{100}, C_{010}, C_{110}, C_{001}, C_{101}, C_{011}, C_{111}.

Next, we perform linear interpolation between C_{000} and C_{100} to find C_{00}, C_{001} and C_{101} to find C_{01}, C_{011} and C_{111} to find C_{11}, C_{010} and C_{110} to find C_{10}.

Now, we do interpolation between C_{00} and C_{10} to find C_0, C_{01} and C_{11} to find C_1. Finally, we calculate the value C via linear interpolation of C_0 and C_1

In practice, a suggested interpolation is identical to two bilinear interpolation combined with a linear interpolation. figure (2.6) illustrates a geometric visualization of suggested interpolation. The product of the value at the desired point and the entire volume is equal to the sum of the products of the value at each corner and the partial volume diagonally opposite the corner.

$$C \approx l(b(C_{000}, C_{010}, C_{100}, C_{110}), b(C_{001}, C_{011}, C_{101}, C_{111}))$$

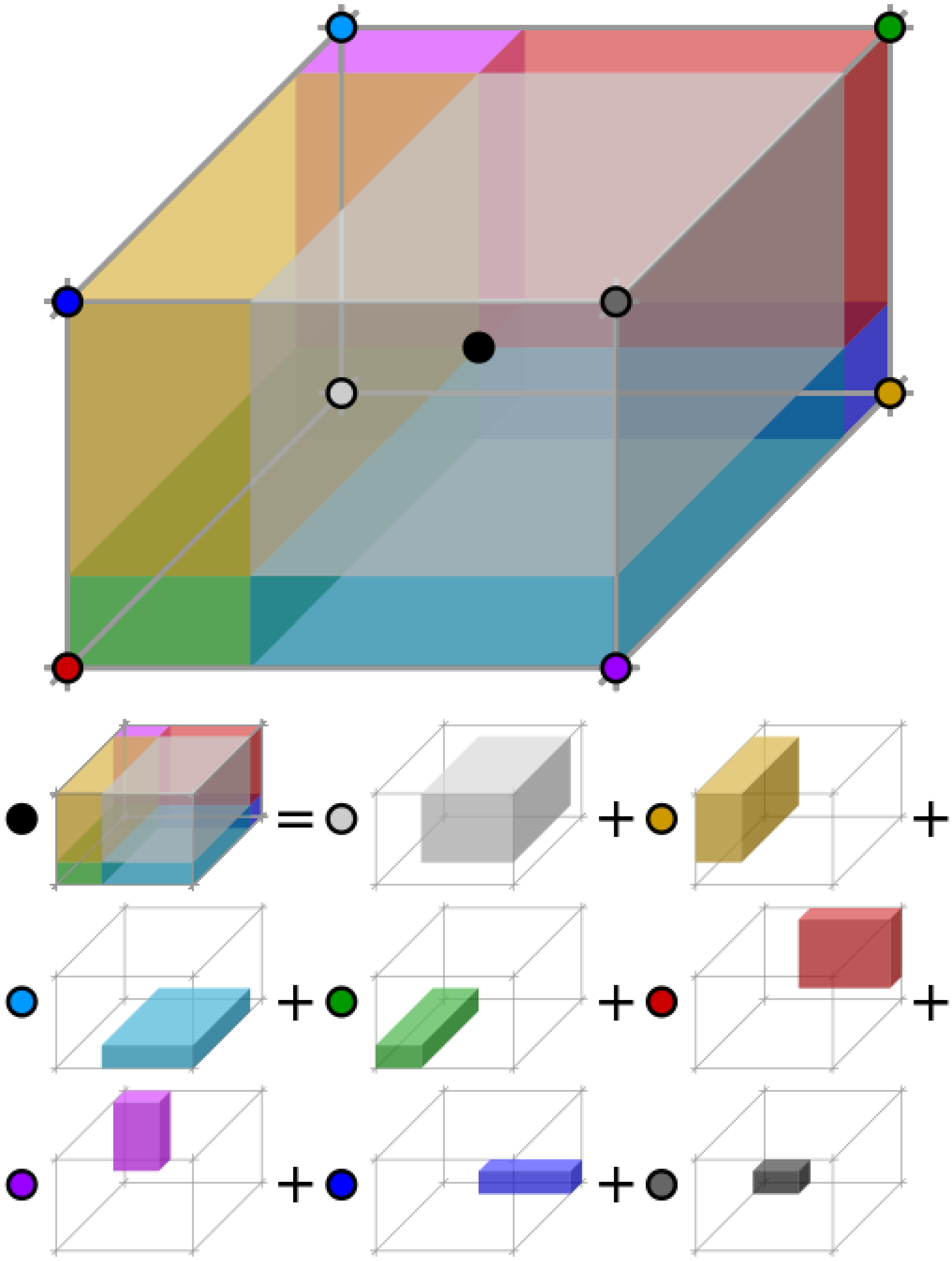


Figure 2.6: A geometric visualization of suggested interpolation.

For a given set of points in space, a decomposition of space into cells, one for each given point, so that anywhere in space, the closest given point is inside the cell. By assigning the function value at the given point to all the points inside the cell. figure (2.7) illustrate by color the shape of the cells. Each colored cell indicates the area in which all the points have the black point in the cell as their nearest black point. figure (2.8) illustrate interpolation by suggested method of a random set.

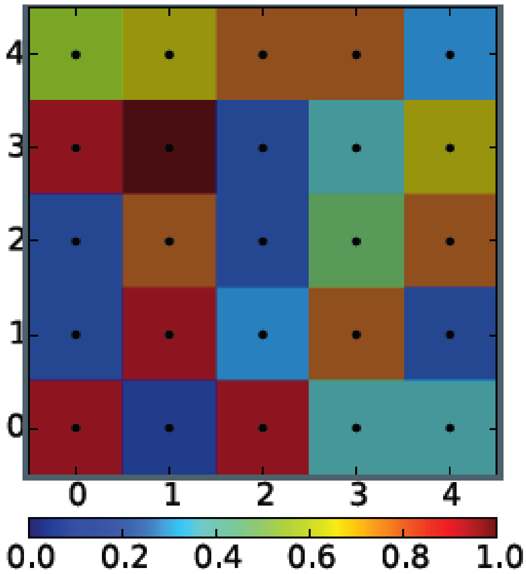


Figure 2.7: Interpolation on a uniform 2D grid (black points).

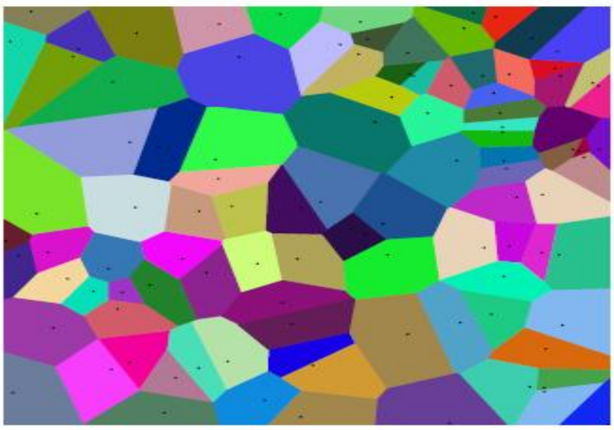


Figure 2.8: Interpolation of suggested method a random set of points (black dots) in 2D.

Chapter Three

Applications

3.1 Introduction

In this chapter, we introduce an application for the suggested numerical method which describes dilation in previous chapters about the soil contamination by heavy metals for different zones in Diyala Governorate. Our aim to estimate the concentration of the heavy metals such Cd , Cu, Pb, Cr, Ni, Fe, Zn and Co by our suggested method then compares the results with Standard Universal. The difference between them represents the rate of contamination in soil.

3.2 Samples

Diyala governorate located in the middle of Iraq, its area about (17685 km^2) and represents (4.1 %) from the area of Iraq, consist of 19 administrative units. For the purpose of collection of soil samples, the study area was divided into five main types of land use viz. residential, commercial, agricultural, main roads and industrial; and two main source areas, within each land use type viz. roadside and open areas. The areas are illustrated by geographic information system (GIS).

By using a stainless steel spatula, the sample was collected carefully from each source area in the land. They were air–dried in the laboratory, homogenized and sieved through a 2mm polyethylene sieve to remove large debris, stones and pebbles, after they were disaggregated with a porcelain pestle and mortar. Then these samples were stored in clean self–sealing plastic bags for further analysis.

Metal determinations were done by X- Ray Fluorescence analysis (XRF) these samples represent the initial data will be used in the suggested method in previous chapters to get the concentration of these metals for any depth and time. So, the samples were carefully chosen from each source area to get more accuracy results.

Firstly, studying the contamination of soil by heavy metal was done in Khalis zone; where the sampling was collected in December 2016. Figure (3.1) gives an indication for the character of the zones, from which samples were taken.

The 8 soil samples were collected with depth (0- 20 cm) using soil core, and then all the samples were put in plastic bags to measure the concentration of heavy metals Cd, Pb, and Zn. The soil samples were taken on a dry day from various categories of gardens on road sides near factories, children's playgrounds, schools situated in the residential area, tanning leather factories and brick factories.

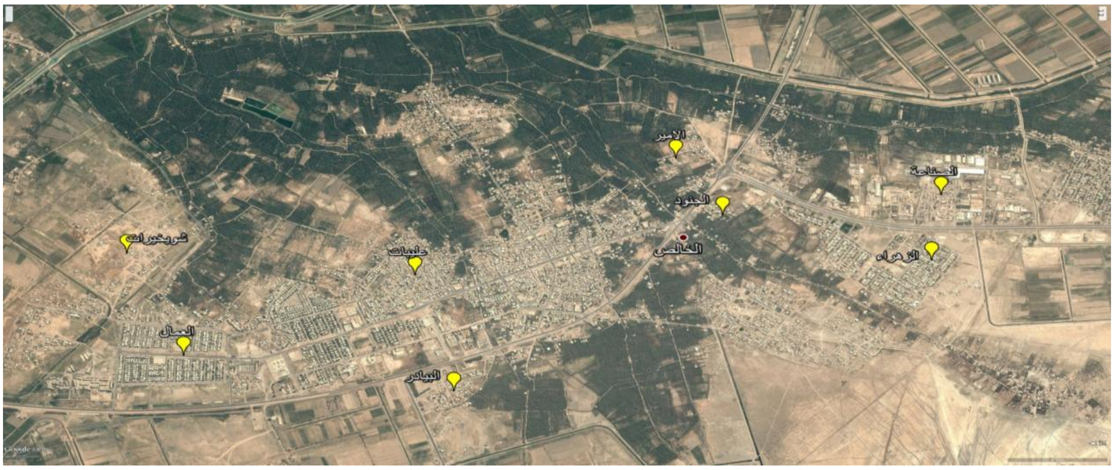


Figure 3.1: Location of samples in Al- Khalis

" In the laboratory, samples were sieved in a 2-mm sieve to remove stones, glass and large plant roots and subsequently dried at room temperature for 3 days. The dried samples were then homogenized with a mortar and a pestle. The procedure described by" [39] was followed to digest the samples with some modifications.

Then these samples were stored in clean self–sealing plastic bags for further analysis. Metal determinations were done by XRF.

These samples represent the initial data given in Table 3.1a, the center of Khalis city and Table 3.1b,the agricultural zone of Alkhuyls dorp in Khalis city, which used to get interpolation function that is substituted in the suggested formula.

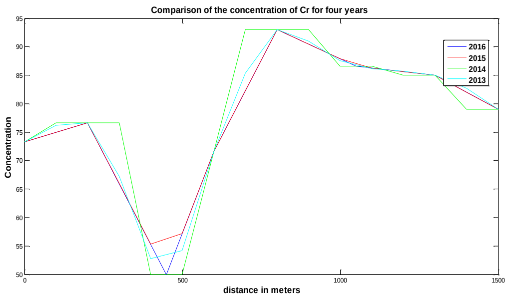
Table 3.1a: Laboratory results & Standard Universal (S.M.C.) [41]for concentration of HM in Khalis soil with measure units PPm

No	Cities of khalis	Cd (S.M.C.=1)		Pb (S.M.C.=50)		Zn (S.M.C.=70)	
		Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.
1	Hay al-senaay	7.504	6.504	18.1	-31.9	3	-67
2	Al –eummal	5.879	4.879	37.1	-12.9	5.2	-64.8
3	Hay Zahra	7.733	6.733	18	-32	3.8	-66.2
4	Al-junud	6.546	5.546	35.3	-14.7	6.2	-63.8
5	Al-amir	7.380	6.380	28.12	-21.88	3.2	-66.8
6	eulaybat	7.467	6.467	30.4	-19.6	5.9	-64.1
7	Bayader	7.836	6.836	23.51	-26.49	2.1	-67.9
8	shawaykhirat	8.123	7.123	38.3	-11.7	3.6	-66.4

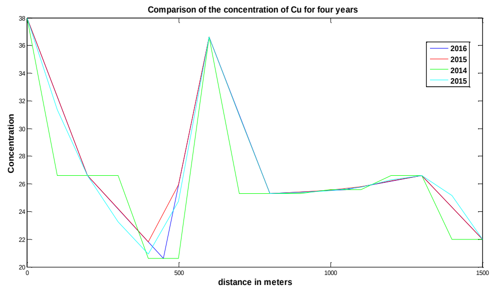
Table 3.1b: Laboratory results & Standard Universal (S.M.C.) [41]for concentration of HM in Alkhuyls dorp with measure units PPm

measures of heavy metals by XRF														
Samples	PH	Pb (S.M.C.=50)		Ni (S.M.C.=50)		Zn (S.M.C.=70)		Fe (S.M.C.=38000)		Cr (SMC.=100)		Cu (S.M.C=20)	
		Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.
Sample(1)	7.24	62.6	12.6	104.6	54.6	240	170	12820	-25180	73.3	-26.7	38	18
Sample(2)	6.79	44	-6	100	50	110	40	12054	-25946	76.6	-23.4	26.6	6.6
Sample(3)	7.13	38	-12	76.6	26.6	103	33	8325	-29675	50	-50	20.6	0.6
Sample(4)	7.56	44.3	-6.3	90	40	166.5	96.5	12404	-25596	71.6	-28.4	36.6	16.6
Sample(5)	7.28	93	43	121	71	93	23	16650	-21350	93	-7	25.3	5.3
Sample(6)	7.21	37.3	-12.7	118	68	100	30	16317	-21683	86.6	-13.4	25.6	5.6
Sample(7)	7.19	36	-14	120	70	86.6	16.6	15651	-22349	85	-15	26.6	6.6
Sample(8)	6.98	42	-8	120	70	100	30	15451	-22549	79	-21	22	2
average		49.65		106.3		124.9		13709		76.9		27.7	

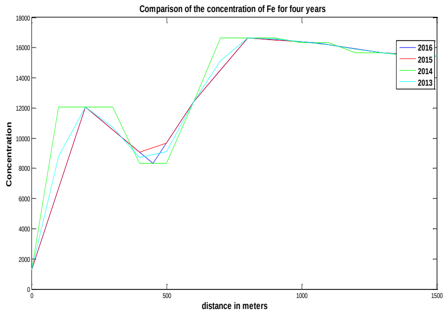
The results of suggested method which illustrate the concentration for each metal of four years given in Figure (3.2) for Alkhuyls dorp. Also, Figure (3.3), illustrates the concentration for each metal by the suggested method for Khalis city.



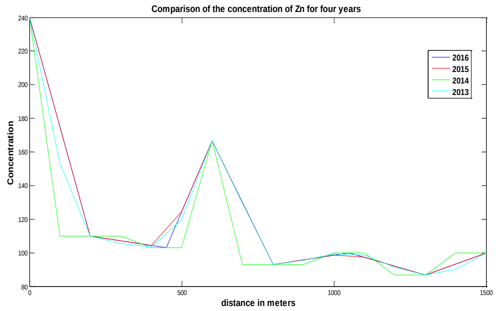
(a) Cr



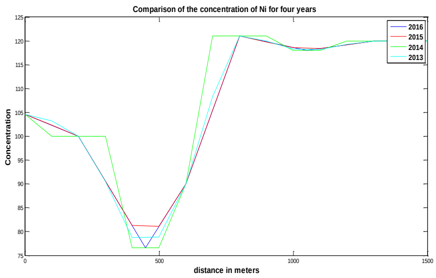
(b) Cu



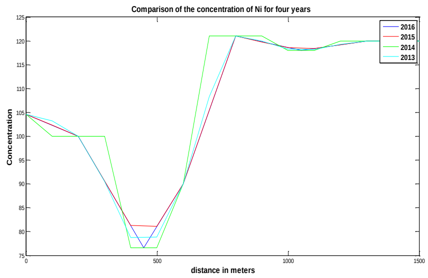
(c) Fe



(d) Zn



(e) Pb



(f) Ni

Figure 3.2: concentration of heavy metals by suggested method for Alkhuyls city

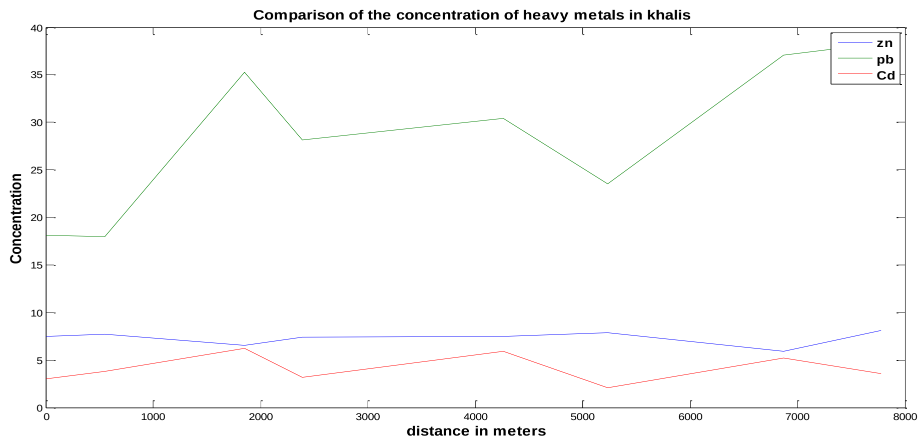


Figure 3.3: The concentration HM by suggested method in Khalis city

Now, our suggested method will be applied to determine the concentration of heavy metal in the soil of Baquba city. The Data and information on soil contaminants were collected from 20 zones located on different part for Baquba city. We tried to cover most areas of the city, with a focus on the type of each area as commercial, main roads, residential, industrial and agricultural; as illustrated in Figure (3.4- 3.6).

The 20 soil samples were collected in January 2017 with depth (0- 20 cm). Using iron shovel (the quantity of each sample was 1 kg), foreign materials such as plant leaves, debris etc. were isolated and removed from the collected soil samples then all the samples were put in plastic bags to measure the concentration of Cd, Co, Cr, Fe, Ni, Pb and Zn. The laboratory results illustrated in Table (3.3) which represent the initial data that will be used to get interpolation function which is substituted in the suggested formula.

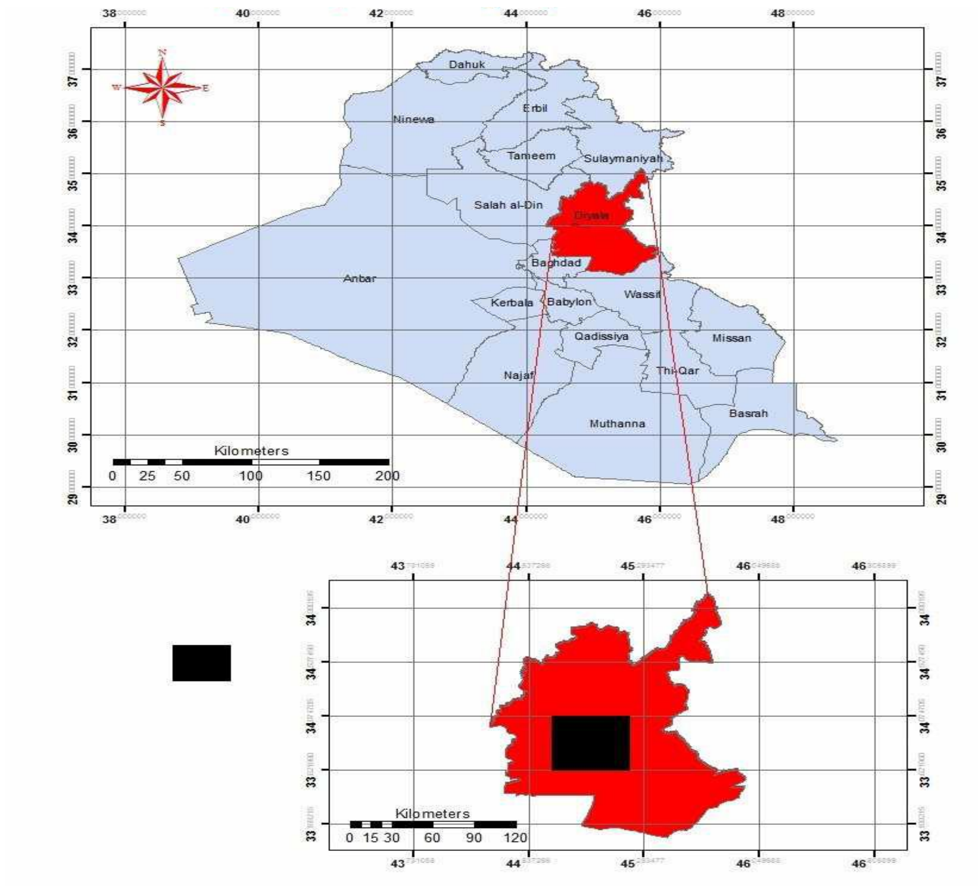


Figure 3.4: **Map of Iraq illustrate Diyala Governorate**

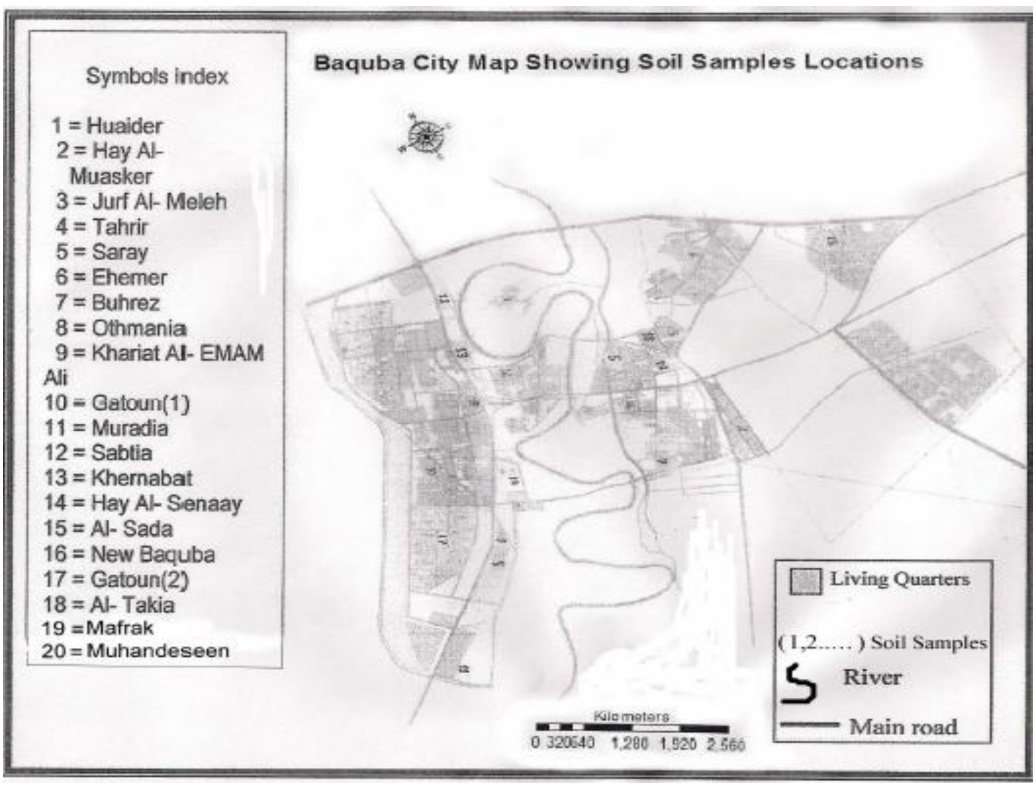


Figure 3.5: Study area and soil samples for Baquba city

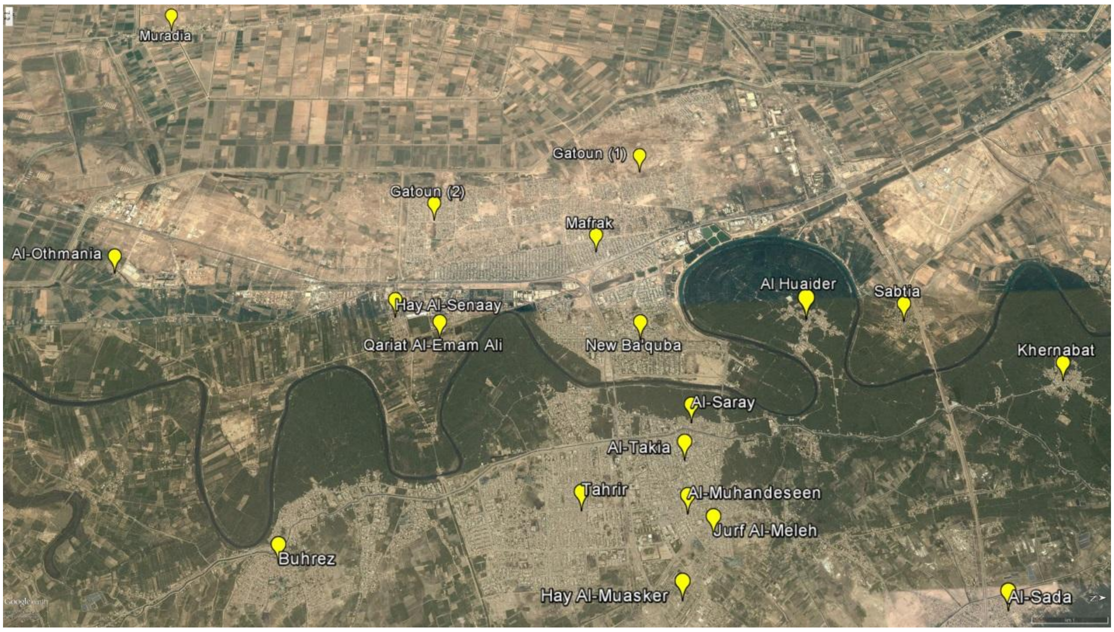


Figure 3.6: Location of samples in Baquba city

The data obtained from laboratory dissecting Table (3.2) represent the concentration of heavy metals for selected soils which compared with standard universal for concentration of heavy metals in soil depending on [41] to acknowledge the rate of contamination in soil in that time.

Then, the laboratory data are substituted in the suggested model in chapter two to get the concentration of heavy metals in those soil for any future times and any neighboring area. Thus, we can estimate the rate of contamination in that soil for any times and any neighboring area without laboratory dissecting. Figure (3.6), illustrates the results of interpolation by suggested method for each metal and Figure (3.7), illustrates the accuracy of suggested method by comparing between the results of suggested method, laboratory dissecting and standard universal. Figure (3.8) illustrates the concentrations of heavy metals in Baquba city.

Table 3.2: The concentrations of the studied heavy metals in the soil of Baquba city

No.	cities of Baquba	measures of heavy metals by unit PPm where S.M.S represent to standard universal													
		Cd (SMC=1)		Co (S.M.C=1)		Cr (SMC.=100)		Fe (S.M.C.=38000)		Zn (S.M.C.=70)		Ni (S.M.C.=50)		Pb (S.M.C.=50)	
		conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.
1	Al Huaider	2.5	1.5	14	13	127	27	16040	-21960	111	41	126	76	75	25
2	Hay Al-Muasker	1.8	0.8	12	11	119	19	24750	-13250	121	51	121	71	128	78
3	Jurf Al-Meleh	1.5	0.5	13.5	12.5	275	175	26600	-11400	96	26	58	8	17	-33
4	Tahrir	2	1	15	14	118	18	16500	-21500	98	28	126	76	45	-5
5	Al-Saray	1.3	0.3	11	10	146	46	17330	-20670	82	12	123	73	56	6
6	Al-Ehemer	1	0	12.2	11.2	101	1	21223	-16777	83	13	127	77	38	-12
7	Buhrez	2.7	1.7	12.5	11.5	105	5	23202	-14798	156	86	100	50	83	33
8	A-lOthmania	2.9	1.9	13	12	111	11	25801	-12199	122	52	118	68	91	41
9	Kariat Al-EMAM ALI	3	2	17	16	191	91	15304	-22696	104	34	115	65	48	-2
10	Gatoun (1)	2.1	1.1	13	12	229	129	24262	-13738	99	29	178	128	47	-3
11	Muradia	3.2	2.2	11	10	122	22	20998	-17002	86	16	134	84	85	35
12	Sabtia	2.2	1.2	16	15	115	15	21325	-16675	80	10	108	58	21	-29
13	Khernabat	2	1	14	13	112	12	16878	-21122	91	21	111	61	32	-18
14	Hay Al-Senaay	1.1	0.1	12	11	100	0	18543	-19457	83	13	98	48	24	-26
15	Al-Sada	2.3	1.3	15	14	96	-4	19664	-18336	94	24	103	53	15	-35
16	New Baquba	2.5	1.5	14.5	13.5	121	21	18774	-19226	97	27	93	43	33	-17
17	Gatoun (2)	2.6	1.6	13.3	12.3	133	33	23546	-14454	103	33	87	37	21	-29
18	Al-Takia	3.5	2.5	15.5	14.5	116	16	29885	-8115	90	20	83	33	42	-8
19	Mafrak	4.2	3.2	16	15	97	-3	27353	-10647	88	18	103	53	68	18
20	Al-Muhandeseen	3.6	2.6	15	14	124	24	22654	-15346	92	22	115	65	77	27

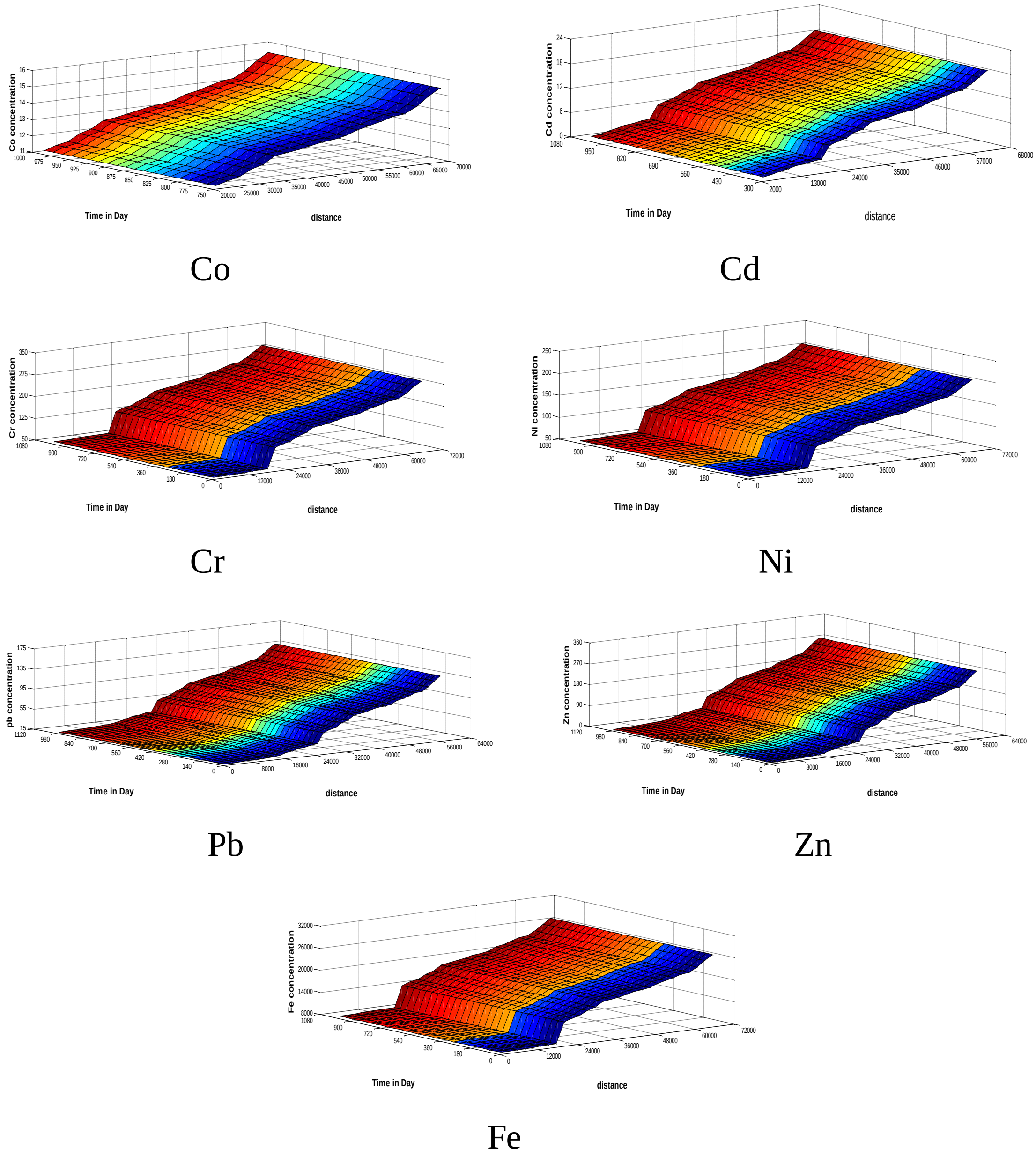


Figure 3.6: Concentration of HM measured by suggested method for Baquba Soils

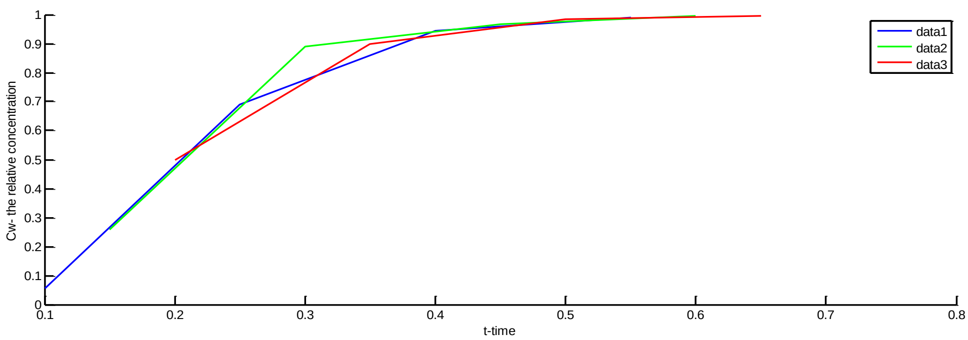


Figure 3.7: Comparison between standard universal (data1), suggested method (data2), and laboratory dissecting (data3)

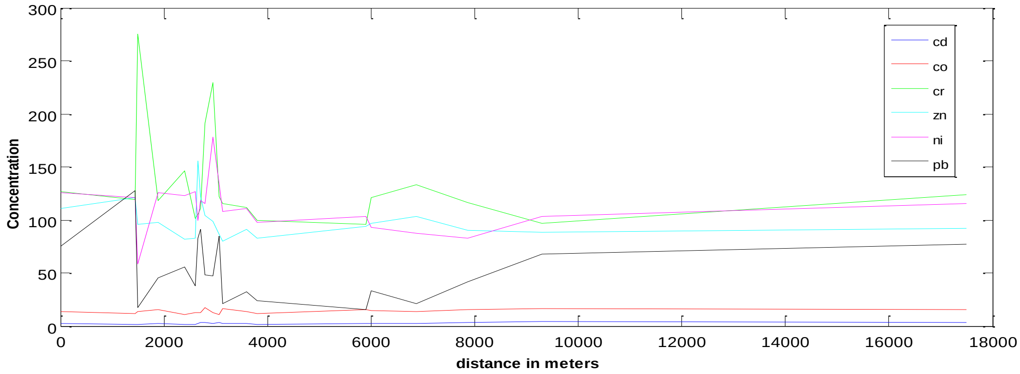


Figure 3.8: Concentrations of HM in Baquba soil

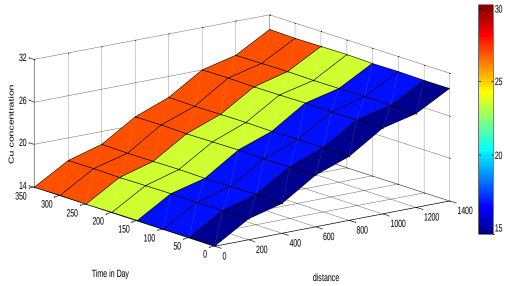
Finally we study the rate of contaminated in the Muasker Saad displaced people camp in Diyala Governorate; where the sampling was carried out in January 2017. The 8 soil samples were collected with depth (0- 10 cm) in the same manner used in the previous applications to measure the concentration of Cd, Cu, Cr, Fe, Ni, Cr, Pb and Zn.

Table (3.3) gives the laboratory results, which represents the initial data used in the suggested method to get the concentration of these heavy metals in any desire time and any neighboring area. We compared the results with standard universal for concentration of heavy metals in soil to determine the rate of contamination in that time. The results illustrated in Figure (3.7) for each heavy metals. Also, we measured pH for all samples then applied in the suggested method. The results illustrated in Figure (3.8).

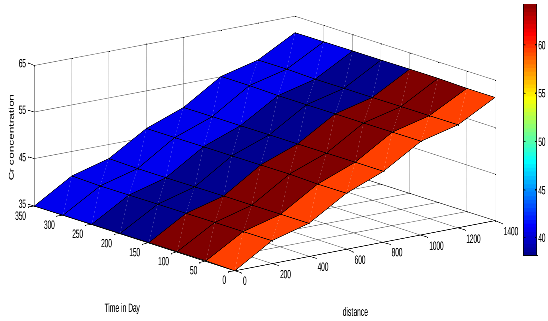
Figure (3.9) illustrates the accuracy of the suggested method by using a comparison between the results of the suggested method and laboratory dissecting for pH. Figure (3.10) illustrates the concentration of heavy metal in soil's for study area. Figure (3.11) illustrates the concentration of heavy metals by the suggested method for different years.

Table 3.3: Laboratory results & Standard Universal (S.M.C.) for concentration of HM in Muasker Saad with measure units PPm

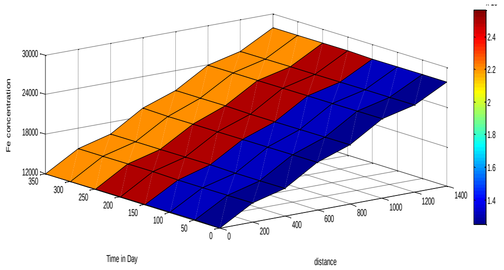
Samples	measures of heavy metals													
	PH	Pb (S.M.C.=50)		Ni (S.M.C.=50)		Zn (S.M.C.=70)		Fe (S.M.C.=38000)		Cr (SMC.=100)		Cu (S.M.C=20)		Cd (SM C=1)
		Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc.	Poll. Am.	Conc	Poll. Am.	Conc.	Poll. Am.	conc.
Sample 1	6	20	-30	64	14	124	54	12500	-25500	59.3	-40.7	14.6	-5.4	ND
Sample2	6.2	20.6	-29.4	64.2	14.2	126.5	56.5	13320	-24680	64.6	-35.4	16.6	-3.4	ND
Sample3	6.2	20.6	-29.4	64.2	14.2	126.5	56.5	13320	-24680	64.6	-35.4	16.6	-3.4	ND
Sample4	6.4	26.9	-23.1	96.6	46.6	113.2	43.2	24975	-13025	38.2	-61.8	23.6	3.6	ND
Sample5	6.4	26.9	-23.1	96.6	46.6	113.2	43.2	24975	-13025	38.2	-61.8	23.6	3.6	ND
Sample6	6.6	28.6	-21.4	104.9	54.9	126.5	56. 5	21978	-16022	40.9	-59.1	26.9	6.9	ND
Sample7	6.6	28.6	-21.4	104.9	54.9	126.5	56.5	21978	-16022	40.9	-59.1	26.9	6.9	ND
Sample8	6.6	30.5	-19.5	128	78	128	58	25650	-12350	45	55	30.3	10.3	ND



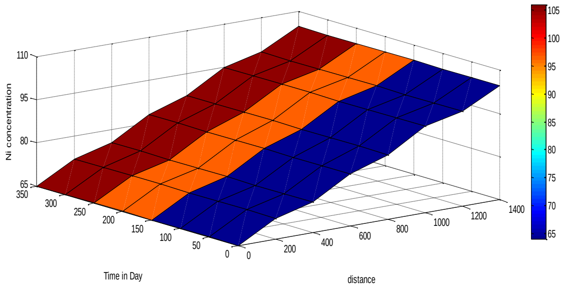
Cu



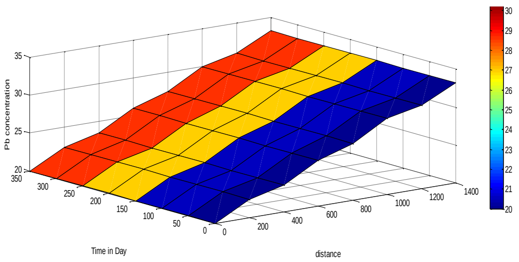
Cr



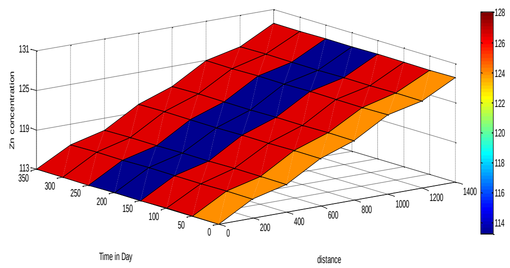
Fe



Ni



Pb



Zn

Figure 3.7: Concentration of HM by suggested method for muasker Saad displaced people camp

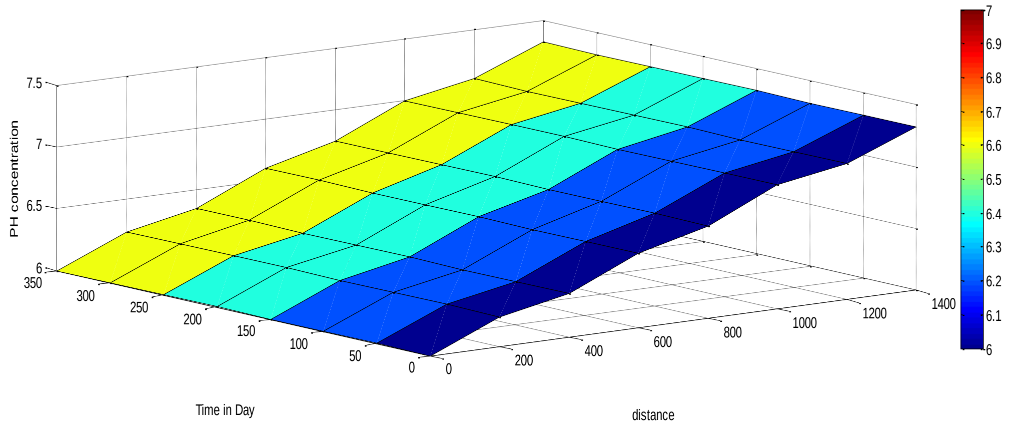


Figure 3.8: Results of suggested method for PH in muasker Saad displaced persons camp

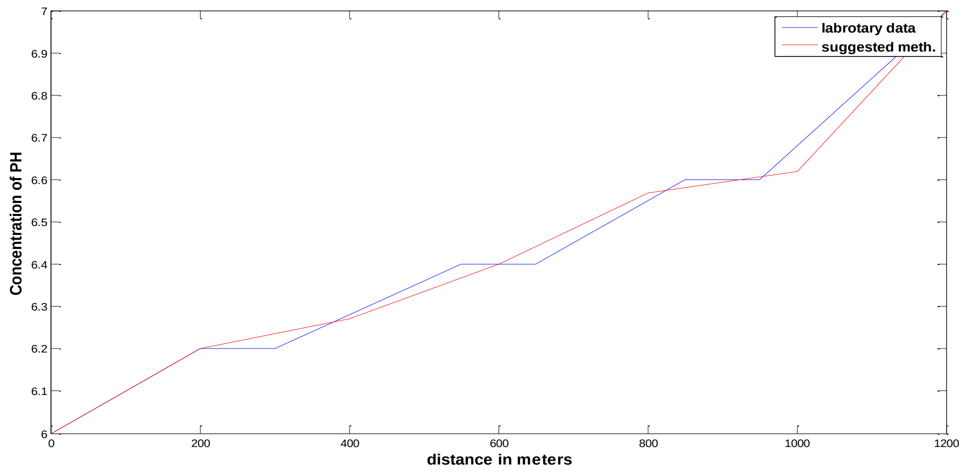


Figure 3.9: Comparison between suggested method and laboratory dissecting for pH

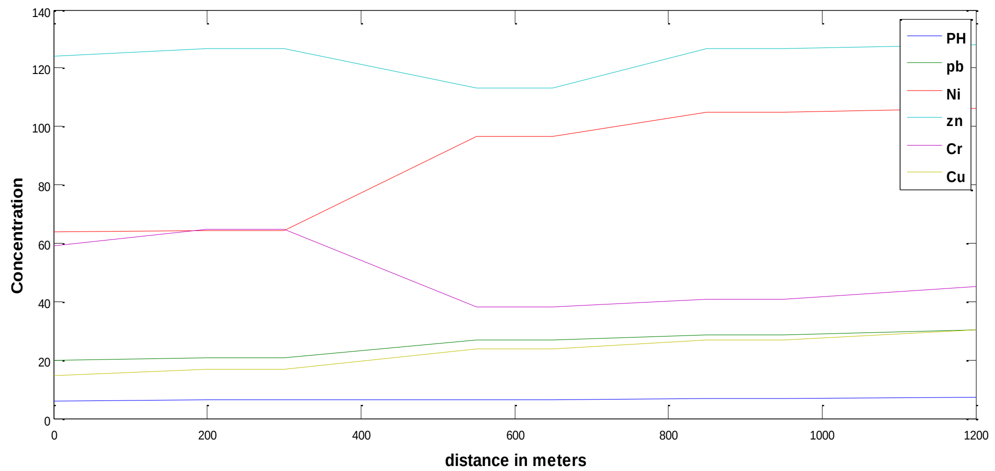


Figure 3.10: Concentration of HM in muasker Saad displaced persons camp

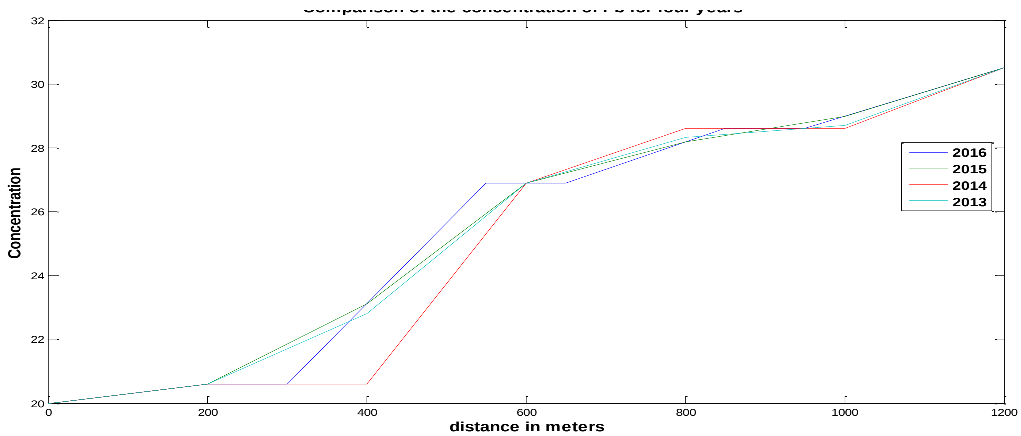


Figure 3.11: Concentration of HM by suggested method for different years

3.3. Results with Discussion

Now, we discuss the results for each heavy metal in Baquba city as follow.

Cadmium

The concentration of Cadmium (Cd) in the soil samples was ranging (1- 4.2ppm) with a mean (2.4 ppm), so within the ordinary range (5 ppm). Highest value of Cd is in the sample of (Mafrak zone, see Table 3.2). Increasing of Cd due to the concentration of plastic materials that can cause an increasing the concentration of Cd in the atmosphere and then accumulates on the soil.

Cobalt

The concentration of Cobalt (Co) in this study ranging (11- 17ppm) with mean (13.8ppm). The mean was within the range in the soil. The highest values were in the samples of (Kariat Al- Emam Ali , see Table 3.2).

Chromium

The concentration of Chromium (Cr) in the study area was ranging (96- 275 ppm) with a mean (133ppm). The lowest concentrations of Cr in the (Al- Sada , Mafrak and Hay Al- Senaay) and the other samples were over the international limits (see Table 3.2). The increasing of Cr in the Iraq soils due to the transported classics sediments which formed the stratigraphic column of Mesopotamian plain. These classics sediments come from the north and north- east of Iraq as a result of weathering processes in these areas. Weathering processes move with the Tigris River and its tributaries which are rich with Cr metals and Serpentine metals and Olivine as Cr (Cr_2O_4).

Iron

The concentration of Iron (Fe) in the present work was ranging (15304ppm) in the sample of (Kariat Al- Emam Ali) and (29885 ppm) in the sample of (Takia)), with a mean (21532 ppm), (see Table 3.2). All the values in the soil samples in this study were under their limits. The increasing of the concentration of Iron in the soil or exist the Iron within the limits in the soil depends on their concentration in the mother rocks that forming this soil.

Lead

The values of lead (Pb) in the soil samples ranging (15- 128ppm) with the mean (51ppm), (see Table 3.2). Concentration of lead in all soil samples was over the limits (10ppm). Highest concentration is in the sample of (Hay Al- Muasker) and the lowest concentration of lead in (Al-sada). Increasing the concentration of lead is due to different sources and the most probable source is fuel combustion in the automobiles which adding for the fuel of automobiles as tetraethyl lead. Studies of heavy metals in ecosystem have indicated that many areas near urban complexes or major road systems contain enormous high concentrations of the elements. In particular, soils in such regions have been contaminated from wide range of sources with Pb, Cd and other heavy metals.

Nickel

The value of nickel (Ni) is ranging (58- 178ppm), with mean (111ppm) in all soil samples and were exceed the limits in soil (40ppm), (see Table 3.2). The highest value of Ni in Hay Al- Muasker and the lowest value in Jurf Al- Meleh. It is known that untreated or not filtered emissions from most types of combustion and incineration will carry different trace metals.

Zinc

The value of Zinc (Zn) in the soil samples is ranging (80- 156ppm) with mean (99ppm) (see Table 3.2). All the values of Zn were above limits in the soil (70ppm). The highest one was in Buhrez while the lowest value in Sabtia. Zinc connecting by a strong positive relation with (Cd, Cr, Cu, Ni, and Pb), organic matter and the ratio of clays, this strong relation with these elements due to their origin from basic and ultra-basic igneous rocks and weathering of different sedimentary rocks and the influence of anthropogenic activities.

Now, we discuss the results of Alkhuyls.

From Table (3.1b) we see the average concentration of Pb is 49.65 PPm which is less than concentration Pb in the standard universal that attains 50 PPm.

The average concentration of Ni is 106.3 PPm which is higher than the concentration of Ni in the standard universal that attain 50 PPm (see Table 3.1b). The reason for the increased concentration of Ni in the soil is due to the increase in human's industrial activity as the combustion of fuel generates.

The average of concentration of Cr is 67.9 PPm which is less than its value in the standard universal that attains 100 PPm. As well as for the average concentration of Fe is 13709PPm less than the standard universal that attain 38000 PPm.

The average concentration of Zn is 124.9 PPm which is higher than in the standard universal that attain 70 PPm. The average concentration of Cu is 27.7 PPm which is exceeding the standard universal by 7.7 PPm.

Chapter Four

Error Estimation and Stability

4.1. Introduction

This chapter, illustrates how the suggested method is well-suited for describes model, by discussion the global error. This thesis, suggest a new modification of the error estimation which helps to reduce the computational time of classic estimation of error. It also helps to perform well for a given data and existed samples

4.2. Error Estimate and Default Weights

Every known software package scales either the maximum relative defect or relative error. The weights used to scale the maximum defect. In this thesis we modify this package to consist: interpolation simulink named "pythINSIM", which is defind as where; $C_l(x, t)$ is the value of concentration of heavy metals getting from laboratory inspecting, and $C_s(x, t)$ is the value of concentration of heavy metals getting from suggested method.

Also, two dimensions interpolation simulink named "pythTDINSIM" defind as

and multivariate interpolation simulink named "pythMDINSIM" which defind as

The application of these packages for the results of suggested method in Chapter three is given in Table (4.1) for Khalis city, Table (4.2) for Khuylis city, Table (4.3) for Baquba city and Table (4.4) for Muasker Saad displaced people camp.

Table 4.1: Maximum defect for Khalis city

Heavy metals	pythINSIM
Cr	0.00483
Cu	0.0009
Fe	0.00204
Ni	0.00183
Pb	0.00087
Zn	0.00012
pH	0.0074

Table 4.2: Maximum defect for Khuylis city

Heavy metals	pythINSIM
Cr	0.00483
Cu	0.0009
Fe	0.00204
Ni	0.00183
Pb	0.00087
Zn	0.00012
pH	0.00074

Table 4.3: Maximum defect for Baquba city

Heavy metals	pythINSIM
Cd	0.01319
Co	0.02833
Cr	0.01373
Fe	0.01916
Zn	0.04222
Pb	0.02303
Ni	0.01804

Table 4.4: Maximum defect for Muasker Saad displaced persons camp.

Heavy metals	pythINSIM
Cr	0.00064
Cu	0.00166
Fe	0.00184
Ni	0.00121
Pb	0.00086
Zn	0.00039
pH	0.00163

4.3. Global Error Estimation

The global error can be estimated by using the software packages COLSYS and COLNEW to assess the accuracy of the numerical results. There are many different algorithms that can be used to estimate the global error effectively; most of them perform four algorithms: Richardson extrapolation, higher-order formulae, deferred corrections, and a conditioning constant. The first and second algorithms are modified and described in next subsections.

4.3.1. Richardson Extrapolation

Many software packages use Richardson extrapolation to estimate the global error [1]. This algorithm starts with a discrete numerical results for a given mesh C_s. Next, the software determines a more accurate numerical result $C_{s/2}$ by halving all subintervals of the original mesh. Then, an estimate of the norm for the global error, e_{RE}, is given by

$$e_{RE} = \left\| \left(\frac{2^n}{2^n - 1}\right) \left(C_s - C_{s/2}\right) \right\|_\infty,$$

where n is the number of data in the sample.

The application of this package for the results of suggested method will be given in Table (4.5) Khalis city, Table (4.6) for Khuylis city, Table (4.7) for Baquba city, and Table (4.8) for Muasker Saad displaced people camp.

4.3.2. Higher Order Formula

Higher order formula can be used to determine a more accurate numerical result with the same mesh for the original result, specifically, the global error can be estimated by

$$e_{HO} = \| C_s - C_l \|_{\infty} .$$

The application of this package for the results of suggested method will be given in Table (4.5) for Khalis city, Table (4.6) for Khuylis city, Table (4.7) for Baquba city and Table (4.8) for Muasker Saad displaced people camp.

Table 4.5: Estimation of global error for Khalis city

Heavy metals	e_{ER}	e_{HO}
Cr	0.42633	0.423
Cu	0.03528	0.035
Fe	3.426772	3.40
Ni	0.22476	0.223
Pb	0.08265	0.082
Zn	0.03024	0.03
pH	0.0615	0.061

Table 4.6: Estimation of global error for Khuylis city

Heavy metals	e_{ER}	e_{HO}
Cr	0.42633	0.423
Cu	0.03528	0.035
Fe	3.426772	30.40
Ni	0.22476	0.223
Pb	0.08265	0.082
Zn	0.03024	0.03
pH	0.0615	0.061

Table 4.7: Estimation of global error for Baquba city

Heavy metals	e_{RE}	e_{HO}
Cd	0.0686001	0.32
Co	0.5100009	0.51
Cr	0.3790007	0.379
Fe	57.260109	57.26
Zn	0.662801	0.6628
Pb	0.2971006	0.2971
Ni	0.320300062	0.323

Table 4.8: Estimation of global error for Muasker Saad displaced persons camp.

Heavy metals	e_{RE}	e_{HO}
Cr	0.04233	0.042
Cu	0.05241	0.052
Fe	472.7165	472
Ni	0.13102	0.13
Pb	0.02721	0.027
Zn	0.00312	0.0031
pH	0.21165	0.21

Chapter Five

Conclusions and Future Work

5.1. Conclusions

This Thesis demonstrates a numerical model based on interpolation depending on divided difference method. The method was generalized in multi- dimensions then modified to simplify the implementation. The application of the suggested method is to evaluate the contaminated soil by heavy metals for any distance and time are introduced to illustrate the importance, efficiency, and accuracy of suggested method. Thus, the results provided many of the features, such as

- The suggested modification for interpolation in multi dimensions is more efficient, easy implemented and rapid compared with the other methods, also it can be considered to be a good representation of that application.
- New approaches to interpolation of multivariable is proposed by generalized one variable divided difference method then developing it to increase the accuracy of results.

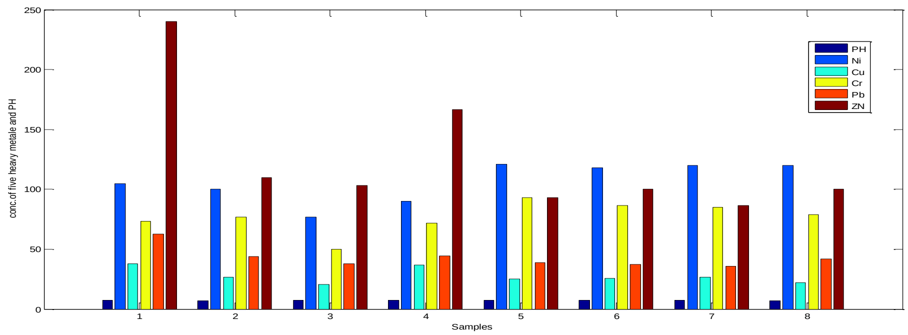
- The results which obtained from the present work show that**:**

1. The average of the concentrations of heavy metals in soil for any time and zones in Diyala governorate are increase with the time, posing a great risk to the environment contamination.

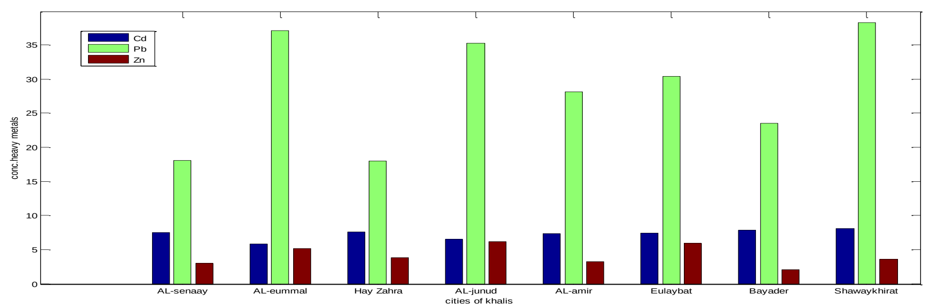
2. There are different causes for increasing the concentrations of heavy metals in soil such as: the big traffic jams resulting from the great number of cars lately which use gasoline that contains a lot of fourth lead Ethylene which cause big problems to the environment. This creates dangers to human beings. In addition, the increase in the amount of litter and how to get rid of industry waste in sewerage and the decrease in the green region which participate in lessening the damage of heavy metals on the environment.

3. As a result of the increasing in the population during the late years which results in converting the regions of vegetation to residential regions. The technological development also causes contamination because of the prolife ration of plants and workshops scattered everywhere.

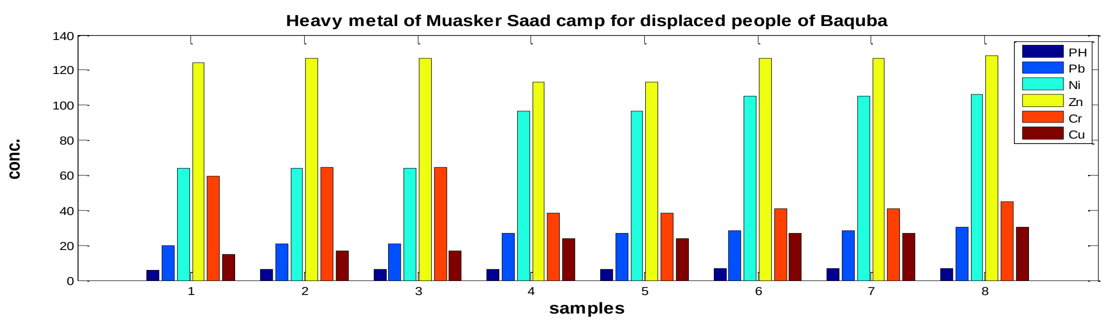
4. To illustrate the rate of contamination in the Diyala governorate soil, see Figure (5.1).



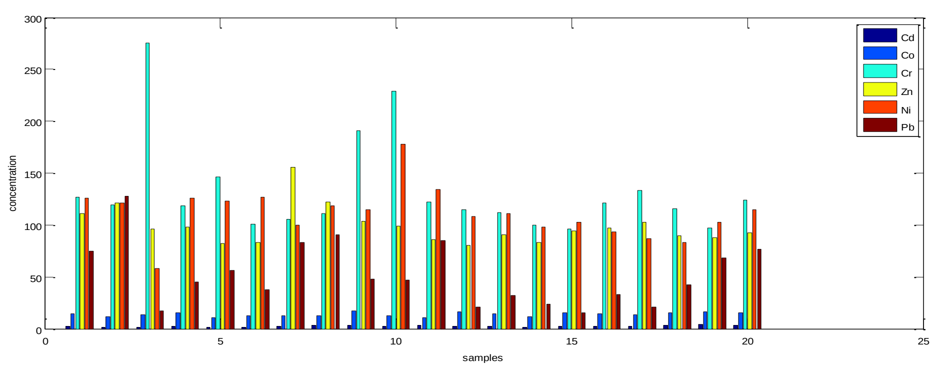
(a) Alkhuyls



(b) Al Khalis city



(c) muasker Saad displaced persons camp



(d) Baquba city

Figure 5.1: Concentrations of heavy metals in the soils of the different zones in Diyala governorate

- Advantages of the suggested modification is:

1. Numerically stable: since it has nice property such:

- Directly uses the given data

- Not need to represent the polynomial in the basis 1, t, t^2, ……

2. Fast and easy implementation

3. Simple structure since it has parallel property and strip off first and last indices

4. Can be update easily.

- We introduce suggested method as replacement of traditional methods, for estimating the concentration of heavy metals in any times, since its less cost, rapid and practical.

5.2 Future Works

The studies for future investigations are suggested below:

1- Develop a model equation that describes other types of soil.

2- The researchers can use other numerical methods as most suitable technique in applications and data processing.

3- Study the error analysis in details and modify the error estimation for multivariable model.

4- Study the effect of atmospheric pressure and winds, as variables when counting the concentrations on the surface of soil.

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Appendix

Contamination

This appendix present a brief overview of environmental contamination and soil contamination in particular of heavy metals, which is at the core of the thesis.

1. Environment Contamination

A contamination can be defined generally as: the presence of any material in the environment in quantities that results directly or indirectly, Alone or reacting with other materials, harmful effects on health of organisms.

Contamination is a major problem faced by modern man, and even more dangerous. It needs to combine all efforts to address and reduce them. The problem is compounded by the fact that human beings have the obvious role of increasing their seriousness through their various activities that have become threatening to human life. As well as their impact on other living organisms, which causes a change in the natural balance of the environment and its various components, both living and non-living.

Contamination is divided into three main types: air Contamination, water contamination, soil contamination. These include, however, two other types of contamination that are classified as separate variables: noise contamination and radiation contamination. In the end, it can be said that any of the main types of contamination cannot be separated from the other two types, but that one of them leads to the other**.**

2. Soil Contamination

Soil contamination: The presence of pollutants in the soil in quantities that allow directly or indirectly damage to the public health of the human being, destruction of ecosystems, or influence on plant environments or animal, or impact on surface or groundwater. The most important soil pollutants are: minerals Heavy, radioactive materials, pesticides, solid waste and sewage.

We will only discuss soil pollution with heavy metals because this is the subject of the thesis.

3. Heavy Metals [25]

Means all heavy metals that increase density 5 g/cm^3, and at density less it is called light metals. Some of these metals play an important role in the lives of the living and have different biological effects. For example the iron well-known enzymes in the blood and the installation of importance are all of the elements manganese, zinc, and copper enzymatic catalysts. However, these metals are toxic and dangerous to be in certain concentrations. Adding to that the seriousness of these metals, it is not possible to be analyzed by bacteria and other natural processes as well as the authenticity of which enable them to spread over long distances for ton or sources sites. Perhaps the most dangerous thing is due to susceptibility to each bio-accumulate in the tissues and organs of living organisms in the environment water or land. In addition, some heavy metals serve as radioactive isotopes, therefore, these metals will be charged double the risk to the environment in terms of being toxic and radioactive at the same time, as is the case in 65 of radioactive zinc, uranium 235.

Heavy metals such as lead, cadmium, zinc, mercury, arsenic and others have a significant impact on Soil, and some other metals such as chromium, copper, iron and others have limited impact on soil most often confined to their places of deployment. Human toxicity occurs with heavy metals by directing entry with air, water or food to the body as biochemical compounds, or their accumulation in the human body Through low concentrations over long periods of time (occupational poisoning), or by accidental entry with concentration Very high exceeds the concentrations permitted in specifications.

4. Harms of Heavy Metals

Now, we introduce the harms of each heavy metal

4.1. The Harms of Copper (Cu)

To lead a healthy life is wonderful but is not an easy matter achieve. For example irritation of the eyes, nose or mouth can be cussed by constant exposure to Cu. This exposure may cause headaches, stomach-ache, dizziness, vomiting and diarrhea. It is very possible that the high take-up of copper cause liver and kidney damage and even in certain cases cause death. Be that as it may, regardless of whether copper is cancer-causing, i.e causing tumor has not been resolved yet. Metal smoke fever with atrophy's changes in mucous membrane films in the nose might be made modern introduction copper tidies, fogs or exhaust. Interminable Cu harming brings about Wilson's Disease, described by a hepatic cirrhosis, mind harm, demoralization, renal sickness, and Cu affidavit in the cornea.

4.2. The Harms of Lead (Pb)

The numerous applications of lead have been for a long time. This is so because lead is a soft metal. In fact, people have widely used lead since 5000 BC in metal alloys, pipelines and cables. They have also used it paints and pesticides. Leads lead along with other harmful metal effects, directly affecting human health. It might enter the human body through take-up of air (15%), water (20%) and nourishment (65%).

Air such as dust, fume or cigarette smoke, e.g. soft drink and win, food, i.e. vegetables, fruit, meat, seafood and grains may contain significant amounts of Pb to varying degrees. For as far as it is known lead can only do harm to human body, It has no essential function in the human body.

Several un favorite serious can be caused by lead such as:

- Behavioral disruption of children, such as aggression, impulsive behavior and hyperactivity.
- Brain damage.
- Diminished learning abilities of children.
- Declined fertility of men through sperm damage.
- Miscarriages and subtle abortions.
- A rise in blood pressure.
- Disruption of nervous systems.
- Disruption of the biosynthesis of hemoglobin and anemia.
- Kidney damage.

And Pb can enter a fetus through the placenta of the mother. Because of this it can cause serious damage to the nervous system and the brains of unborn children.

4.3. The Harms of Nickel (Ni)

Nickel is a metal that occurs in the environment only at very low levels. Ni is used for many different purpose and application by people. One of the applications it is use as an ingredient of steel and other metal products. Ni is found in many popular metal products such as jewelry. Naturally, small amounts of Ni can be traced in foodstuffs. Fats and chocolate are known to contain high quantities. The rate of Ni taken into our body increases when people eat large quantities of vegetables from contaminated soils. Plants are known to accumulate Ni and as result the Ni uptake from vegetables will be eminent. Smokers have a higher Ni uptake through their lungs. Finally, we can find it in detergents. Like copper human beings may be exposed to Ni by drinking water, breathing air, smoking cigarettes or eating food. Skin contact with Ni-contaminated soil or water may also result in exposure. However Ni is essential when the intake is small, but when the uptake is too high it cause a danger to human health.

When Ni is taken in great quantities, it may have the following consequences:

- Birth defects.
- Higher chances of development of lung cancer, larynx cancer, nose cancer and prostate cancer.
- Heart disorders
- Sickness and dizziness after exposure to Ni gas.
- Asthma and chronic bronchitis.
- Lung embolism.
- Allergic reactions such as skin rashes, mainly from jewelry.
- Respiratory failure.

4.4. The Harms of Cadmium (Cd)

Food is the major source of cadmium uptake by human beings. Foodstuffs that are rich in Cd can greatly increase the concentration of Cd in human bodies. For example of such foodstuffs are liver, shellfish, mushrooms, dried, mussels seaweed and cocoa powder. When people smoke, they expose themselves to significantly high level of Cd intake because tobacco smoke transports cadmium into the lungs. From lungs to blood and afterward from blood to whatever remains of the body where it can expand impacts by probability Cd that is as of now introduce from Cd – rich nourishment. Other high rates of exposures can happen when individuals who live close dangerous waste destinations or industrial facilities that discharge Cd into the air and when they work in the metal refinery industry. When individuals take in Cd, it can seriously harm the lungs. This may even cause death. The principal transported of Cd is to the liver through the blood. There it is bound to proteins to form edifices that are transported to the kidneys. It aggregates in kidneys, where it harms sifting systems while separating components neglect to work. This causes the discharge of basic proteins and sugars from the body and further kidney harm. This procedure takes quite a while before Cd that has collected in kidneys is discharged from a human body.

Different impacts on health that can be caused by Cd are:

- Bone fracture.
- Possibly DNA damage or cancer development.
- Diarrhea, stomach pains and severe vomiting.
- Damage to the immune system.
- Reproductive failure and possibly even infertility.
- Reproductive disorders.

4.5. The Harms of Cobalt (Co)

Like most of the metals that affect human health, Co is broadly scattered in nature, human, might be presented to it by drinking water, breathing air and eating sustenance that contains Co. When the skin is in contact with contaminated water or soil with Co, this may also enhance exposure end in turn our health is affected. We can't discover Co is uninhibitedly accessible in nature. However, Co particles are will undoubtedly soil or deposit particles, they take-up by plants and creatures is higher and aggregation in plants and creatures may happen. Humans can benefit from Co because it is a part of vitamin B12, which is important for the health of humans. It is used to treat anemia with pregnant women, because it stimulates the production of red blood cells. The total daily intake of Co is variable and may be as much as 1mg, but almost all will pass through the body unabsorbed, except that in vitamin B12. However, human health may be highly damaged by too high concentrations of Co. When people breathe Co in too high concentrations through air, they experience lung effects, such as pneumonia and asthma. This mainly occurs with people who work with Co.

The effects Co on health that are a result of the uptake of high concentrations are:

- Thyroid damage.
- Heart problems.
- Vision problems.
- Vomiting and nausea.

It may be also effect on health they caused by radiation of radioactive Co isotopes. This can cause vomiting diarrhea, hair loss, sterility, coma, bleeding and even death. This radiation is sometimes used with cancer patients to destroy tumors. These patients also suffer from loss hair, diarrhea, and vomiting.

4.6. The Harms of Zinc (Zn)

Zinc is a metal which is essential for human health. When people do not absorb enough amount of Zn, they can experience a loss of appetite, decreased sense of smell and taste, slow wound healing and skin sores. Birth defects can be caused by Zn- shortages. Although, humans can handle proportionally large concentrations of Zn can still cause eminent health problems, such as skin irritations, stomach cramps, vomiting, anemia and nausea. Excessive amount of Zn can damage the pancreas and disturb the protein metabolism, and cause arteriosclerosis. Too much exposure to Zinc chloride can cause respiratory disorders. In the work place environment Zn contagion can lead to a flu-like condition known as metal fever. This condition will pass after two days and it is caused by over sensitivity. Zn can be dangerous to unborn or newborn children. When their mothers have absorbed large concentrations of Zn the children may be exposed to through blood or milk of their mothers.

5. Sources of Heavy Metals

Excess heavy metals in the soil originate from many source, while include atmospheric deposition, sewage irrigation, improper stacking of industrial solid waste, mining activities, the use of pesticides and fertilizers (Zhang et al., 2011), etc. Table 1 shows various sources of heavy metals contaminating soil in the world (Qin et al., 2008).

Table 1:Different sources of heavy metals contaminating soils annually in the world (1000t*a^{-1})

sources	As	Cd	Cr	Cu	Hg	Ni	Pb	Zn
Agriculture and food waste	0~ 0.6	0~0.3	4.5~90	3~38	0~1.5	6~45	1.5~27	12~150
Farmyard manure	1.2~4.4	0.2~1.2	10~60	14~80	0~0.2	3~36	3.2~20	150~320
Logging and timber Industry wastes	0~3.3	0~2.2	2.2~18	3.3~52	0~2.2	2.2~23	6.6~82	13~65
Municipal wastes	0.09~0.7	0.88~7.5	6.6~33	13~40	0~0.26	2.2~10	18~62	22~97
Municipal sludge	0.01~0.24	0.02~0.34	1.4~11	4.9~21	0.01~0.8	5.0~22	2.8~9.7	18~57
Organic wastes	0~0.25	0~0.01	0.1~0.48	0.04~0.61		0.17~3.2	0.02~1.6	0.13~2.1
Metal processing solid wastes	0.01~0.21	0~0.08	0.65~2.4	0.95~7.6	0~0.08	0.84~2.5	4.1~11	2.7~19
Coal ash	6.7~37	1.5~13	149~446	93~335	0.37~4.8	56~269	45~242	112~484
Fertilizer	0~0.02	0.03~0.25	0.03~0.38	0.05~0.58		0.20~3.5	0.42~2.3	0.25~1.1
Marl	0.04~0.5	0~0.11	0.04~0.19	0.15~2.0	0~0.02	0.22~3.5	0.45~2.6	0.15~3.5
Commodity impurities	36~41	0.78~1.6	305~610	395~790	0.55~0.82	6.5~32	195~390	310~620
Atmospheric deposition	8.4~18	2.2~8.4	5.1~38	14~36	0.63~4.3	11~37	202~263	49~135
Total	52~112	5.6~38	484~1309	541~1367	1.6~15	106~544	479~1113	689~2054

Table 2: The content of heavy metals in urban soils (mg/kg^{-1})

City/country	Cr	Cu	Pb	Zn	Ni	Cd	Reference
Beijing	35.60	23.70	28.60	65.60	27.80	0.15	Zheng et al.,2008
Guangzhou	-	62.57	108.55	169.24	25.67	0.50	Lu et al. 7007
Shanghai	107.90	59.25	70.69	301.40	31.14	0.52	Shi et al., 2008
Changsha	121.00	51.41	89.40	276.00	-	6.90	Xi et al., 2008
Hong Kong	23.10	23.30	94.60	125.00	12.40	0.62	Li et al., 2004
Qingdao	54.00	55.00	62.22	201.00	17.30	0.30	Yao et al., 2008
Baoji	102.40	112.14	25380.55	1964.14	72.10	-	Li and Huang, 2007
Luoyang	71.42	85.40	65.92	215.75	-	1.71	Lu et al., 2007
Wenzhou	-	34.59	65.22	169.40	-	-	Chen et al., 2007
Nanjing	84.70	66.10	107.30	162.60	-	-	Lu et al., 2003
Cincinnati	37.00	26.00	41.00	60.00	19.00	-	Turer et al., 2001
Syria	57.00	34.00	17.00	103.00	39.00	-	Moller ei al., 2005
France	42.08	20.06	43.14	43.14	14.47	0.53	Hernandez et al., 2003
Spain	-	57.01	1505.45	596.09	-	3.76	Rodrguez et al., 2009
Iran	63.79	60.15	46.59	94.09	37.53	1.53	Sayadi and Rezaei, 2014
Turku,Finland	59.00	23.00	17.00	90.00	24.10	0.17	Salonen and Korkka-Niemi, 2007
Range	23.10~121	20.06~112.14	17~25380.55	60~1964.12	12.40~72.10	0.15~6.90	
Average	66.08	49.60	1733.94	289.78	29.14	1.52	
Background	61.00	22.60	26.00	100	26.90	0.10	CEPA 1990
Environ. capacity	200.00	100.00	300.00	250	50.00	0.30	CEPA, 1995

Table 3: The content of heavy metals in the agricultural soil (mg/ kg^{-1})

City/Countary	Cr	Cu	Zn	Ni	Cd	Hg	As	Pb	Reference
Beijing	75.74	28.05	81.10	-	0.18	-	-	18.48	Liu et al., 2009
Guangzhou	64.65	24.00	162.60	-	0.28	0.73	10.90	58.00	Li et al., 2009
Yangzhou	77.20	33.90	98.10	38.50	0.30	0.20	10.20	35.70	Huang et al., 2007
Wuxi	58.60	40.40	112.90	-	0.14	0.16	14.30	46.70	Zhao et al., 2007
Chengdu	59.50	42.52	227.00	-	0.36	0.31	11.27	77.27	Liu et al., 2006
Kunshan	87.73	34.27	105.93	31.08	0.20	0.20	8.15	30.48	Chen and Pu, 2007
Xuzhou	-	35.28	149.68	-	2.57	-	-	56.20	Liu et al., 2006
Changde	-	-	-	-	-	-	92.7	-	PSTV, 2014
Spain	63.48	107.65	427.80	34.75	1.42	-	-	213.93	Zimakowska-Gnoinska et al., 2000
America	-	95.00	-	57.00	0.78	-	-	23.00	Han et al., 2002
Korea	-	2.98	4.78	-	0.12	0.05	0.78	5.25	Kim and Kim, 1999
Slovakia	-	65.00	140.00	29.00	-	-	-	139.00	Wilcke, 2005
USA	48.5	48	88.5	29	13.5	-	-	55	Jean- Philippe etal., 2012
India	2.19	1.20	28.24	4.34	0.82	-	-	0.95	Raju et al., 2013
India	1.23	2.62	4.65	0.14	0.05	-	-	2.82	Prajapati and Meravi, 2014
Iran	10.36	9.62	11.56	11,28	0.43	-	-	5.17	Sayyed and Sayadi, 2011
Iran	11.15	-	-	-	-	-	-	-	Zojaji et al., 2014
Range	1.23~87.73	1.20~107.65	4.65~427.8	0.14~57	0.05~ 13.50	0.05~ 0.73	0.78~ 92.7	0.95~ 213.93	
Average	46.69	38.03	117.35	26.12	1.50	0.28	21.19	51.19	
Background	61	22.6	74.2	26.9	0.097	0.065	11.2	26	CEPA 1995
Environ. capacity	200	100	250	50	0.3	0.3	30	300	Zheng et al.,2008

Remediation of heavy metals contamination soils

1. **Engineering remediation**

   Engineering remediation refers to using physical or chemical methods to control heavy metal contamination of soils.

   1.1 Replacement of contaminated soil, soil removal and soil isolation.

   1.2 Electrokinetic remediation

   1.3 Soil leaching .

   1.4 Adsorption

   1.5 Other methods

   Other engineering methods include washing and compounding , heat treatment, physical solidification, chemical improvers, chemical curing lamp remediation, etc.

2. **Bioremediation**

   2.1 Phytoremediation

   2.2 Microbial remediation

المُستخلص

في هذه الرسالة تم اقتراح نهج مطوراً للطريقة العددية استندت على نظرية الاندراج للدالة ذات المتغيرين حيث قمنا بتطوير طريقة الفروقات المقسمة متعددة المتغيرات بالإضافة إلى ذلك، تم تحدي الصيغ الصريحة من خلال ربط معاملات الاندراج الكلاسيكي للفروق المقسمة ذو المتغير الواحد مع معاملات الاندراج المتعدد المتغيرات.

لتوضيح دقة وكفاءة الطريقة المقترحة استخدمت لتخمين نسبة تلوث التربة بالمعادن الثقيلة بمعنى تخمين تركيز المعادن الثقيلة في تربة محافظة ديالى . على وجه الخصوص، يتم تطبيق طرق الاندراج على نطاق واسع في نماذج التي تصف الظواهر المختلفة ذات البيانات التجريبية التي يجب استخدام الدراسات الحاسوبية فيها والتي تقتضي وصف لتلك البيانات و عليه اقترحنا طريقة مناسبة لتخمين معدل التلوث.

طريقة الفروق المقسمة تم توسيعها لتشمل بعدين و اقترح تطويرها ثم طبقت لتخمين تأثير مخيمات النازحين في محافظة ديالى على زيادة تلوث التربة بالمعادن الثقيلة .

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة بغداد
كلية التربية للعلوم الصرفة /ابن الهيثم

نموذج رياضي لتخمين تلوث التربة بالمعادن الثقيلة في محافظة ديالى

رسالة

مقدمة إلى كلية التربية للعلوم الصرفة / ابن الهيثم – جامعة بغداد

وهي جزء من متطلبات نيل شهادة ماجستير علوم

في الرياضيات

من قبل

اسراء نجم عبود

بإشراف

أ. د. لمى ناجي محمد توفيق

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