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Comparison Different Estimation Methods for System Reliability in Stress-Strength Models

A Thesis

Submitted to Council of College of Education for Pure Science (Ibn-AL-Haitham)/ University of Baghdad in partial Fulfillment of the Requirements for the Degree of Master of Science In Mathematic

By

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بسم الله الرحمن الرحيم

﴿وَقُلِ اعْمَلُوا فَسَيَرَى اللَّهُ عَمَلَكُمْ وَرَسُولُهُ وَالْمُؤْمِنُونَ ۖ
وَسَتُرَدُّونَ إِلَىٰ عَالِمِ الْغَيْبِ وَالشَّهَادَةِ فَيُنَبِّئُكُم بِمَا
كُنتُمْ تَعْمَلُونَ﴾

صدق الله العظيم

سوره التوبة- 105

Supervisor Certification

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We certify that we have read this thesis entitled " **Comparison Different Estimation Methods for System Reliability in Stress-Strength Models** "and as examining committee examined the student (**Bareq Baqe Selman**)in its contains and what is connected with it, and that in our opinion it is adequate for the partial fulfillment of the requirement for the degree of Master of Science in Mathematics.

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AUTHOR'S PUBLICATIONS

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3-Alaa.M.Hamad, Bareq Baqe Selman "Estimation the Shape Parameter for Power Function Distribution" The Journal of the Indian Mathematical Society.(Accepted).

DEDICATION

TO

MY FATHER AND MY MOTHER , MAY GOD HAVE MERCY ON THEM….

TO ALL FREE MARTYRS OF IRAQ WHO SACRIFICED FOR OUR BELOVED COUNTRY ….

TO MY BROTHERS AND SISTERS WITH THEIR FAMILIES….

TO ALL MY FRIENDS…..

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I GIVE MY THANKS AND PRIDE TO ALL TEACHERS DEPARTMENT OF MATHEMATICS AT THE UNIVERSITY OF BAGHDAD COLLEGE OF EDUCATION IBN-AL-HAITHAM ....

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BAREQ

ABSTRACT

This thesis concerned with finding the mathematical formula for the reliability of system content one component which has a strength X subject to stress Y [R=P(y<x)] and for a system content two series components have a strength X_1, X_2 subject to stress Y [R_S=P(y<min x_1, x_2)] in stress- strength model (S-S) also has been formulated, when the stress and strength are independent random variables, studying of three distributions, Power function distribution, modified Exponentiated Lomax distribution and Power Lomax distribution, In addition including the estimation of two models of system reliability R and R_S using different estimation methods namely, Maximum likelihood method (MLE), Moment method (MOM), Least square method (Ls) and Shrinkage (shrinkage weight function (sh1), constant shrinkage factor (sh2) and beta shrinkage factor (sh3)), and involving a comparison between the previous methods for three distributions using Monte-Carlo simulation depend on mean square error (MSE) criteria.

Through the Monte-Carlo simulation, it can be noted that, the shrinkage method using constant shrinkage factor (sh2) can be considered as a best method in all cases for all distributions, than according to the outcome results.

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LIST OF SYMBOLS AND ABBREVIATIONS

The symbols	The meaning
α, λ	The shape parameter, The scale parameter
Pow	Power function distribution
MELD	Modified exponentiated Lomax distribution
PoLo	Power Lomax distribution
Pdf	Probability density function
Cdf	Cumulative distribution function
R	Reliability function
h(x)	Hazard function
H(x)	Cumulative hazard function
$E(x^r)$	r^{th} moment
E(x)	1^{st} moment
$E(x^2)$	2^{nd} moment
var(x)	Variance
$M_t(x)$	Moment generating function
c.v(x)	Coefficient of variation
x_{med}	Median

n.m	Samples size
$\hat{\alpha}$	Estimator the shape parameter
MLE	Maximum likelihood method
MOM	Moment method
Sh	Shrinkage method
Sh_1	Shrinkage weight function
Sh_2	Constant shrinkage factor
Sh_3	Beta shrinkage factor
Ls	Least square method
(S-S)	Stress-strength model
$\hat{R}$	Estimate the reliability
$\hat{\alpha}_{mle}$	Maximum likelihood estimator the shape parameter α
$\hat{\alpha}_{mom}$	Moment estimator the shape parameter α
$\hat{\alpha}_{sh1}$	Shrinkage weight function estimator the shape parameter α
$\hat{\alpha}_{sh2}$	Constant shrinkage factor estimator the shape parameter α
$\hat{\alpha}_{sh3}$	Beta shrinkage factor estimator the shape parameter α

$\hat{\alpha}_{Ls}$	Least square estimator of the shape parameter α
$\hat{R}_{mle}$	Maximum likelihood estimator with one component of R
$\hat{R}_{mom}$	Moment estimator with one component of R
$\hat{R}_{sh1}$	Shrinkage weight function estimator with one component of R
$\hat{R}_{sh2}$	Constant shrinkage factor estimator with one component of R
$\hat{R}_{sh3}$	Beta shrinkage factor estimator with one component of R
$\hat{R}_{Ls}$	Least square estimator with one component of R
R_s	Reliability of series system
$\hat{R}_{s(mle)}$	Maximum likelihood estimator with series system of R_s
$\hat{R}_{s(mom)}$	Moment estimator with series system of R_s
$\hat{R}_{s(\mathrm{sh1})}$	Shrinkage weight function estimator with series system of R_s
$\hat{R}_{s(\mathrm{sh2})}$	Constant shrinkage factor estimator with series system of R_s
$\hat{R}_{s(\mathrm{sh3})}$	Beta shrinkage factor estimator with series system of R_s
$\hat{R}_{s(\mathrm{Ls})}$	Least square estimator with series system of R_s

Chapter one

Introduction, literature of Review,

Thesis objective, Thesis layout

Chapter one

Introduction, literature of Review, Thesis objective, Thesis layout

(1-1) Introduction

The reliability function, which is defined as a monotonically decreasing function of the lifetime, including the study of reliability represent an important task to develop future plans and performance of the quality for the equipment [30].

Reliability theory basically related withe the determination of the probability $p(x<y)$ for the applied a system, which may be consisted of several components. The reliability function can be considered as probability for failure of process until a given time [41][59].

Actually, the reliability for the given system is the probability when operating under stated environmental conditions, or the probability of the system is to overcome the stress imposed in which, the term stress is defined: as the failure inducing variables, it is also defined as stress which tends to produce a failure of a component or a device of a material while, the term strength is defined as: the ability of component or a material to accomplish it is required function satisfactorily without failure when subjected to the external loading and environment therefore the strength resisting variable[35][58][60].

We will study the estimation of the unknown parameters, when inaccurate and biased estimates can be misleading, so the estimation of any parameters probability distribution is vital. Also the estimator approach can be considered as very useful method in the real world and a non-conventional instrument in statistical inference [44][54].

The problem of estimating unknown parameters in statistical distribution is one of the most important problems. It was used in statistical applications to study the certain phenomena. At present time there are many methods for estimating parameters that may be analyzed such as maximum likelihood method, moments method, least square method as well as shrinkage technique or Bayesian method [8][46].

"The estimation of the reliability is a very common problem in the statistical literature. The widest approach has been applied for reliability estimation in the well-known stress–strength model" This model is used in many applications of physics and engineering such as strength failure and the system collapse beside in some engineering systems, that have more than components there may fail separately or together [32].

The power function distribution was often used for the reliability analysis , income distribution data and lifetime, It was worth mentioning that, there is of generalized the power function distribution defined by (cordeiro and decastro) based on the Kumaraswamy distribution, (Meniconi and Barry) proved the power function distribution is the most efficient method by discussing it is application in different fields, also in finding hazard function and reliability [6][38][50].

The power function distribution can be considered one of the most important distributions because it was widely used in electrical system and applied mathematics, It is more accurate about hazard rate and reliability, so the statisticians prefer it from many distributions [51].

The Lomax distribution is also called Pareto II distribution, presented by the Lomax (1954). It was used frequently applied in statistical literature and used to study business failure data. It was applied in many fields such as actuarial sciences, biological sciences and engineering [23].

Extended and modified versions of the Lomax distribution have been studied; examples include the exponentiated Lomax distribution, Poisson

Lomax distribution, Weibull Lomax distribution, exponential Lomax distribution, gamma Lomax distribution, McDonald Lomax distribution and power Lomax distribution [39].

(1-2) Review of Literature

Many of the researchers was interested in statistical distribution and studied the different estimation methods, finding the reliability of stress-strength has been also discussed widely in literature including have some special distribution, such as power function distribution and generalized Lomax distribution, some of these literature references and studied of the estimated reliability can be introduced as follows:

In (1967) Malik.H.J, Studied the exact moments for power function distribution by using order statistics. He found a precise expression of moment order statistics for P. F. distribution. He also had recurrence relations between product moments for order statistics [34].

In (1991) Ashanullah.M, studied record values of Lomax distribution. He mentioned the Lomax distribution is also called the pareto 2 distribution and introduced by k .s Lomax in 1954. He found Moments of record values based on Lomax distribution function, these moments (shape, scale and location) parameters [4].

In (1996) Dilip.R and Gupta.R.P, studied bivariate extension of Lomax and finite rang distributions, They mentioned the benefits of Lomax distribution in the study of biological analysis, queuing theory and labor turnover, They also mentioned the Lomax flex and belongs to family of pareto distribution [47].

In (1998), Daved .D, estimation for system reliability for parallel system in stress- strength model for different distributions (Gamma, Weibull and pareto), he obtain the a asymptotic normal distributions [24].

In (2004) Pandey .J.S.A discuss the estimation parameters of a power function distribution and described by properties of the K-TH record values [48].

In (2006) Sikora.T and Kral.V, discuss of the times for the failure of system components using power function distribution described hazard rate and time to failure of power function distribution [56].

In (2007) Chang S.K, studied characterization of the power function distribution using the independence of record values. He gave description of sufficient and necessary the shape parameter $\alpha=1$ for the power function distribution depend on the value of lower record [18].

In the same year Sarhan.A.M, Alkhedhairi.A and Tadj. studied the estimator of the parameters (shape and scale) of the Generalized Rayleigh distribution using different estimation methods, maximum likelihood method is used for derive point and estimates of unknown parameters for asymptotic or faience [8].

In (2010) Blondeau.C, Canteaut.A and Charpih.P, studied differential properties for Power Function distribution and the relationships between the might enumerator of cyclic code and the spectrum of power permutation [17].

In (2012) Rahman .H, Roy.M.K and Baizid.A.R, studied the estimation of the shape parameter using different methods, maximum likelihood estimation, Bayes estimator for (squared error loss function, quadratic loss function, NLINEX loss function , MLINEX loss function) for power function distribution [44].

In the same year, Hameed.H.F.A, and Moniem.I.B.A, studied a new family for distributions called exponentiated Lomax distribution, They discussed some properties for this family and the estimation for unknown parameters using moments methods , L- moment methods and maximum likelihood [50].

In the same year, Gu.X and Shi.Y.M, interested dependent masked data to estimate Generalized Rayleigh components reliability in system and defined Generalized Rayleigh distribution is a type of the (exponential Weibull) distribution [28].

In (2013) Ahsanullah.M, Shakil.M and GolamKibiria .B.M, studied a new characterization of Power function distribution depending on lower records and discussed their properties and application important of statistical research [5].

In the same year, Shams .T.M, studied and introduced the Kumaraswamy generalized Lomax distribution. He provide the density function of order statistic and obtain their moment and he used the maximum likelihood method for estimating the model parameters, He derived the observed information matrix, The new distribution an including special sub –model like pareto distribution , and studied of the maximum likelihood estimate (MLEs) for new distribution [55].

In the same year, Nada.S, studied the estimations for system reliability of one component, two component and s-out of k stress- strength system for non-identical component by using exponentiated exponential distribution with scale parameter[35].

In (2014) G.Arsalan, studied the order statistics properties used model reliability growth system of power function distribution (PFD). He used order statistics from independent and obtained results considered as one of the first characterization results [12].

In the same year Pathak and Ajit .C, estimate of the reliability function for two parameter exponentiation Rayleigh or Burr type x distribution using two methods, uniform minimum variance unbiased estimator (UMVUE), and maximum likelihood estimator (MLEs) of reliability R(t)=P(X>T) of power function distribution [41].

In the same year , Fatima . K, Jan.V and Ahmed .S .P, Studied properties of Rayleigh Lomax distribution. They discussed the reliability measures and mentioned the different characteristics of the distribution. Also they used different methods to estimates [23].

In the same year, Zaka. A, Akhter.A.S and Farooqi .N, studied different methods for estimating two parameters (shape and scale) of the Power Function distribution. They used several methods to estimate two parameters of Power Function distribution such as Ridges regression, Lest Square methods and Relative Lest Square. They compare between these methods to discover the most accurate method [6].

In the same year, Pathak.A and Chaturvedi.A, estimated the reliability function for exponentiated generalized Lomax distribution, They compare between the maximum likelihood estimator and uniformly minimum variance unbiased for the reliability function R(t) of four parameter in exponentiated generalized Lomax distribution[42] .

In the same year , Attia .A.F, Shaban .S.A and Amer.Y.M, estimate the bivariate generalized Lomax distribution parameter depending on censored samples, They said the generalized Lomax distribution can be used to analyze lifetime in one dimension [15].

In (2015) Salman.A.N and Amen.M.M, used Bayesian – Shrinking Technique to estimate the shape parameter of Generalized Rayleigh Distribution, when the scale parameter λ is known, addition a prior knowledge about the shape parameter α is available as initial value α_0 [52].

In the same year, EL-Bassiouny.A.H, Abdo.N.F and Shahen.H. S, Studied Exponential Lomax distribution called new generalized of Lomax distribution. They discussed properties for distribution such as hazard function, moment. Median ….etc [20].

In the same year ,Salman.A.N and Hamed A.M, studied estimating the shape parameter for the Power Function Distribution .They mentioned the importance of statistical distribution different fields such as engineering, economics and applied mathematics, They discussed the estimating of the unknown shape parameter with the scale (ɵ=1) parameter using shrinkage estimation method[51].

In the same year, Ashour.S.K and Eltehiwy.M.A, studied Generalized Power Lindley Distribution for (one parameter – two parameter – three parameter). They derived (RTH) Moments and the Moment Generating Function. They used Maximum \likelihood Methodist Square Estimation Method to estimate distribution parameters[13].

In the same year, Oguntunede.P.E, Odetumibi.O.A, Okagbue.H.I Bobatund.O.S and Ugwoke, discussed the Kasmaswamy Power Function Distribution Parameter (A Generalization of the Power Distribution). They gave some properties of K- Power Distribution and estimating the parameters of the model using maximum likelihood estimation [38].

In the same year Shahzad.M. N, Asghar Z. Shahzad.F and Shahzad M, studied estimation the parameters of Power Function Distribution with TL-Moments. They said the Power Function Distribution is flexible distribution and considered one of the best distributions. They using many the Method of Moments (MM), L - Moments (LM), LH-Moments (LHM), LL- Moments, LL –Moment (LLM), Trimmed L- moment (TLM). They also derived the first four moments by (LM), (LHM) , (LLM) and (TLM) of the P.D.F [36].

In the same year , AL-Zahrani .B, investigated and introduced extension of Poisson Lomax distribution. He studied behavior for the density function and the shape parameter for the fuiler rate function ,He estimate parameter by using maximum likelihood estimator[11].

In (2016) Shakeel.M .Haq, A. M.Hussein, L.Abdulhamis.A.M and Faisal M, compare between a new two modern rebuts parameters estimation methods for the Power Function Distribution is probability weighted moments and generalized probability weighted methods, usually. They are useful for electrical reliability. Standard Pareto Distribution has opposite relationship with the Power Function Distribution [54].

In the same year, El-Houssainy.A, Hassanein.W, and Elhaddad.T, studied the power Lomax distribution (POLO) with three parameter , they derived the hazard rate, survival statistical and reliability they used the maximum likelihood method and method of moments to the point estimation and the real data application to bladder cancer data[43].

In the same year, Fares.A.S and Gopal.V, introduced the generalized double Lomax distribution and studied some general properties, they estimate parameters of generalized double Lomax by maximum likelihood method [21].

In (2017) Al-Mutaira.A.O, studied Bayesian Estimating of the loss function of shape parameter for Generalized Power Function Distribution. They mentioned Power Function Distribution Parameter is kind of Pareto function Distribution. As well explained exists relate between (Rayleigh, Weibull, Gamma) distribution and power function distribution[9]

In the same year, Dhanya. M, and Jeevavand .E .S, studied the estimation of stress- strength reliability $R=p\ (y<x)$ for power function distribution with shape parameter and same scale parameter based on record. The bayes estimator under linex loss function and squared error loss function and the maximum likelihood estimator [19].

In the same year , Oquntunde .P,Abdullah .M ,Taha .M and Adejume .A, studied generalized of the Lomax distribution by decreasing , increasing and constant failure rate, They mentioned extended and modified of the Lomax distribution such as gamma Lomax distribution, power Lomax distribution, Poisson Lomax distribution …….etc , The result indicate comports Lomax distribution the best and follows by, beta Lomax distribution, Weibull Lomax distribution and Kumaraswamy Lomax distribution [39].

In the same year, Ibrahim.B, present recurrence relations between product and single moment of order statistics for power Lomax distribution, he studied some properties for power Lomax distribution and tabulated some results for mean and variance[2].

In (2018), Mohammad .I.K studied generalized order statistics for power Lomax distribution (POLO), He mentioned some of his characteristics, one of the properties of this distribution the recurrence relations of moments for order statistics[31].

In the same year, Sanaa Al-Marzouki, studied exponentiated of power Lomax distribution called generalization Lomax distribution, she mentioned some properties for this distribution and provided, estimate parameters this model by using maximum likelihood method [10].

In the same year, Claudio.C, Leo.O and Romanus.O, they studied parameter estimation for power Lomax distribution to compute the maximum likelihood estimator they used expectation maximization algorithm and simulation depending on mean square error [29].

In the same year, Raj.K and Yogesh.M, consider the reliability estimation in a multi component stress- strength model of Burr distribution when beta unknown by using Bayes estimates and maximum likelihood are also derived under the squared error loss function[16].

In (2019), Ashok.P, Swathi.N, Tirumala.M and Uma.T, derived the reliability for multi component series system, parallel system and considered standby system for stress- strength Weibull distribution, they obtained the general expression for the reliability for multi component[14].

(1-3) Thesis Objective

The main aims of this thesis can be summarized as follows:

- Obtain the official reifies of the best performance the systems either contain on component function or series systems component, this process include the estimator of the parameter and the reliability of stress-strength (S-S) models for the system with applied power function distribution, modified Exponentiated Lomax distribution and the power Lomax distribution.

- Applying different methods of estimation such as Maximum likelihood (MLE), Moment (MOM), Shrinkage methods (SH) and Least square (LS) with obtaining the results for different sample sizes (30,50, 100) accordingly to the selected initial value using simulation technique.
- Comparisons between the proposed estimation method for system reliability of both considered model depending on Mean squared Error (MSE) indicator.

(1-4) Thesis Layout

This thesis includes the following chapters and appendix:

Chapter one includes, Introduction, Literature of review, Thesis objective and Thesis layout.

Chapter two includes, the principle of the concepts of the studied distributions.

Chapter three includes, estimation parameter methods [Maximum likelihood method (MLE), Moment method (MOM), Least square method (LS) and Shrinkage estimation method (SH)] for the studied distributions.

Chapter four includes, Experimental aspect.

Chapter five includes, conclusion and recommendations.

Chapter two

The Principle Concepts of the Studied Distributions

Chapter two

The Principle Concepts of the Studied Distributions

(2-1)Introduction

This chapter represent some properties and the genesis of power function distribution, Exponentiated Lomax distribution, modified Exponentiated Lomax distribution and the power Lomax distribution with parameters as well as the reliability system in stress- strength model for one component and system content two series components will be drived accordingly for distribution studied.

(2-2) Power Function Distribution

(2-2-1)Genesis of Power Function Distribution

The power function distribution is one of the most important distribution and can be express it by $X \sim pow\ (\alpha, \varphi)$. The power function distribution considered a special case for Pearson kind I distribution represent simplicity the moments of power function distribution, it is simple distribution flexible and always used in electrical component reliability, Menicani and Barry proved the power function distribution is the most efficient by discussing it is application in different fields, also by finding hazard function and reliability [6][44][54].

The relationship of the power function distribution with some other distributions can be describe all in Figure below

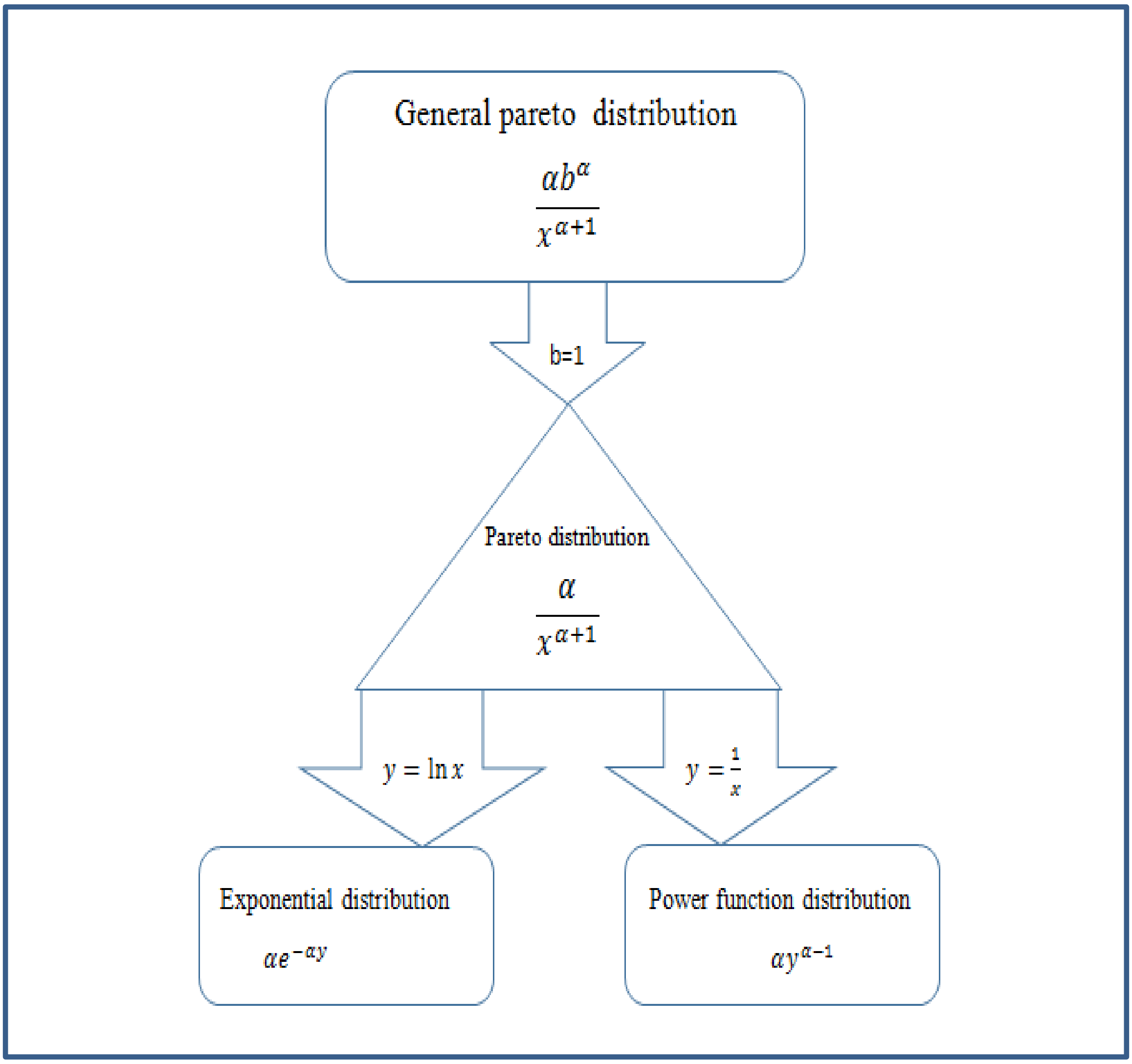

Figure 1:Relationship of power function distribution with some distributions (by researcher)

Lemma1: If $X \sim Gp(a,b)$ **and b=1 than**

1-if y=Ln x than $Y \sim ex(\alpha)$

2-if $y = \frac{1}{x}$ **than** $Y \sim pow(\alpha)$

(Derivation Using Transformation)

Let $X \sim Gp(a,b)$ General pareto distribution

$$f(x)=\frac{\alpha b^{\alpha}}{x^{\alpha+1}} \qquad 1<x<\infty \qquad \alpha, b>0$$

As special case b=1 become pareto distribution $\frac{\alpha}{x^{\alpha+1}}$

Now,

When $y = Lnx$, and using transformation

$$x = e^{y}$$

$$\frac{dx}{dy}=e^{y}=|J|$$

Hence , the (pdf) of y will be

$$f(y)=\alpha e^{-\alpha y} \qquad 0<y<\infty \qquad \text{(Exponential distribution)}$$

Also, when $y=\frac{1}{x}$ using transformation

$$x=\frac{1}{y}$$

$$\frac{dx}{dy}=\frac{1}{y^{2}}=|J|$$

$$\frac{\alpha}{\left(\frac{1}{y}\right)^{\alpha+1}} \cdot \frac{1}{y^{2}}$$

$$f(y)=\alpha y^{\alpha-1} \qquad 0<y<1 \qquad \text{(Power function distribution)}$$

(2-2-2)Probability Function and Some Important Properties for Power Function Distribution [8] [18]

The probability density function (pdf) of random variable X which is follow the two parameters power function distribution $X \sim pow(\alpha,\theta)$ given as:

$$f(x,\alpha,\theta)=\begin{cases}\alpha\theta^{\alpha}x^{\alpha-1} & 0<x<\theta^{-1} \\ o & o.w\end{cases} \qquad \alpha,\theta>0 \qquad (2\text{-}1)$$

And, The cumulative function for power function distribution

$$F(x,\alpha,\theta)=\theta^{\alpha}x^{\alpha} \qquad 0<x<\theta^{-1} \qquad (2\text{-}2)$$

In this thesis, we will study special case of power function distribution when $\theta=1$, The density function for power function distribution

$$f(x,\alpha)=\begin{cases}\alpha x^{\alpha-1} & 0<x<1 \\ 0 & o.w\end{cases} \qquad \alpha>0 \qquad (2\text{-}3)$$

And the cumulative function for power function distribution

$$F(x,\alpha)=x^{\alpha} \qquad 0<x<1 \qquad (2\text{-}4)$$

And the reliability function become

$$R(x)=1-x^{\alpha} \qquad 0<x<1 \qquad (2\text{-}5)$$

Also the hazard function will be

$$h(x)=\frac{f(x)}{R(x)}=\frac{\alpha x^{\alpha-1}}{1-x^{\alpha}} \qquad (2\text{-}6)$$

In the following table we present the main properties of power function distribution

Table (2-1): Some important properties for power function distribution

Term	Symbol	Formal
Cumulative hazard function	$H(x)$	$-Ln(1-x^{\alpha})$
Median	x_{med}	$(0.5)^{\frac{1}{\alpha}}$
k-moment	$E(x^{k})$	$\frac{\alpha}{\alpha+K}$
First moment	$E(x)$	$\frac{\alpha}{\alpha+1}$
Second moment	$E(x^{2})$	$\frac{\alpha}{\alpha+2}$
Variance	$V(x)$	$\frac{\alpha}{(\alpha+2)(\alpha^{2}+2\alpha+1)}$
Moment generating function	$M_{x}(t)$	$1+\sum_{n=1}^{\infty}\frac{\alpha}{\alpha+n}\frac{t^{n}}{n!}$
The coefficient of variation	c.v(x)	$\frac{1}{\sqrt{(\alpha+2)(\alpha)}}$

Now, the sketch of the probability function graphs listed as below,

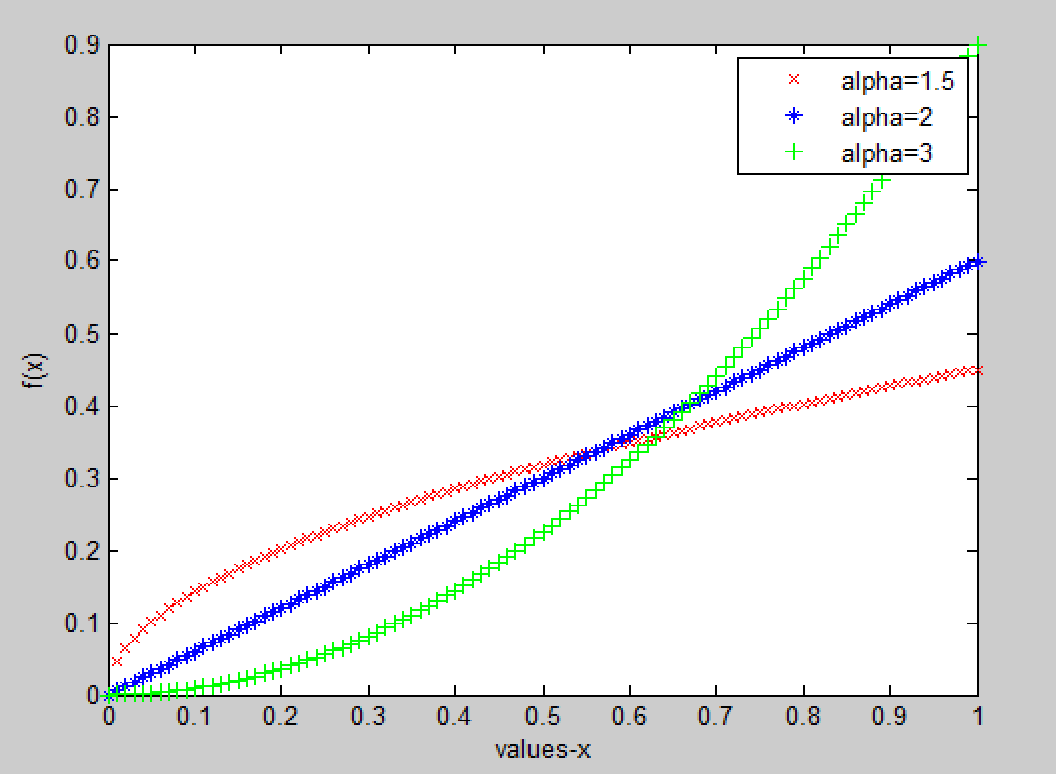

Figure 2: pdf for power function distribution

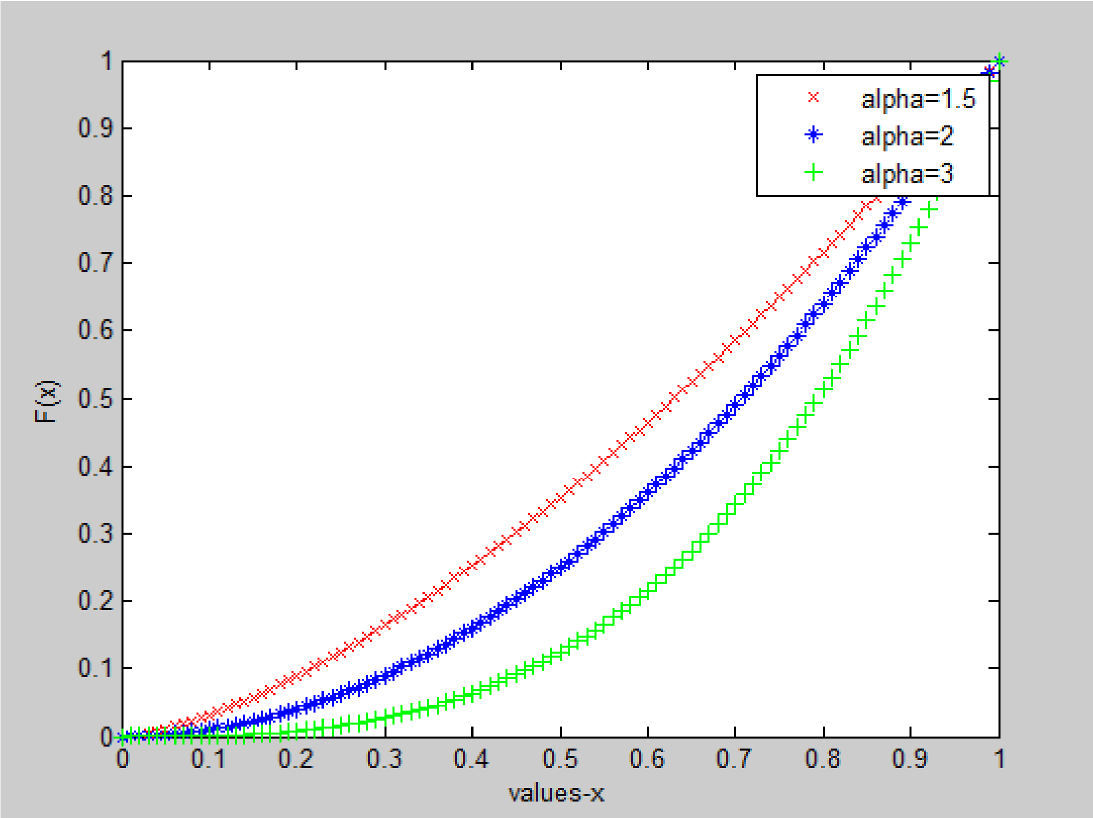

Figure 3: cdf for power function distribution

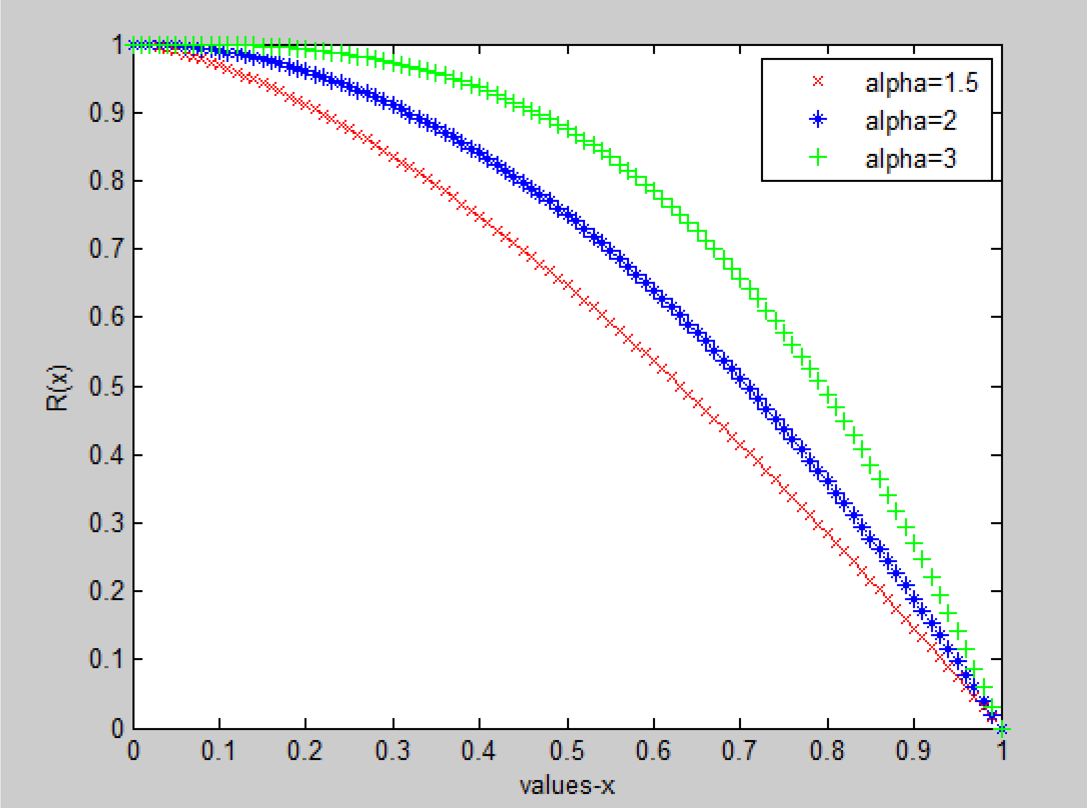

Figure 4: Reliability for power function distribution

(2-3) <u>Exponentiated Lomax Distribution</u>

(2-3-1) <u>Genesis of Exponentiated Lomax Distribution</u>

It also called Pareto II distribution proposed by K.S Lomax in 1954 and it belongs to sets of decreasing hazard rate. Can be express it by $X \sim lom(\alpha, \lambda, \theta)$ as hamullah (1991) studied record values of Lomax distribution [4].

The Lomax distribution function is widely applicable in many fields like reliability engineering, biological sciences, lifetime modeling and actuarial sciences [23].

It is possible to describe a relationship of the Exponentiated Lomax Distribution with other distributions as in the following figure

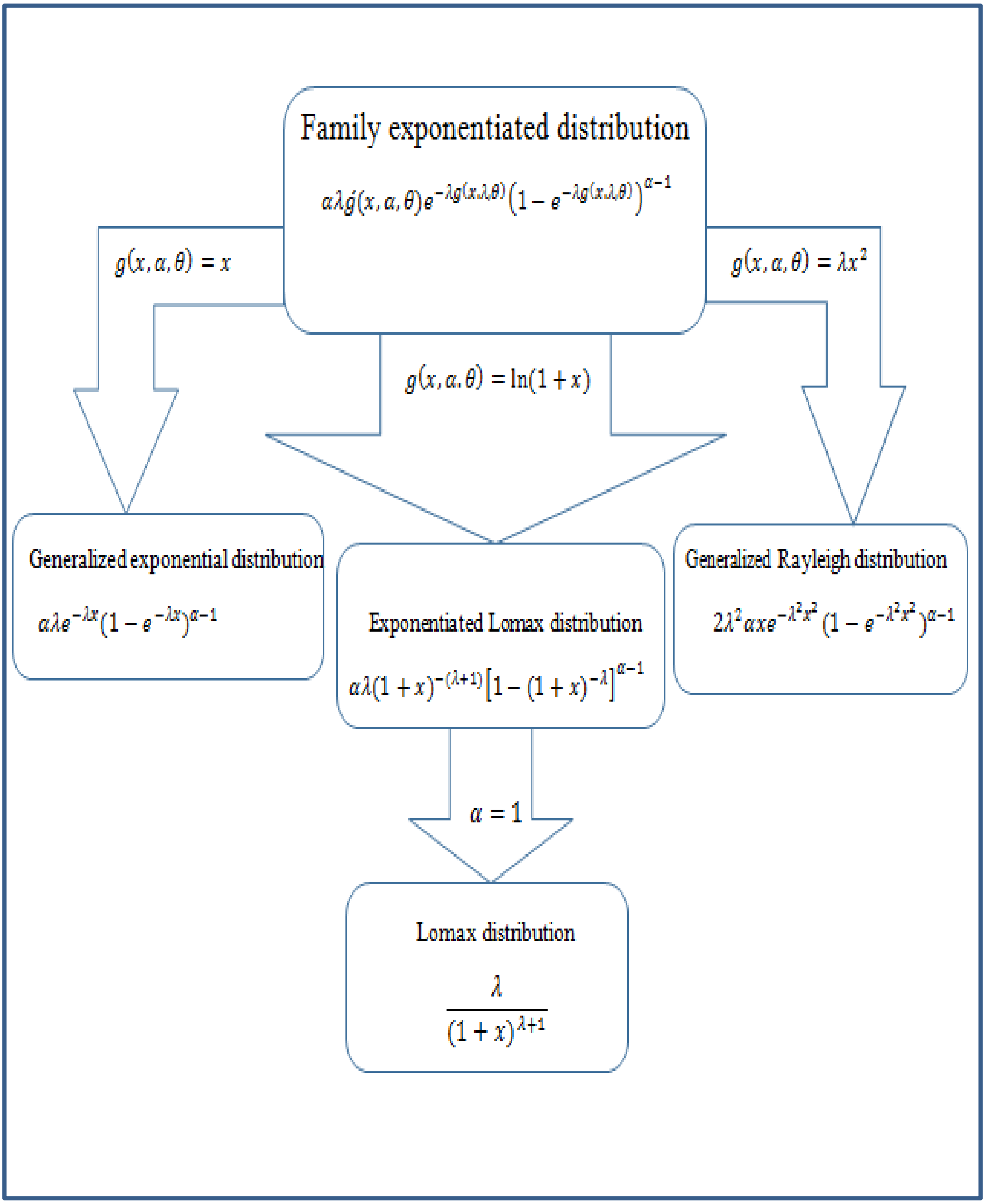

Figure 5:the relationship of the Exponentiated Lomax distribution with other distribution (by researcher)

(2-3-2) Probability Function and Some Important Properties for Exponentiated Lomax Distribution [3] [39]

Let X be random variable such as $X \sim ELD(\alpha, \theta, \lambda)$ then the probability density function of X

$$f(x,\alpha,\theta,\lambda)=\begin{cases}\alpha\theta\lambda\left[1-(1+\lambda x)^{-\theta}\right]^{\alpha-1}(1+\lambda x)^{-(\theta+1)} & 0<x<\infty \\ 0 & o.w\end{cases} \quad (2\text{-}7)$$

Where $\alpha,\theta,\lambda>0$, represent the shape parameters

The Cumulative function of Exponentiated Lomax distribution as bellows:

$$F(x,\alpha,\theta,\lambda)=\left[1-(1+\lambda x)^{-\theta}\right]^{\alpha} \qquad 0<x<\infty \quad (2\text{-}8)$$

Special case when $\lambda=1$, than

The density function for two parameters Exponentiated Lomax distribution

$$f(x,\alpha,\theta)=\begin{cases}\alpha\theta\left[1-(1+x)^{-\theta}\right]^{\alpha-1}(1+x)^{-(\theta+1)} & 0<x<\infty \\ 0 & o.w\end{cases} \quad (2\text{-}9)$$

When $\alpha,\theta>0$

The cumulative function for Exponentiated Lomax distribution

$$F(x,\alpha,\theta)=\left[1-(1+x)^{-\theta}\right]^{\alpha} \qquad 0<x<\infty \quad (2\text{-}10)$$

The reliability function become

$$R(x)=1-\left[1-(1+x)^{-\theta}\right]^{\alpha} \quad (2\text{-}11)$$

The hazard function will be

$$h(x)=\frac{f(x)}{R(x)}=\frac{\alpha\theta\left[1-(1+x)^{-\theta}\right]^{\alpha-1}(1+x)^{-(\theta+1)}}{1-\left[1-(1+x)^{-\theta}\right]^{\alpha}} \tag{2-12}$$

In the following table we present the main properties Exponentiated Lomax distribution

Table (2-2) Some important properties for Exponentiated Lomax distribution [3]

Term	Symbol	Formal
Cumulative hazard function	$H(x)$	$-Ln[1-(1-(1+x)^{-\theta})^{\alpha}]$
Median	x_{med}	$\left[1-(0,5)^{\frac{1}{\alpha}}\right]^{-\frac{1}{\theta}}-1$
k-moment	$E(x^r)$	$\alpha\sum_{i=0}^{r}\binom{r}{i}(-1)^i B\left(1-\frac{(r-i)}{\theta},\alpha\right)$
First moment	$E(x)$	$\alpha\left[B\left(1-\frac{1}{\theta},\alpha\right)-B(1,\alpha)\right]$
Second moment	$E(x^2)$	$\alpha\left[B\left(1-\frac{2}{\theta},\alpha\right)-2B(1-\frac{1}{\theta},\alpha)+B(1,\alpha)\right]$
Variance	$V(x)$	$\alpha\left[B\left(1-\frac{2}{\theta},\alpha\right)-2B(1-\frac{1}{\theta},\alpha)+B(1,\alpha)\right]$ $-\alpha^2\left[B\left(1-\frac{1}{\theta},\alpha\right)-B(1,\alpha)\right]^2$
The coefficient of variation	c.v(x)	$\frac{\sqrt{\alpha[B(1-2/\theta,\alpha)-2B(1-1/\theta,\alpha)+B(1,\alpha)]-\alpha^2[B(1-1/\theta,\alpha)-B(1,\alpha)]^2}}{\alpha\left[B\left(1-\frac{1}{\theta},\alpha\right)-B(1,\alpha)\right]}$

(2-4) Modified Exponentiated Lomax distribution

As a special case of Exponentiated Lomax distribution when $\theta = 2$ and $\lambda = 1$, we obtain modified Exponentiated Lomax distribution donated by $X \sim \text{MELD}(\alpha)$ in this part of the chapter we will examine some of the properties of this distribution.

(2-4-1) Probability Function and Some Properties for Modified Exponentiated Lomax distribution

Let X random variable and $X \sim \text{MELD}(\alpha)$ then the density function for modified exponentiated Lomax distribution

$$f(x,\alpha) = \begin{cases} 2\alpha\left[1-(1+x)^{-2}\right]^{\alpha-1}(1+x)^{-3} & 0 < x < \infty \\ & \alpha > 0 \\ 0 & \text{o.w} \end{cases} \tag{2-13}$$

The cumulative function for modified exponentiated Lomax distribution

$$F(x,\alpha) = \left[1-(1+x)^{-2}\right]^{\alpha} \qquad 0 < x < \infty \tag{2-14}$$

The reliability function become

$$R(x) = 1-\left[1-(1+x)^{-2}\right]^{\alpha} \tag{2-15}$$

The hazard function will be:

$$h(x) = \frac{f(x)}{R(x)} = \frac{2\alpha\left[1-(1+x)^{-2}\right]^{\alpha-1}(1+x)^{-3}}{1-\left[1-(1+x)^{-2}\right]^{\alpha}} \tag{2-16}$$

In the following table, the main properties of modified Exponentiated Lomax distribution are presence

Table (2-3) Some important properties for modified Exponentiated Lomax distribution[3]

Term	Symbol	Formal
Cumulative hazard function	$H(x)$	$-Ln(1-[1-(1+x)^{-2}]^{\alpha})$
Median	x_{med}	$\left[1-(0.5)^{\frac{1}{\alpha}}\right]^{-\frac{1}{2}}-1$
r-moment	$E(x^r)$	$\alpha\sum_{i=0}^{r}\binom{r}{i}(-1)^i B\left(1-\frac{(r-i)}{2},\alpha\right)$
First moment	$E(x)$	$\alpha\left[B\left(\frac{1}{2},\alpha\right)-B(1,\alpha)\right]$
Second moment	$E(x^2)$	$\alpha\left[B(0,\alpha)-2B\left(\frac{1}{2},\alpha\right)+B(1,\alpha)\right]$
Variance	$v(x)$	$\alpha\left[B(0,\alpha)-2B\left(\frac{1}{2},\alpha\right)+B(1,\alpha)\right]$ $-\alpha^2\left[B\left(\frac{1}{2},\alpha\right)-B(1,\alpha)\right]^2$
The coefficient of variation	c.v(x)	$\frac{\sqrt{\alpha[B(0,\alpha)-2B(0.5,\alpha)+B(1,\alpha)]-\alpha^2[B(0.5,\alpha)-B(1,\alpha)]^2}}{\alpha\left[B(0.5,\alpha)-B(1,\alpha)\right]}$

Now, the sketch of the probability function graphs listed as below,

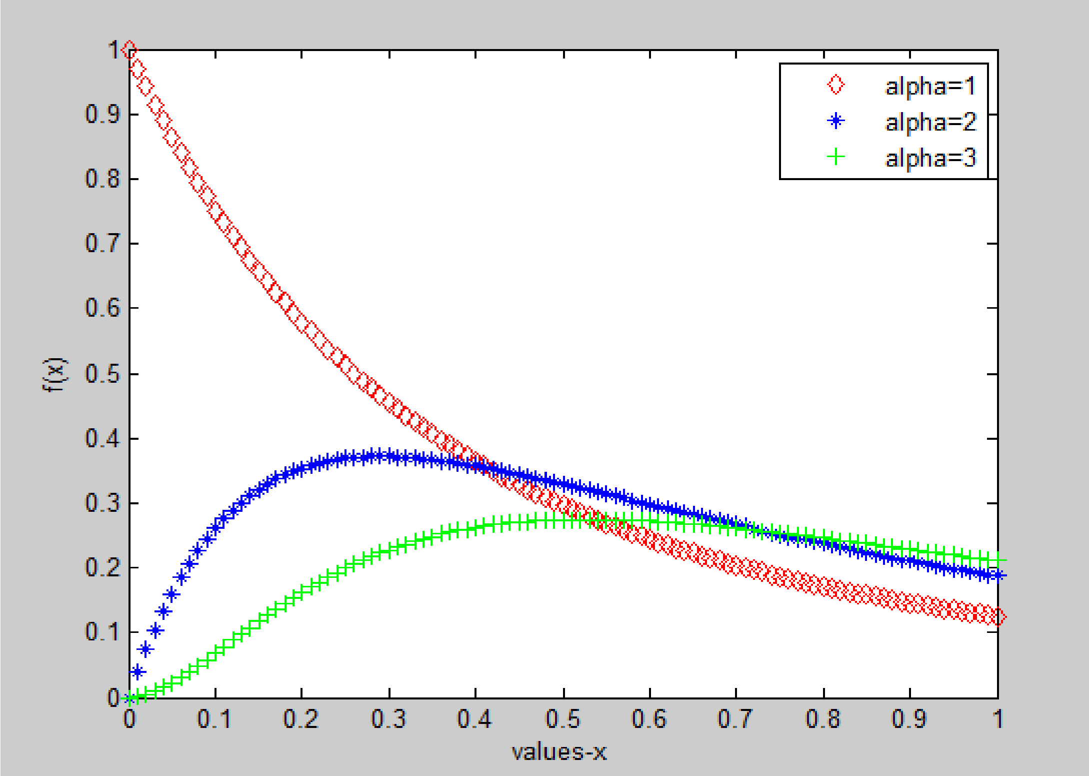

Figure 6:pdf for modified exponentiated Lomax distribution

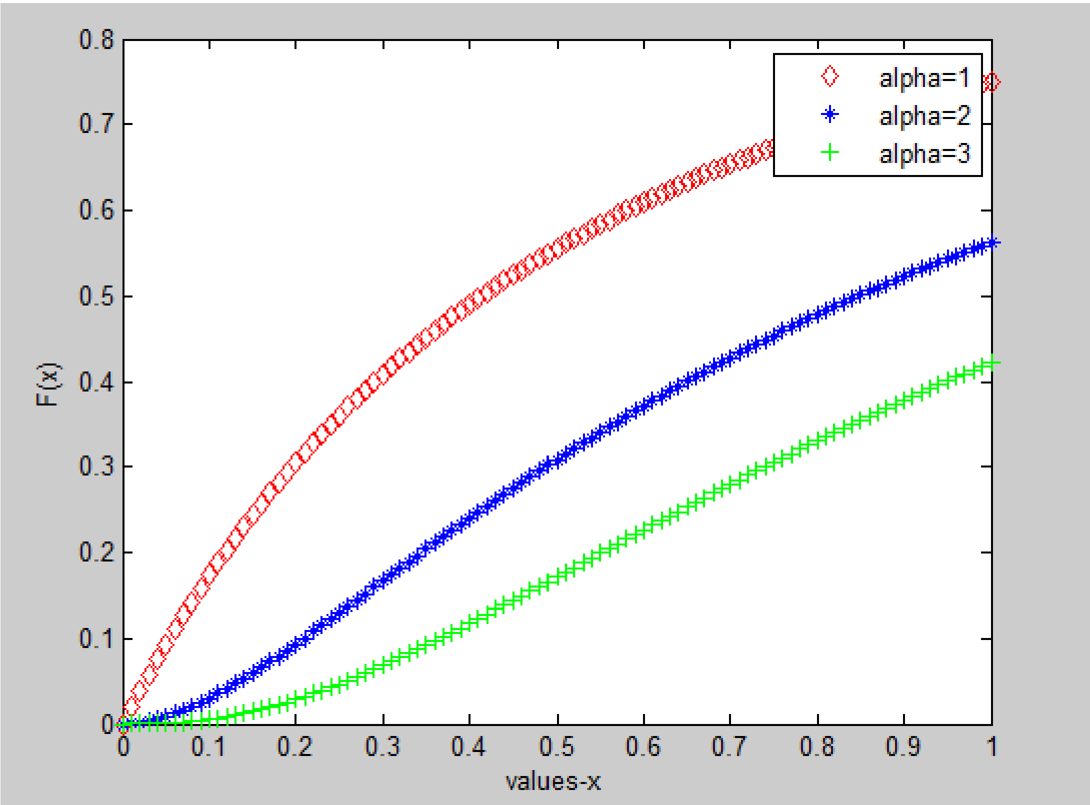

Figure 7: cdf for modified exponentiated Lomax distribution

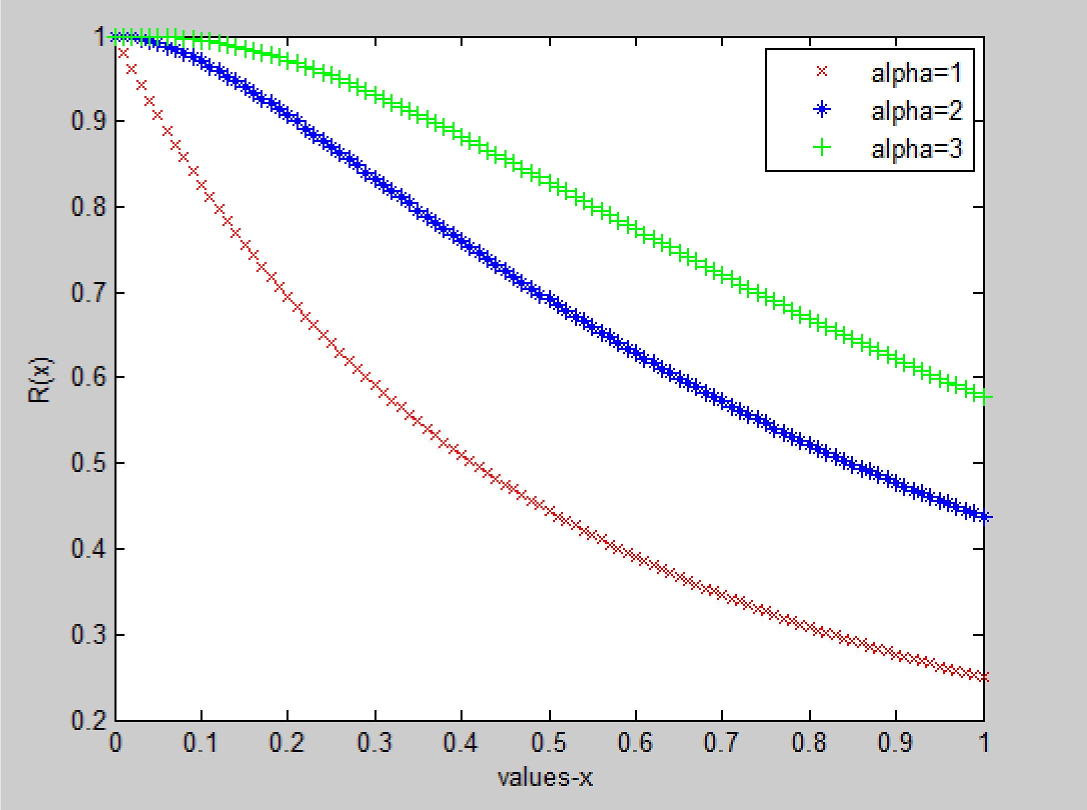

Figure 8: Reliability for modified exponentiated Lomax distribution

(2-5) <u>Power Lomax Distribution(PoLo)</u>

(2-5-1)<u>Genesis of Power Lomax Distribution[1][10]</u>

The Lomax distribution is flexible and found in many application such medical, biological science, engineering, reliability model, income and life time, it is belongs to family increasing hazard rate and with related to the burr family distribution.

The Lomax distribution used for several generalization as the biases , on other hand in (1980) Tadikamall relates Burr family to Lomax distribution and in (2011) Punathumparambath introduced double Lomax distribution.

There are many of extended for Lomax distribution has been studied by statisticians, one of these distributions the power Lomax distribution which we will study in this section and express by $X \sim PoLo(\alpha,\beta,\lambda)$

(2-5-2)<u>Probability Function and Some Important Properties for Power Lomax Distribution[2][29][43]</u>

Let X random variable and $X \sim PoLo(\alpha,\beta,\lambda)$ then the density function for power Lomax distribution (pdf)

$$f(x,\alpha,\beta,\lambda)=\begin{cases}\alpha\beta\lambda^{\alpha}x^{\beta-1}(\lambda+x^{\beta})^{-(\alpha+1)} & 0<x<\infty\\ 0 & o.w\end{cases} \qquad (2\text{-}17)$$

When $\alpha,\lambda,\beta>0$

Hence the cumulative function (cdf) of power distribution which is

$$F(x,\alpha,\beta,\lambda)=1-\lambda^{\alpha}(\lambda+x^{\beta})^{-\alpha} \qquad 0<x<\infty \qquad (2\text{-}18)$$

We will study special case when $\beta=1,\lambda=1$

Probability density function for power Lomax distribution

$$f(x,\alpha)=\begin{cases}\alpha(1+x)^{-(\alpha+1)} & 0<x<\infty\\ 0 & o.w\end{cases} \qquad \alpha>0 \qquad (2\text{-}19)$$

And the cumulative function for power Lomax distribution

$$F(x,\alpha)=1-(1+x)^{-\alpha} \qquad 0<x<\infty \qquad (2\text{-}20)$$

Implies reliability function for power Lomax distribution become

$$R(x) = (1+x)^{-\alpha} \tag{2-21}$$

And the hazard function become:

$$h(x) = \frac{f(x)}{R(x)} = \frac{\alpha}{(1+x)} \tag{2-22}$$

In the following table has some important properties for power Lomax distribution

Table (2-4) Some important properties for power Lomax distribution

Term	Symbol	Formal
Cumulative hazard function	$H(x)$	$\alpha \ln(1+x)$
Median	x_{med}	$2^{\frac{1}{\alpha}} - 1$
r-moment	$E(x^r)$	$\frac{\alpha\Gamma(\alpha - r)\Gamma(r+1)}{\Gamma(1+\alpha)}$
First moment	$E(x)$	$\frac{1}{\alpha - 1}$
Second moment	$E(x^2)$	$\frac{2}{(\alpha-1)(\alpha-2)}$
Variance	$v(x)$	$\frac{\alpha}{(\alpha-1)^2(\alpha-2)}$
The coefficient of variation	c.v(x)	$\sqrt{\frac{\alpha}{\alpha-2}}$

Now, the graph of the probability functions are listed below,

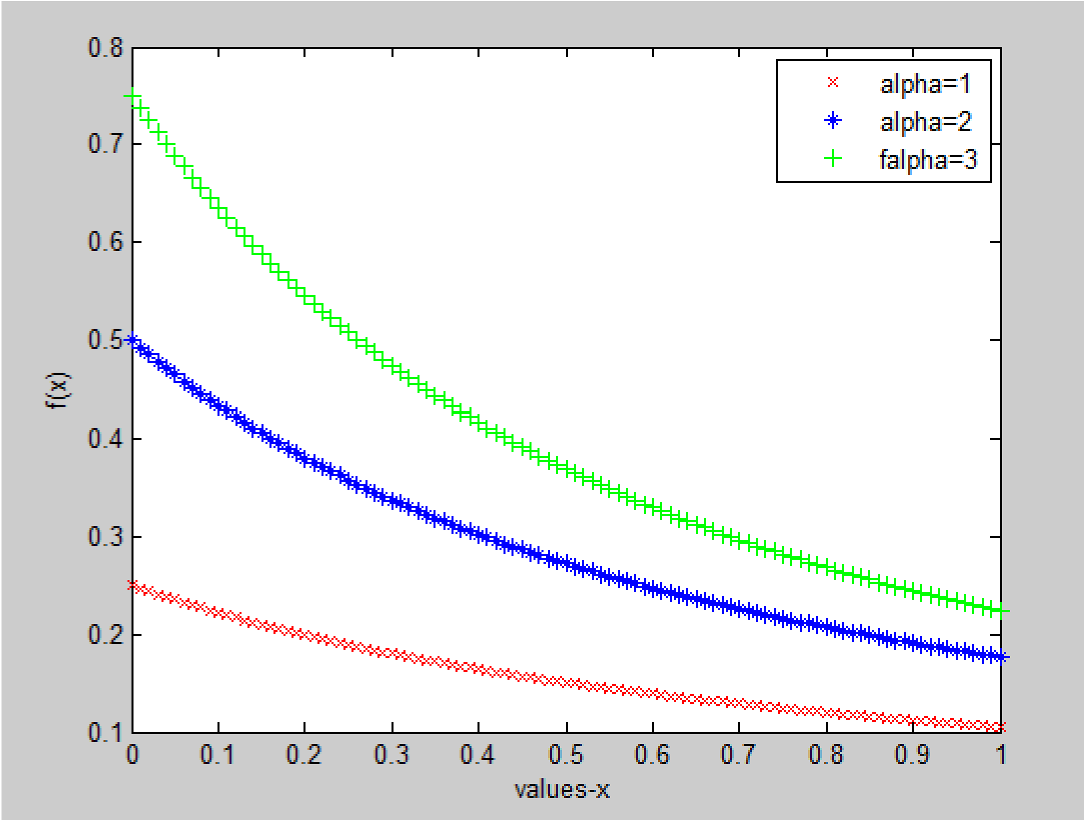

Figure 9: pdf for power Lomax distribution

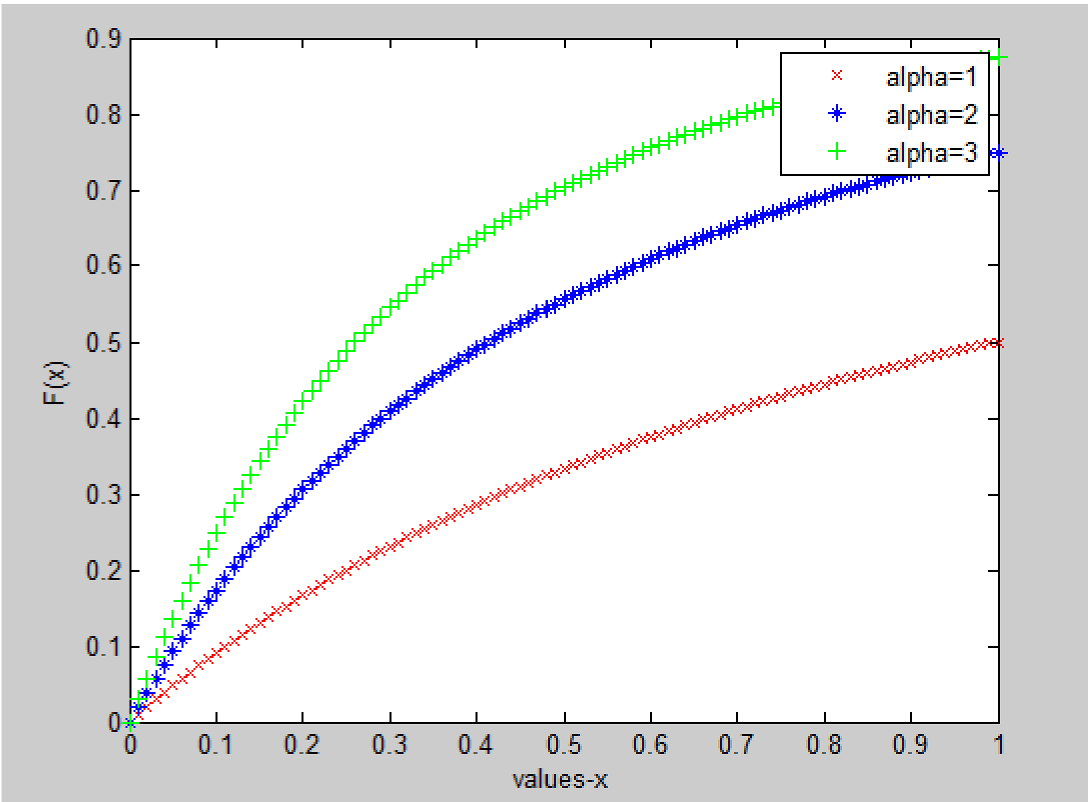

Figure 10: cdf for power Lomax distribution

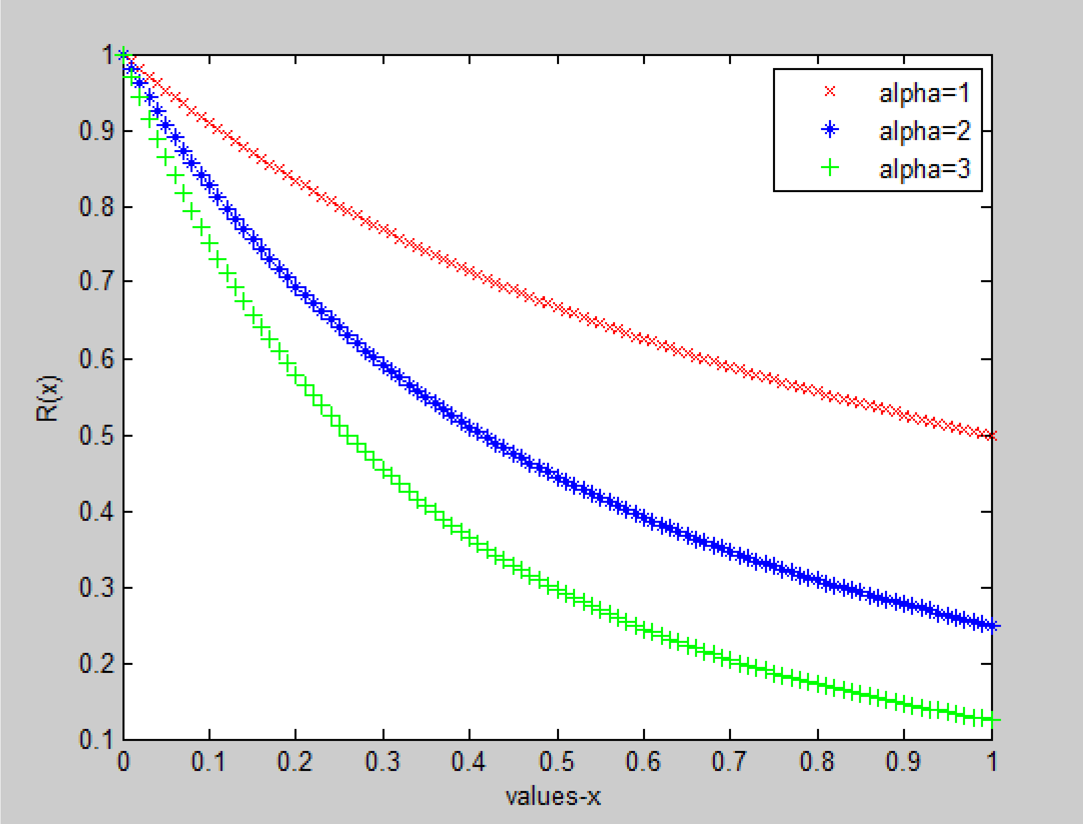

Figure 11: Reliability for power Lomax distribution

(2-6) <u>The System Reliability in Stress-Strength (S-S) Model[1][49]</u>

In this section we consider the problem of finding the reliability of system content one component in stress-strength model $R = P(Y < X)$ when $X and\ Y$ independent random variable for power function distribution, modified Exponentiated Lomax distribution and power Lomax distribution. And to finding the reliability system model R_S, implies that the system make series system. The first to discuss this problem is Mccool in (1991) then discussed by Kundu and Cupta (2006).

(2-6-1) The System Reliability of Stress-Strength (S-S) Model for Power Function Distribution

Now, assume the two random variables $X \, and \, Y$ such as $X \sim Pow(\alpha_1)$ And $Y \sim Pow(\alpha_2)$ as strength and stress respectively

The probability density function (pdf) for each random variable $X \;\; and \;\; Y$ are gave as below

$$f(x,\alpha_1) = \begin{cases} \alpha_1 x^{\alpha_1 - 1} & for \quad 0 < x < 1 \\ 0 & o.w \end{cases} \tag{2-23}$$

$$f(y,\alpha_2) = \begin{cases} \alpha_2 y^{\alpha_2 - 1} & for \quad 0 < y < 1 \\ 0 & o.w \end{cases} \tag{2-24}$$

Hence The cdf of $X \; and \; Y$ given as

$$F(x,\alpha_1) = x^{\alpha_1} \qquad 0 < x < 1 \tag{2-25}$$

$$F(y,\alpha_2) = y^{\alpha_2} \qquad 0 < y < 1 \tag{2-26}$$

The (s-s) reliability R for this model define as

$$R = p(y < x)$$

$$R = \int_0^1 \int_0^x f(x) f(y) dy dx$$

$$R = \int_0^1 \int_0^x \alpha_1 x^{\alpha_1 - 1} \alpha_2 y^{\alpha_2 - 1} dy dx$$

$$= \int_0^1 \alpha_1 x^{\alpha_1 - 1} \left[x^{\alpha_2} - 0 \right] dx$$

$$= \int_0^1 \alpha_1 x^{\alpha_1+\alpha_2-1} dx$$

$$R = \frac{\alpha_1}{\alpha_1 + \alpha_2} \qquad (2\text{-}27)$$

(2 - 6- 2) The System Reliability of Stress – Strength (S-S) Model for Modified Exponentited Lomax Distribution

Assume the two random variables X and Y such as $X \sim \text{MELD}(x, \alpha_1)$ *and* $Y \sim \text{MELD}(y, \alpha_2)$ as strength and stress respectively

The probability density function (PDF) for each random variable X and Y is given as:

$$f(x,\alpha_1) = \begin{cases} 2\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1-1}(1+x)^{-3} & 0 < x < \infty \\ 0 & o.w \end{cases} \qquad (2\text{-}28)$$

$$f(y,\alpha_2) = \begin{cases} 2\alpha_2\left[1-(1+y)^{-2}\right]^{\alpha_2-1}(1+y)^{-3} & 0 < x < \infty \\ 0 & o.w \end{cases} \qquad (2\text{-}29)$$

The C. D. F of X and Y given as:

$$F(x,\alpha_1) = \left[1-(1+x)^{-2}\right]^{\alpha_1} \qquad (2\text{-}30)$$

$$F(y,\alpha_2) = \left[1-(1+y)^{-2}\right]^{\alpha_2} \qquad (2\text{-}31)$$

The (S – S) system reliability R for this model define as

$$R = P(y < x)$$

$$R=\int_0^{\infty}\int_0^{x} f(x)f(y)\,dy\,dx$$

$$=\int_0^{\infty}\int_0^{x} 2\,\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1-1}(1+x)^{-3}\,2\alpha_2\,\left[1-(1+y)^{-2}\right]^{\alpha_2-1}(1+y)^{-3}\,dydx$$

$$=\int_0^{\infty}2\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1-1}(1+x)^{-3}\int_0^{x}2\alpha_2\left[1-(1+y)^{-2}\right]^{\alpha_2-1}(1+y)^{-3}\,dy\,dx$$

$$=\int_0^{\infty}2\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1-1}(1+x)^{-3}\left[\left[1-(1+y)^{-2}\right]^{\alpha_2}\Big|_0^{x}\right]dx$$

$$=\int_0^{\infty}2\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1-1}(1+x)^{-3}\left[1-(1+x)^{-2}\right]^{\alpha_2}dx$$

$$=\int_0^{\infty}2\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1+\alpha_2-1}(1+x)^{-3}dx$$

$$=\left.\frac{\alpha_1\left[1-(1+x)^{-2}\right]^{\alpha_1+\alpha_2}}{\alpha_1+\alpha_2}\right|_0^{\infty}$$

$$R=\frac{\alpha_1}{\alpha_1+\alpha_2} \qquad (2\text{-}32)$$

(2 - 6- 3) <u>The System Reliability of Stress – Strength (S-S) Model for Power Lomax Distribution</u>

Assume the two random variable X and Y such as, $X \sim PoLo(\alpha_1)$ and $Y \sim PoLo(\alpha_2)$

The probability density function (PDF) for each random variable X and Y is given as

$$f(x,\alpha_1)=\begin{cases}\alpha_1(1+x)^{-(\alpha_1+1)} & 0<x<\infty \\ 0 & o.w\end{cases} \tag{2-33}$$

$$f(y,\alpha_2)=\begin{cases}\alpha_2(1+y)^{-(\alpha_2+1)} & 0<y<\infty \\ 0 & o.w\end{cases} \tag{2-34}$$

and the cumulative function (CDF) for random variable X and Y given as

$$F(x,\alpha_1)=1-(1+x)^{-\alpha_1} \qquad 0<x<\infty \tag{2-35}$$

$$F(y,\alpha_2)=1-(1+y)^{-\alpha_2} \qquad 0<y<\infty \tag{2-36}$$

The Stress-Strength (S-S) reliability for power Lomax distribution as below

$$R=P(y<x)$$

$$R=\int_0^{\infty}\int_0^{x} f(x)f(y)\,dy\,dx$$

$$R=\int_0^{\infty}\int_0^{x}\alpha_1(1+x)^{-(\alpha_1+1)}\alpha_2(1+y)^{-(\alpha_2+1)}\,dy\,dx$$

$$R=\int_0^{\infty}\alpha_1(1+x)^{-(\alpha_1+1)}[-(1+x)^{-\alpha_2}+1]\,dx$$

$$R=[-\alpha_1\frac{(1+x)^{-(\alpha_1+\alpha_2)}}{-(\alpha_1+\alpha_2)}]_0^{\infty}+[-(1+x)^{-\alpha_1}]_0^{\infty}$$

$$R=\frac{\alpha_2}{\alpha_1+\alpha_2} \tag{2-37}$$

(2-7)The Reliability System (R_S) for the Series Components in the Stress-Strength (S-S)Model[1][40][45][49][60]

This section concern with finding formula for the reliability of the system contains two series components in stress-strength model when (X_1,X_2) are strength and Y is stress for power function distribution, modified exponentiated Lomax distribution, power Lomax distribution with shape parameters $\alpha_1, \alpha_2, and\ \alpha_3$.

Stress-strength model: when (PDF) for both strength(X_1,X_2) and stress Y are known, the reliability of the component can be analyzed ,in other words, the variation in strength and stress results in natural scatter and statistical distribution in these variables when the two distributions interference.

We will deriving the reliability system of two series components in stress-strength when the stress and strength are independent random variables of the aforementioned distributions.

(2-7-1) The Reliability System for Series Components in Stress-Strength for Power Function Distribution

Assume X_1, X_2 arises strength have (pow) with shape parameters α_1, α_2 and Y is stress have (pow) with shape parameter α_3

The probability density function for each random variables X_1, X_2 and Y is given as

$$f\left(x_1,\alpha_1\right)=\begin{cases}\alpha_1 x_1^{\alpha_1-1} & 0<x_1<1\\ 0 & \text{o}w\end{cases} \tag{2-38}$$

$$f\left(x_2,\alpha_2\right)=\begin{cases}\alpha_2 x_2^{\alpha_2-1} & 0<x_2<1 \\ 0 & \text{o}w\end{cases} \tag{2-39}$$

$$f\left(\text{y},\alpha_3\right)=\begin{cases}\alpha_3 y^{\alpha_3-1} & 0<y<1 \\ 0 & \text{o}w\end{cases} \tag{2-40}$$

The cumulative function for random variables X_1, X_2 and Y is given is

$$F\left(x_1,\alpha_1\right)=x_1^{\alpha_1} \qquad 0<\text{x}_1<1 \tag{2-41}$$

$$F\left(x_2,\alpha_2\right)=x_2^{\alpha_2} \qquad 0<\text{x}_2<1 \tag{2-42}$$

$$F\left(\text{y},\alpha_3\right)=y^{\alpha_3} \qquad 0<\text{y}<1 \tag{2-43}$$

The reliability system of two series components in the stress-strength model for power function distribution has the following from

Suppose $Z=\min(x_1,x_2)$

$$R_s=P(\text{y}<z)=\int_0^1\int_0^z f(y)f(z)dydz$$

$$R_s=\int_0^1 F_y(\text{z})\ \text{f(z)dz} \tag{2-44}$$

$$F_z(\text{z})=p(Z<z)=1-P(Z>z)$$

$$F_z(z)=1-p(\min x>z)$$

$$F_z(z)=1-p(x_1>z)p(x_2>z)$$

$$F_z(\text{z})=1-p(x_1>z)p(x_2>z)$$

$$F_z(\text{z})=1-(1-z^{\alpha_1})(1-z^{\alpha_2})$$

$$F_z(z) = z^{\alpha_1} + z^{\alpha_2} - z^{\alpha_1+\alpha_2} \tag{2-45}$$

We derive the equation (2-45) , we get

$$f(z) = \alpha_1 z^{\alpha_1 - 1} + \alpha_2 z^{\alpha_2 - 1} - (\alpha_1 + \alpha_2) z^{\alpha_1+\alpha_2-1} \tag{2-46}$$

We substituting equation (2-43)and (2-46) in (2-44), we get

$$R_s = \int_0^1 [\alpha_1 z^{\alpha_1-1} + \alpha_2 z^{\alpha_2-1} - (\alpha_1+\alpha_2) z^{\alpha_1+\alpha_2-1}] z^{\alpha_3}\, dz$$

$$R_s = [\frac{\alpha_1}{\alpha_1+\alpha_3} z^{\alpha_1+\alpha_3}]_0^1 + [\frac{\alpha_2}{\alpha_2+\alpha_3} z^{\alpha_2+\alpha_3}]_0^1 - [\frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2+\alpha_3} z^{\alpha_1+\alpha_2+\alpha_3}]_0^1$$

$$R_s = \frac{\alpha_1}{\alpha_1+\alpha_3} + \frac{\alpha_2}{\alpha_2+\alpha_3} - \frac{\alpha_1+\alpha_2}{\alpha_1+\alpha_2+\alpha_3} \tag{2-47}$$

(2-7-2) <u>The Reliability System for Series Components in Stress-Strength for Modified Exponentiated Function Distribution</u>

Assume X_1, X_2 arises strength have (MELD) with shape parameters α_1, α_2 and Y is stress have (MELD) with shape parameter α_3

The probability density function for each random variables X_1, X_2 and Y

$$f(x_1,\alpha_1) = \begin{cases} 2\alpha_1\left[1-(1+x_1)^{-2}\right]^{\alpha_1-1}(1+x_1)^{-3} & 0 < x_1 < \infty \\ & \alpha_1 > 0 \\ 0 & o w \end{cases} \tag{2-48}$$

$$f(x_2,\alpha_2) = \begin{cases} 2\alpha_2\left[1-(1+x_2)^{-2}\right]^{\alpha_2-1}(1+x_2)^{-3} & 0 < x_2 < \infty \\ & \alpha_2 > 0 \\ 0 & o w \end{cases} \tag{2-49}$$

$$f(\mathrm{y},\alpha_3)=\begin{cases}2\alpha_3\left[1-(1+y)^{-2}\right]^{\alpha_3-1}(1+y)^{-3} & 0<y<\infty \\ & \alpha_3>0 \\ 0 & \mathrm{o.}w\end{cases} \quad (2\text{-}50)$$

The cumulative function for random variables X_1, X_2 and Y

$$F(x_1,\alpha_1)=\left[1-(1+x_1)^{-2}\right]^{\alpha_1} \qquad 0<x_1<\infty \qquad (2\text{-}51)$$

$$F(x_2,\alpha_2)=\left[1-(1+x_2)^{-2}\right]^{\alpha_2} \qquad 0<x_2<\infty \qquad (2\text{-}52)$$

$$F(\mathrm{y},\alpha_3)=\left[1-(1+y)^{-2}\right]^{\alpha_3} \qquad 0<y<\infty \qquad (2\text{-}53)$$

The reliability system of two series components in the stress-strength model for modified exponentiated Lomax distribution has the following form:

Suppose $Z=\min(x_1,x_2)$

$$R_s=P(\mathrm{y}<z)=\int_0^{\infty}\int_0^{z}f(y)f(z)\,dydz$$

$$R_s=\int_0^{\infty}F_y(\mathrm{z})\ \mathrm{f(z)dz} \qquad (2\text{-}54)$$

$$F_z(\mathrm{z})=p(Z<z)=1-P(Z>z)$$

$$F_z(z)=1-p(\min x>z)$$

$$F_z(z)=1-p(x_1>z)p(x_2>z)$$

$$F_z(\mathrm{z})=1-p(x_1>z)p(x_2>z)$$

$$F_z(\text{z}) = 1 - [1 - (1 - (1 + z)^{-2})^{\alpha_1}][1 - (1 - (1 + z)^{-2})^{\alpha_2}]$$

$$F_z(\text{z}) = [1 - (1 + z)^{-2}]^{\alpha_1} + [1 - (1 + z)^{-2}]^{\alpha_2} - [1 - (1 + z)^{-2}]^{\alpha_1 + \alpha_2} \qquad (2\text{-}55)$$

We derive the equation (2-55) , we get

$$f(z) = \alpha_1[1 - (1 + z)^{-2}]^{\alpha_1 - 1} 2(1 + z)^{-3} + \alpha_2[1 - (1 + z)^{-2}]^{\alpha_2 - 1} 2(1 + z)^{-3}$$
$$-(\alpha_1 + \alpha_2)[1 - (1 + z)^{-2}]^{\alpha_1 + \alpha_2 - 1} 2(1 + z)^{-3} \qquad (2\text{-}56)$$

We substituting equation (2-53)and (2-56) in (2-54), we get

$$R_S = \int_0^\infty [\alpha_1[1 - (1 + z)^{-2}]^{\alpha_1 - 1} 2(1 + z)^{-3} + \alpha_2[1 - (1 + z)^{-2}]^{\alpha_2 - 1} 2(1 + z)^{-3}$$
$$-(\alpha_1 + \alpha_2)[1 - (1 + z)^{-2}]^{\alpha_1 + \alpha_2 - 1} 2(1 + z)^{-3}] \left[1 - (1 + z)^{-2}\right]^{\alpha_3} \text{dz}$$

$$R_s = [\frac{\alpha_1}{\alpha_1 + \alpha_3}(1 - (1 + z)^{-2})^{\alpha_1 + \alpha_3}]_0^\infty + [\frac{\alpha_2}{\alpha_2 + \alpha_3}(1 - (1 + z)^{-2})^{\alpha_2 + \alpha_3}]_0^\infty$$
$$-[\frac{\alpha_1 + \alpha_2}{\alpha_1 + \alpha_2 + \alpha_3}(1 - (1 + z)^{-2})^{\alpha_1 + \alpha_2 + \alpha_3}]_0^\infty$$

$$R_s = \frac{\alpha_1}{\alpha_1 + \alpha_3} + \frac{\alpha_2}{\alpha_2 + \alpha_3} - \frac{\alpha_1 + \alpha_2}{\alpha_1 + \alpha_2 + \alpha_3} \qquad (2\text{-}57)$$

(2-7-3) <u>The Reliability System for Series Components in Stress-Strength for Power Lomax Distribution</u>

Assume X_1, X_2 arises strength have (PoLo) with shape parameters α_1, α_2 and Y is stress have (PoLo) with shape parameter α_3

The probability density function for each random variables X_1, X_2 and Y is given as

$$f(x_1,\alpha_1)=\begin{cases}\alpha_1(1+x_1)^{-(\alpha_1+1)} & 0<x_1<\infty \\ 0 & o.w\end{cases} \quad (2\text{-}58)$$

$$f(x_2,\alpha_2)=\begin{cases}\alpha_2(1+x_2)^{-(\alpha_2+1)} & 0<x_2<\infty \\ 0 & o.w\end{cases} \quad (2\text{-}59)$$

$$f(y,\alpha_3)=\begin{cases}\alpha_3(1+y)^{-(\alpha_3+1)} & 0<y<\infty \\ 0 & o.w\end{cases} \quad (2\text{-}60)$$

The cumulative function for random variables X_1, X_2 and Y is given is

$$F(x_1,\alpha_1)=1-(1+x_1)^{-\alpha_1} \qquad 0<x_1<\infty \quad (2\text{-}61)$$

$$F(x_2,\alpha_2)=1-(1+x_2)^{-\alpha_2} \qquad 0<x_2<\infty \quad (2\text{-}62)$$

$$F(y,\alpha_3)=1-(1+y)^{-\alpha_3} \qquad 0<y<\infty \quad (2\text{-}63)$$

The reliability system of two series components in the stress-strength model for Power Lomax distribution has the following from

Suppose $Z=\min(x_1,x_2)$

$$R_s=P(y<z)=\int_0^\infty\int_0^z f(y)f(z)\,dy\,dz$$

$$R_s=\int_0^\infty F_y(z)\ f(z)dz \quad (2\text{-}64)$$

$$F_z(z)=p(Z<z)=1-P(Z>z)$$

$$F_z(z)=1-p(\min x>z)$$

$$F_z(z) = 1 - p(x_1 > z)p(x_2 > z)$$

$$F_z(\mathrm{z}) = 1 - p(x_1 > z)p(x_2 > z)$$

$$F_z(\mathrm{z}) = 1 - (1+z)^{-\alpha_1}(1+z)^{-\alpha_2}$$

$$F_z(\mathrm{z}) = 1 - (1+z)^{-(\alpha_1+\alpha_2)} \tag{2-65}$$

We derive the equation (2-65) , we get

$$f(z) = (\alpha_1+\alpha_2)(1+z)^{-(\alpha_1+\alpha_2)-1} \tag{2-66}$$

We substituting equation (2-63)and (2-66) in (2-64), we get

$$R_s = \int_0^\infty (\alpha_1+\alpha_2)(1+z)^{-(\alpha_1+\alpha_2)-1}[1-(1+z)^{-\alpha_3}]\mathrm{dz}$$

$$R_s = [\frac{\alpha_1+\alpha_2}{-(\alpha_1+\alpha_2)}(1+z)^{-(\alpha_1+\alpha_2)}]_0^\infty - [\frac{\alpha_1+\alpha_2}{-(\alpha_1+\alpha_2+\alpha_3)}(1+z)^{-(\alpha_1+\alpha_2+\alpha_3)}]_0^\infty$$

$$R_s = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3} \tag{2-67}$$

Chapter Three

Estimation Methods for System Reliability

Chapter Three

Estimation Methods for system Reliability

(3-1) Introduction

There are many methods of estimation and often analytic methods are the most accurate of other methods, we will study some estimation methods for the studied distribution, and compare them depending on mean square error (MSE).

An estimator was statistic that specifies how to use the sample data to estimate an unknown parameter of the population, in the area of system reliability estimates are considered very important and useful because they reflect the operational ability of the individual component in the system [25][28].

(3-2) Estimation Methods the Reliability System of Power Function Distribution

(3-2-1) Maximum Likelihood Estimation Method (MLE) [19][50][51]

This method was considered one of the most important and most widely used methods and can be obtaining appoint estimator of parameters in any distribution.

A maximum likelihood method has many excellent statistical properties such as consistent, sufficient statistic and asymptotically normally distribution, so it is popular and favorite method [7].

Let $x_1, x_2, ..., x_n$ be a random sample of size n and a probability density function

$$f\left(x_i, \alpha_1\right) = \alpha_1 x_i^{\alpha_1 - 1} \qquad 0<x_i<1 \qquad i=1,2.......n$$

And let $y_1, y_2, ..., y_m$ be a random sample of size m and a probability density function

$$f\left(y_j, \alpha_2\right) = \alpha_2 y_j^{\alpha_2 - 1} \qquad 0<y_j<1 \qquad j=1,2......m$$

And let $z_1, z_2, ..., z_w$ be a random sample of size w and a probability density function

$$f\left(z_q, \alpha_3\right) = \alpha_3 z_q^{\alpha_3 - 1} \qquad 0<z_q<1 \qquad q=1,2......w$$

The maximum likelihood for α_1, α_2 and α_3 as follow

$$L = L(x, y, z, \alpha_1, \alpha_2, \alpha_3) = \prod_{i=1}^{n} f\left(x, \alpha_1\right) \prod_{j=1}^{m} f\left(y, \alpha_2\right) \prod_{q=1}^{w} f\left(z, \alpha_3\right)$$

$$L = \left(\alpha_1\right)^n \prod_{i=1}^{n} x_i^{\alpha_1 - 1} \left(\alpha_2\right)^m \prod_{j=1}^{m} y_j^{\alpha_2 - 1} \left(\alpha_3\right)^w \prod_{w=1}^{q} z_q^{\alpha_3 - 1}$$

Taking(ln)of L we get

$$LnL = Ln\ \alpha_1^n + Ln \prod_{i=1}^{n} x_i^{\alpha_1 - 1} + Ln\ \alpha_2^m + Ln \prod_{j=1}^{m} y_j^{\alpha_2 - 1} + Ln\ \alpha_3^w + Ln \prod_{q=1}^{w} z_q^{\alpha_3 - 1}$$

$$LnL = nLn\ \alpha_1 + (\alpha_1 - 1)\sum_{i=1}^{n} Lnx_i + mLn\ \alpha_2 + (\alpha_2 - 1)\sum_{j=1}^{m} Lny_j$$

$$+wLn\ \alpha_3 + (\alpha_3 - 1)\sum_{q=1}^{w} Lnz_q$$

The partial derivative for Ln-L with respect to parameter α_1

$$\frac{\partial LnL}{\partial \alpha_1} = \frac{n}{\alpha_1} + \sum_{i=1}^{n} Lnx_i$$

We equate partial derivation with zero

$$\frac{\partial LnL}{\partial \alpha_1} = 0$$

$$\frac{n}{\alpha_1} + \sum_{i=1}^{n} Lnx_i = 0$$

$$\frac{n}{\alpha_1} = -\sum_{i=1}^{n} Lnx_i$$

We get

$$\hat{\alpha}_{1mle} = \frac{-n}{\sum_{i=1}^{n} Lnx_i} \qquad (3\text{-}1)$$

By the same way. We get

$$\hat{\alpha}_{2mle} = \frac{-m}{\sum_{j=1}^{m} Lny_j} \qquad (3\text{-}2)$$

$$\hat{\alpha}_{3mle} = \frac{-w}{\sum_{q=1}^{w} Lnz_q} \quad (3\text{-}3)$$

By substituting equations (3-1) and (3-2) in equation (2-27) , we get the maximum likelihood estimator of the reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{1mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle}} \quad (3\text{-}4)$$

The maximum likelihood estimator for reliability system R_S for series component of system for power function distribution ,and by substituting equations (3-1), (3-2) and (3-3) in equation (2-47) as follows

$$\hat{R}_{s(mle)} = \frac{\hat{\alpha}_{1mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{3mle}} + \frac{\hat{\alpha}_{2mle}}{\hat{\alpha}_{2mle} + \hat{\alpha}_{3mle}} - \frac{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle} + \hat{\alpha}_{3mle}} \quad (3\text{-}5)$$

(3-2-2) Moment Method (MOM) [54][57]

The method of moments was another technique commonly used in the field for parameter estimation. It is also express as (MoM), and can be summarized by equating the population moments $M_k = E\left(x_i^k\right)$

With the sample moment

$$M_k = \frac{1}{n}\sum_{i=1}^{n} x_i^k$$

$k = 1, 2, ...$ is the number of unknown parameter to be estimate, then solving the resultant equation to obtain the moment estimator.

The first moment for power function distribution

$$E(x)=\frac{\alpha_1}{\alpha_1+1}$$

Equating the population moments with the sample moments

$$E(x)=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\frac{\alpha_1}{\alpha_1+1}=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\alpha_1=\alpha_1\frac{1}{n}\sum_{i=1}^{n}x_i+\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\alpha_1-\alpha_1\frac{1}{n}\sum_{i=1}^{n}x_i=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\alpha_1\left(1-\frac{\sum_{i=1}^{n}x_i}{n}\right)=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\hat{\alpha}_{1mom}=\frac{\sum_{i=1}^{n}x_i}{n-\sum_{i=1}^{n}x_i} \qquad (3\text{-}6)$$

By the same way

$$\hat{\alpha}_{2mom}=\frac{\sum_{j=1}^{m}y_j}{m-\sum_{j=1}^{m}y_j} \qquad (3\text{-}7)$$

$$\hat{\alpha}_{3mom}=\frac{\sum_{q=1}^{w}z_q}{w-\sum_{q=1}^{w}z_q} \qquad (3\text{-}8)$$

By substituting equations (3-6) and (3-7) in equation (2-27) , we get the moment estimator of reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{1mom}}{\hat{\alpha}_{1mom} + \hat{\alpha}_{2mom}} \tag{3-9}$$

The moment estimator for reliability system R_S for series component of system for power function distribution, and by substituting equations (3-6), (3-7)and (3-8) in equation (2-47) as follows

$$\hat{R}_{s\,(\mathrm{mom})} = \frac{\hat{\alpha}_{1mom}}{\hat{\alpha}_{1mom} + \hat{\alpha}_{3mom}} + \frac{\hat{\alpha}_{2mom}}{\hat{\alpha}_{2mom} + \hat{\alpha}_{3mom}} - \frac{\hat{\alpha}_{1mom} + \hat{\alpha}_{2mom}}{\hat{\alpha}_{1mom} + \hat{\alpha}_{2mom} + \hat{\alpha}_{3mom}} \tag{3-10}$$

(3-2-3) <u>The Least Squares Estimator (LS) [6] [33]</u>

The least squares method can be applied in reliability engineering, mathematics problems and the estimation of probability distribution

Parameter, assume that there is linear relationship between two variables.

By using cdf of power function distribution

$$F\left(x_1\right) = X_i^{\alpha_1}$$

$$X_i = \left[F\left(X_i\right)\right]^{\frac{1}{\alpha_1}}$$

Take logarithm for both sides

$$Ln\left(x_i\right) = \frac{1}{\alpha_1} Ln\left[F\left(x_i\right)\right]$$

$$y = ax + b$$

$$y = Ln(x_i)$$

$$a = \frac{1}{\alpha_1}$$

$$X_i = Ln\left[F(x_i)\right]$$

$$b = 0$$

$$a = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}$$

$$\frac{1}{\alpha_1} = \frac{\sum_{i=1}^{n} LnF(x_i) Lnx_i - \frac{\sum_{i=1}^{n} LnF(x_i) \sum_{i=1}^{n} Lnx_i}{n}}{\sum_{i=1}^{n}\left[LnF(x_i)\right]^2 - \frac{\sum_{i=1}^{n}\left[LnF(x_i)\right]^2}{n}}$$

$$\hat{\alpha}_{1LS} = \frac{\sum_{i=1}^{n}\left[LnF(x_i)\right]^2 - \frac{\left[\sum_{i=1}^{n} LnF(x_i)\right]^2}{n}}{\sum_{i=1}^{n} Ln\ F(x_i) Lnx_i - \frac{\sum_{i=1}^{n} Ln\ F(x_i) \sum_{i=1}^{n} Lnx_i}{n}} \qquad (3\text{-}11)$$

By the same way

$$\hat{\alpha}_{2LS} = \frac{\sum_{j=1}^{m}\left[LnF(y_j)\right]^2 - \frac{\left[\sum_{j=1}^{m} LnF(y_j)\right]^2}{m}}{\sum_{j=1}^{m} Ln\ \mathrm{F}(y_j) Lny_j - \frac{\sum_{j=1}^{m} Ln\ \mathrm{F}(y_j) \sum_{j=1}^{m} Lny_j}{m}} \qquad (3\text{-}12)$$

$$\hat{\alpha}_{3LS} = \frac{\sum_{q=1}^{w}\left[LnF\left(z_q\right)\right]^2 - \frac{\left[\sum_{q=1}^{w} LnF\left(z_q\right)\right]^2}{w}}{\sum_{q=1}^{w} Ln\ \mathrm{F}\left(z_q\right) Lnz_q - \frac{\sum_{q=1}^{w} Ln\ \mathrm{F}\left(z_q\right)\sum_{q=1}^{w} Lnz_q}{w}} \qquad (3\text{-}13)$$

By substituting equations (3-11) and (3-12) in equation (2-27) , we get the least square estimator of the system reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{Ls} = \frac{\hat{\alpha}_{1Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls}} \qquad (3\text{-}14)$$

The least square estimator for reliability system R_S for series component of system for power function distribution , and by substituting equations (3-11), (3-12) and (3-13) in equation (2-47) as follows

$$\hat{R}_{s\,(\mathrm{Ls})} = \frac{\hat{\alpha}_{1Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{3Ls}} + \frac{\hat{\alpha}_{2Ls}}{\hat{\alpha}_{2Ls} + \hat{\alpha}_{3Ls}} - \frac{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls} + \hat{\alpha}_{3LS}} \qquad (3\text{-}15)$$

(3-2-4) Shrinkage Estimation Method (sh)

The shrinkage estimation method is the Bayesin approach depending on prior information, Thompson in 1968, introduced the basic reasons for use prior estimate. He suggested the problem of Shrink the usual estimator $\hat{\alpha}$ of the parameter α toward prior information. It was considered α as initial value α_0 from the past and usual estimator α_{mle} through consolidation them by Shrinkage weight factor $\phi(\hat{\alpha})$, $0 < \phi(\hat{\alpha}) < 1$ [8] [39] as follows:

$$\hat{\alpha}_{sh} = \phi(\hat{\alpha})\hat{\alpha}_{ub} + \left(1 - \phi(\hat{\alpha})\right)\alpha_0 \qquad (3\text{-}16)$$

Noted that : $\hat{\alpha}_{mle} = \frac{-n}{\sum_{i=1}^{n} Lnx_i}$ is biased

To find $\hat{\alpha}_{ub}$ $x \sim pow(\alpha_1)$

Since E($\hat{\alpha}_{mle}$)= $\frac{n}{n-1}\alpha$

$$\hat{\alpha}_{1ub} = \frac{-n}{\sum_{i=1}^{n} Lnx_i} \times \frac{n-1}{n}$$

$$\hat{\alpha}_{1ub} = \frac{-(n-1)}{\sum_{i=1}^{n} Lnx_i} \qquad (3\text{-}17)$$

So, $E(\hat{\alpha}_{ub}) = \alpha$ which is unbiased estimator

And by the same way founded $\hat{\alpha}_{2ub}, \hat{\alpha}_{3ub}$

The Shrinkage weight factor may be function or may be constant. We can be introduced three different estimation as follows

(3-2-4-1) Shrinkage Estimator Function(sh_1) [51][52]

In this case, the Shrinkage weight factor as a function of sample size n as below.

$$\phi_1(\alpha) = \left|\frac{\sin\ n}{n}\right|,\ \phi_2(\alpha) = \left|\frac{\sin\ m}{m}\right|,\ \phi_3(\alpha) = \left|\frac{\sin\ w}{w}\right|$$

It was implemented in the equation (3-16)

$$\alpha_{sh_1} = \left|\frac{\sin(n)}{n}\right|\alpha_{ub} + (1 - \left|\frac{\sin(n)}{n}\right|)\alpha_0 \qquad (3\text{-}18)$$

Hence, the shrinkage estimator using in shrinkage Wight equation (3-16)

We put (3-17) in (3-18)

$$\hat{\alpha}_{1sh_1} = \left|\frac{\sin\ \text{n}}{n}\right| \frac{-(n-1)}{\sum_{i=1}^{n} Lnx_i} + \left(1 - \left|\frac{\sin\ \text{n}}{n}\right|\right)\alpha_0 \qquad (3\text{-}19)$$

By the same way

$$\hat{\alpha}_{2sh_1} = \left|\frac{\sin \text{m}}{m}\right| \frac{-(m-1)}{\sum_{j=1}^{m} Lny_j} + \left(1 - \left|\frac{\sin(\text{m})}{m}\right|\right)\alpha_0 \qquad (3\text{-}20)$$

$$\hat{\alpha}_{3sh_1} = \left|\frac{\sin\ \text{w}}{w}\right| \frac{-(w-1)}{\sum_{q=1}^{w} Lnz_q} + \left(1 - \left|\frac{\sin(w)}{w}\right|\right)\alpha_0 \qquad (3\text{-}21)$$

By substituting equations (3-19) and (3-20) in equation (2-27) , we get the shrinkage estimator function of the reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{sh_1} = \frac{\hat{\alpha}_{1sh_1}}{\hat{\alpha}_{1sh_1} + \hat{\alpha}_{2sh_1}} \qquad (3\text{-}22)$$

The shrinkage estimator for reliability system R_S for series component of system for power function distribution ,and by substituting equations (3-19), (3-20) and (3-21) in equation (2-47) as follows

$$\hat{R}_{s\,(\text{sh1})} = \frac{\hat{\alpha}_{1sh1}}{\hat{\alpha}_{1sh1} + \hat{\alpha}_{3sh1}} + \frac{\hat{\alpha}_{2sh1}}{\hat{\alpha}_{2sh1} + \hat{\alpha}_{3sh1}} - \frac{\hat{\alpha}_{1sh1} + \hat{\alpha}_{2sh1}}{\hat{\alpha}_{1sh1} + \hat{\alpha}_{2sh1} + \hat{\alpha}_{3sh1}} \qquad (3\text{-}23)$$

(3-2-4-2) <u>Constant Shrinkage Estimator (sh_2) [51][52]</u>

In this case using to suggest study of constant shrinkage weight factor will be assumed $\phi(\hat{\alpha}) = k$, k=0.001

And used the equation (3-16)

$$\hat{\alpha}_{1sh2} = k_1\hat{\alpha}_{1ub} + (1-k_1)\alpha_0 \tag{3-24}$$

And by the same way

$$\hat{\alpha}_{2sh2} = k_2\hat{\alpha}_{2ub} + (1-k_2)\alpha_0 \tag{3-25}$$

$$\hat{\alpha}_{3sh2} = k_3\hat{\alpha}_{3ub} + (1-k_3)\alpha_0 \tag{3-26}$$

By substituting equations (3-24) and (3-25) in equation (2-27) , we get the constant shrinkage estimator of the reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{sh_2} = \frac{\hat{\alpha}_{1sh_2}}{\hat{\alpha}_{1sh_2} + \hat{\alpha}_{2sh_2}} \tag{3-27}$$

The constant shrinkage estimator for reliability system R_S for series component of system for power function distribution, and by substituting equations (3-19), (3-20) and (3-21) in equation (2-47) as follows

$$\hat{R}_{s(\mathrm{sh}2)} = \frac{\hat{\alpha}_{1sh2}}{\hat{\alpha}_{1sh2} + \hat{\alpha}_{3sh2}} + \frac{\hat{\alpha}_{2sh2}}{\hat{\alpha}_{2sh2} + \hat{\alpha}_{3sh2}} - \frac{\hat{\alpha}_{1sh2} + \hat{\alpha}_{2sh2}}{\hat{\alpha}_{1sh2} + \hat{\alpha}_{2sh2} + \hat{\alpha}_{3sh2}} \tag{3-28}$$

(3-2-4-3) <u>Beta Shrinkage Estimator (sh_3) [51][52]</u>

In this cases using the Beta shrinkage weight factor will be assumed as $\phi_1(\hat{\alpha}) = B(1,n)$, $\phi_2(\hat{\alpha}) = B(1,\mathrm{m})$ and $\phi_3(\hat{\alpha}) = B(1,\mathrm{w})$,and substituted in the equation (3-16)

$$\alpha_{1sh_3} = B(1,n)\hat{\alpha}_{1ub} + \left(1 - B(1,n)\right)\alpha_0 \tag{3-29}$$

And by the same way

$$\alpha_{2sh_3} = B\left(1,\mathrm{m}\right)\hat{\alpha}_{2ub} + \left(1 - B\left(1,\mathrm{m}\right)\right)\alpha_0 \tag{3-30}$$

$$\alpha_{3sh_3} = B\left(1,\mathrm{w}\right)\hat{\alpha}_{3ub} + \left(1 - B\left(1,\mathrm{w}\right)\right)\alpha_0 \tag{3-31}$$

By substituting equations (3-29) and (3-30) in equation (2-27) , we get beta shrinkage estimator of the reliability in (S-S) model for power function distribution which is contain one component as below

$$\hat{R}_{sh3} = \frac{\hat{\alpha}_{1sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3}} \tag{3-32}$$

The beta shrinkage estimator for reliability system R_S for series component of system for power function distribution, and by substituting equations (3-29), (3-30) and (3-31) in equation (2-47) as follows

$$\hat{R}_{s(\mathrm{sh}3)} = \frac{\hat{\alpha}_{1sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{3sh3}} + \frac{\hat{\alpha}_{2sh3}}{\hat{\alpha}_{2sh3} + \hat{\alpha}_{3sh3}} - \frac{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3} + \hat{\alpha}_{3sh3}} \tag{3-33}$$

(3-3) <u>Estimation Methods the Reliability System of Modified Exponenetiated Lomax Distribution</u>

(3-3-1) <u>Maximum likelihood method (MLE)[3][37]</u>

Let$x_1, x_2 \ldots\ldots x_n$ be a random sample of size n and probability density function

$$f\left(x,\alpha_1\right) = 2\alpha_1\left[1-\left(1+x\right)^{-2}\right]^{\alpha_1-1}\left(1+x\right)^{-3} \qquad 0 < \mathrm{x} < \infty$$

And let$y_1, y_2 \ldots\ldots y_m$ be a random sample of size m and probability density function

$$f\left(y,\alpha_2\right) = 2\alpha_2\left[1-\left(1+y\right)^{-2}\right]^{\alpha_2-1}\left(1+y\right)^{-3} \qquad 0 < \mathrm{y} < \infty$$

And let $z_1, z_2 \ldots\ldots z_w$ be a random sample of size w and probability density function

$$f\left(z,\alpha_3\right)=2\alpha_3\left[1-\left(1+z\right)^{-2}\right]^{\alpha_3-1}\left(1+z\right)^{-3} \qquad 0<z<\infty$$

Then

$$Lf\left(x,y,z,\alpha_1,\alpha_2,\alpha_3\right)=2^n\alpha_1^n\prod_{i=1}^{n}\left[1-\left(1+x_i\right)^{-2}\right]^{\alpha_1-1}\prod_{i=1}^{n}\left(1+x_i\right)^{-3}2^m\alpha_2^m\prod_{j=1}^{m}\left[1-\left(1+y_j\right)^{-2}\right]^{\alpha_2-1}$$

$$\prod_{j=1}^{m}\left(1+y_j\right)^{-3}2^w\alpha_3^w\prod_{q=1}^{w}\left[1-\left(1+z_q\right)^{-2}\right]^{\alpha_3-1}\prod_{q=1}^{w}\left(1+z_q\right)^{-3}$$

Take Ln for both sides

$$LnL=nLn2+nLn\alpha_1+\left(\alpha_1-1\right)\sum_{i=1}^{n}Ln\left[1-\left(1+x_i\right)^{-2}\right]-3\sum_{i=1}^{n}Ln\left(1+x_i\right)+mLn2$$

$$+mLn\alpha_2+\left(\alpha_2-1\right)\sum_{j=1}^{m}Ln\left[1-\left(1+y_j\right)^{-2}\right]-3\sum_{j=1}^{m}Ln\left(1+y_j\right)+wLn2+wLn\alpha_3$$

$$+\left(\alpha_3-1\right)\sum_{q=1}^{w}Ln\left[1-\left(1+z_q\right)^{-2}\right]-3\sum_{q=1}^{w}Ln\left(1+z_q\right)$$

The partial derivative for ln-function with respect to parameter α_1

$$\frac{\partial LnL}{\partial\alpha_1}=\frac{n}{\alpha_1}+\sum_{i=1}^{n}Ln\left[1-\left(1+x_i\right)^{-2}\right]$$

We equate partial derivation with zero

$$\frac{\partial LnL}{\partial\alpha_1}=0$$

$$\frac{n}{\alpha_1}+\sum_{i=1}^{n}Ln\left[1-\left(1+x_i\right)^{-2}\right]=0$$

$$\hat{\alpha}_{1mle} = \frac{-n}{\sum_{i=1}^{n} Ln\left[1-\left(1+x_i\right)^{-2}\right]} \tag{3-34}$$

By the same way

$$\hat{\alpha}_{2mle} = \frac{-m}{\sum_{j=1}^{m} Ln\left[1-\left(1+y_j\right)^{-2}\right]} \tag{3-35}$$

$$\hat{\alpha}_{3mle} = \frac{-w}{\sum_{q=1}^{w} Ln\left[1-\left(1+z_q\right)^{-2}\right]} \tag{3-36}$$

By substituting equations (3-34) and (3-35) in equation (2-32) , we get the maximum likelihood estimator of reliability in (S-S) model for exponentiated modified Lomax distribution which is contain one component as below

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{1mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle}} \tag{3-37}$$

The maximum likelihood estimator for reliability system R_S for series component of system for exponentiated modified Lomax distribution ,and by substituting equations (3-34), (3-35) and (3-36) in equation (2-57) as follows

$$\hat{R}_{s(mle)} = \frac{\hat{\alpha}_{1mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{3mle}} + \frac{\hat{\alpha}_{2mle}}{\hat{\alpha}_{2mle} + \hat{\alpha}_{3mle}} - \frac{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle} + \hat{\alpha}_{3mle}} \tag{3-38}$$

(3-3-2) Moment Method (MOM)

Let $x_1, x_2 \ldots\ldots. x_n$ be a random sample of size n follows MELD(α_1), from the properties, the first moment for modified Exponentiated Lomax distribution of x, and by equating the population moments with the sample moments

$$E(x)=\alpha_1\left[B\left(\frac{1}{2},\alpha_1\right)-\frac{1}{\alpha_1}\right]=\frac{\sum_{i=1}^{n}x_i}{n}$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma(\alpha_1)}{\Gamma\left(\frac{1}{2}+\alpha_1\right)}-\frac{1}{\alpha_1}=\frac{\sum_{i=1}^{n}x_i}{\alpha_1 n}$$

$$\frac{\Gamma\left(\frac{1}{2}\right)\Gamma(\alpha_1)}{\Gamma\left(\alpha_1+\frac{1}{2}\right)}=\frac{\bar{x}}{\alpha_1}+\frac{1}{\alpha_1}$$

$$\frac{\Gamma\left(\frac{1}{2}\right)\Gamma(\alpha_1)}{\Gamma\left(\alpha_1+\frac{1}{2}\right)}=\frac{\bar{x}+1}{\alpha_1}$$

$$\hat{\alpha}_{1mom}=\frac{\Gamma\left(\frac{1}{2}+\alpha_0\right)\left(\frac{\sum_{i=1}^{n}x_i}{n}+1\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(\alpha_0)} \qquad (3\text{-}39)$$

By the first moment of y and z

$$\hat{\alpha}_{2mom}=\frac{\Gamma\left(\frac{1}{2}+\alpha_0\right)\left(\frac{\sum_{j=1}^{m}y_j}{m}+1\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(\alpha_0)} \qquad (3\text{-}40)$$

$$\hat{\alpha}_{3mom} = \frac{\Gamma\left(\frac{1}{2}+\alpha_0\right)\left(\frac{\sum_{q=1}^{w} z_q}{w}+1\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\alpha_0\right)} \tag{3-41}$$

By substituting equations (3-39) and (3-40) in equation (2-32) , we get the moment estimator of the reliability in (S-S) model for modified exponentiated Lomax distribution which is contain one component as below

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{1mom}}{\hat{\alpha}_{1mom}+\hat{\alpha}_{2mom}} \tag{3-42}$$

The moment estimator for reliability system R_S for series component of system for modified exponentiated Lomax distribution , and by substituting equations (3-39),(3-40) and (3-41) in equation (2-57) as follows

$$\hat{R}_{s\,(\mathrm{mom})} = \frac{\hat{\alpha}_{1mom}}{\hat{\alpha}_{1mom}+\hat{\alpha}_{3mom}} + \frac{\hat{\alpha}_{2mom}}{\hat{\alpha}_{2mom}+\hat{\alpha}_{3mom}} - \frac{\hat{\alpha}_{1mom}+\hat{\alpha}_{2mom}}{\hat{\alpha}_{1mom}+\hat{\alpha}_{2mom}+\hat{\alpha}_{3mom}} \tag{3-43}$$

(3-3-3) Least Square Method (LS)[6][33]

Let$x_1, x_2 \ldots\ldots.. x_n$ a random variable the probability density function (p.d.f) of modified Generalized Lomax distribution

$$f\left(x,\alpha_1\right) = 2\alpha_1\left[1-\left(1+x\right)^{-2}\right]^{\alpha_1-1}\left(1+x\right)^{-3} \qquad 0 < x < \infty$$

Now, to estimate parameters (α_1) using least square method

$$F\left(x_i\right) = \left[1-\left(1+x_i\right)^{-2}\right]^{\alpha_1}$$

$$\left[F\left(x_i\right)\right]^{\frac{1}{\alpha_1}}=1-\left(1+x_i\right)^{-2}$$

Take Ln for both sides

$$Ln\left[F\left(x_i\right)\right]^{\frac{1}{\alpha_1}}=Ln\left[1-\left(1+x_i\right)^{-2}\right]$$

$$\frac{1}{\alpha_1}Ln\left[F\left(x_i\right)\right]=Ln\left[1-\left(1+x_i\right)^{-2}\right]$$

$$ax+b=y_i$$

$$a=\frac{1}{\alpha_1}$$
$$b=0$$
$$x_i=Ln\left[F\left(x_i\right)\right]$$
$$y_i=Ln\left[1-\left(1+x_i\right)^{-2}\right]$$

$$a=\frac{\sum_{i=1}^{n}x_iy_i-\frac{\sum_{i=1}^{n}x_iy_i}{n}}{\sum_{i=1}^{n}x_i^{2}-\frac{\left(\sum_{i=1}^{n}x_i\right)^2}{n}}$$

$$\frac{1}{\alpha_1}=\frac{\sum_{i=1}^{n}LnF\left(x_i\right)Ln\left[1-\left(1+x_i\right)^{-2}\right]-\frac{\sum_{i=1}^{n}LnF\left(x_i\right)\sum_{i=1}^{n}Ln\left[1-\left(1+x_i\right)^{-2}\right]}{n}}{\sum_{i=1}^{n}\left[LnF\left(x_i\right)\right]^2-\frac{\left[\sum_{i=1}^{n}LnF\left(x_i\right)\right]^2}{n}}$$

$$\hat{\alpha}_{1ls}=\frac{\sum_{i=1}^{n}\left[LnF\left(x_i\right)\right]^2-\frac{\left[\sum_{i=1}^{n}LnF\left(x_i\right)\right]^2}{n}}{\sum_{i=1}^{n}LnF\left(x_i\right)Ln\left[1-\left(1+x_i\right)^{-2}\right]-\frac{\sum_{i=1}^{n}LnF\left(x_i\right)\sum_{i=1}^{n}Ln\left[1-\left(1+x_i\right)^{-2}\right]}{n}} \qquad (3\text{-}44)$$

By the same way

$$\hat{\alpha}_{2ls} = \frac{\sum_{j=1}^{m}\left[LnF\left(y_j\right)\right]^2 - \frac{\left[\sum_{j=1}^{m}LnF\left(y_j\right)\right]^2}{m}}{\sum_{j=1}^{m}LnF\left(y_j\right)Ln\left[1-\left(1+y_j\right)^{-2}\right] - \frac{\sum_{j=1}^{m}LnF\left(y_j\right)\sum_{j=1}^{m}Ln\left[1-\left(1+y_j\right)^{-2}\right]}{m}} \quad (3\text{-}45)$$

$$\hat{\alpha}_{3ls} = \frac{\sum_{q=1}^{w}\left[LnF\left(z_q\right)\right]^2 - \frac{\left[\sum_{q=1}^{w}LnF\left(z_q\right)\right]^2}{w}}{\sum_{q=1}^{w}LnF\left(z_q\right)Ln\left[1-\left(1+z_q\right)^{-2}\right] - \frac{\sum_{q=1}^{w}LnF\left(z_q\right)\sum_{q=1}^{w}Ln\left[1-\left(1+z_q\right)^{-2}\right]}{w}} \quad (3\text{-}46)$$

By substituting equations (3-44) and (3-45) in equation (2-32) , we get the least square estimator of the reliability in (S-S) model for modified exponentiated Lomax distribution which is contain one component as below

$$\hat{R}_{Ls} = \frac{\hat{\alpha}_{1Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls}} \quad (3\text{-}47)$$

The least square estimator for reliability system R_S for series component of system for modified exponentiated Lomax distribution ,and by substituting equations (3-61), (3-62) and (3-63) in equation (2-57) as follows

$$\hat{R}_{s\,(\mathrm{Ls})} = \frac{\hat{\alpha}_{1Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{3Ls}} + \frac{\hat{\alpha}_{2Ls}}{\hat{\alpha}_{2Ls} + \hat{\alpha}_{3Ls}} - \frac{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls} + \hat{\alpha}_{3LS}} \quad (3\text{-}48)$$

(3-3-4) Shrinkage Estimation Methods (sh)[22][51]

Now, we will study estimation of parameter for modified Exponentiated Lomax distribution by using equation

$$\hat{\alpha}_{sh} = \phi(\hat{\alpha})\hat{\alpha}_{ub} + (1-\phi(\hat{\alpha}))\alpha_0 \qquad (3\text{-}49)$$

Noted that: $\hat{\alpha}_{mle} == \dfrac{-n}{\sum_{i=1}^{n} Ln\left[1-(1+x_i)^{-2}\right]}$ is biased

$$E(\hat{\alpha}_{mle}) = \frac{n}{n-1}\alpha$$

To find α_{ub}

$$\hat{\alpha}_{ub} = \frac{n-1}{n} \times \hat{\alpha}_{mle}$$

$$= \frac{-n}{\sum_{i=1}^{n} Ln\left[1-(1+x_i)^{-2}\right]} \times \frac{(n-1)}{n}$$

$$\hat{\alpha}_{1ub} = \frac{-(n-1)}{\sum_{i=1}^{n} Ln\left[1-(1+x_i)^{-2}\right]} \qquad (3\text{-}50)$$

And by the same way founded $\hat{\alpha}_{2ub}, \hat{\alpha}_{3ub}$

We can be introduced three different estimation as follows

(3-3-4-1) Shrinkage Estimator Function (sh_1) [22][51]

In this case, the Shrinkage weight factor as function $n, \phi_1(\hat{\alpha}) = \left|\dfrac{\sin\ n}{n}\right|$,

$\phi_2(\hat{\alpha}) = \left|\dfrac{\sin\ m}{m}\right|$ and $\phi_3(\hat{\alpha}) = \left|\dfrac{\sin\ w}{w}\right|$ it was implemented in the equation (3-49)

$$\hat{\alpha}_{1sh_1} = \left|\frac{\sin\ n}{n}\right|\hat{\alpha}_{1ub} + \left(1 - \left|\frac{\sin\ n}{n}\right|\right)\alpha_0 \tag{3-51}$$

$$\hat{\alpha}_{2sh_1} = \left|\frac{\sin\ m}{m}\right|\hat{\alpha}_{2ub} + \left(1 - \left|\frac{\sin\ m}{m}\right|\right)\alpha_0 \tag{3-52}$$

$$\hat{\alpha}_{3sh_1} = \left|\frac{\sin\ w}{w}\right|\hat{\alpha}_{3ub} + \left(1 - \left|\frac{\sin\ w}{w}\right|\right)\alpha_0 \tag{3-53}$$

By substituting equations (3-51) and (3-52) in equation (2-32) , we get the shrinkage estimator of the reliability in (S-S) model for modified exponentiated Lomax distribution which is contain one component as below

$$\hat{R}_{sh_1} = \frac{\hat{\alpha}_{1sh_1}}{\hat{\alpha}_{1sh_1} + \hat{\alpha}_{2sh_1}} \tag{3-54}$$

The shrinkage estimator function for reliability system R_S for series component of system for modified exponentiated Lomax distribution, and by substituting equations (3-46), (3-47) and (3-48) in equation (2-57) as follows

$$\hat{R}_{s\,(\mathrm{sh1})} = \frac{\hat{\alpha}_{1sh1}}{\hat{\alpha}_{1sh1} + \hat{\alpha}_{3sh1}} + \frac{\hat{\alpha}_{2sh1}}{\hat{\alpha}_{2sh1} + \hat{\alpha}_{3sh1}} - \frac{\hat{\alpha}_{1sh1} + \hat{\alpha}_{2sh1}}{\hat{\alpha}_{1sh1} + \hat{\alpha}_{2sh1} + \hat{\alpha}_{3sh1}} \tag{3-55}$$

(3-3-4-2) <u>Constant Shrinkage Estimator (sh_2) [22][51]</u>

In this case, using to suggest study of constant shrinkage weight factor will be assumed $\phi(\hat{\alpha}) = k$, k=0.001,and used the equation (3-49)

$$\hat{\alpha}_{1sh_2} = k_1\hat{\alpha}_{1ub} + (1 - k_1)\alpha_0 \tag{3-56}$$

$$\hat{\alpha}_{2sh_2} = k_2\hat{\alpha}_{2ub} + (1 - k_2)\alpha_0 \tag{3-57}$$

$$\hat{\alpha}_{3sh_2} = k_3\hat{\alpha}_{3ub} + (1-k_3)\alpha_0 \tag{3-58}$$

By substituting equations (3-56) and (3-57) in equation (2-32) , we get the constant shrinkage estimator of the reliability in (S-S) model for modified exponentiated Lomax distribution which is contain one component as below

$$\hat{R}_{sh_2} = \frac{\hat{\alpha}_{1sh_2}}{\hat{\alpha}_{1sh_2} + \hat{\alpha}_{2sh_2}} \tag{3-59}$$

The constant shrinkage estimator for reliability system R_S for series component of system for modified exponentiated Lomax distribution, and by substituting equations (3-56), (3-57) and (3-58) in equation (2-57) as follows

$$\hat{R}_{s(sh2)} = \frac{\hat{\alpha}_{1sh2}}{\hat{\alpha}_{1sh2} + \hat{\alpha}_{3sh2}} + \frac{\hat{\alpha}_{2sh2}}{\hat{\alpha}_{2sh2} + \hat{\alpha}_{3sh2}} - \frac{\hat{\alpha}_{1sh2} + \hat{\alpha}_{2sh2}}{\hat{\alpha}_{1sh2} + \hat{\alpha}_{2sh2} + \hat{\alpha}_{3sh2}} \tag{3-60}$$

(3-3-4-3) The Beta Shrinkage Estimator (sh_3) [22][51]

In this case, beta shrinkage weight factor will be assumed $\phi_1(\hat{\alpha}) = B(1,n)$, $\phi_2(\hat{\alpha}) = B(1,\mathrm{m})$ and $\phi_3(\hat{\alpha}) = B(1,\mathrm{w})$,and subsisted in the equation (3-49)

$$\hat{\alpha}_{1sh_3} = B(1,n)\hat{\alpha}_{1ub} + (1-B(1,n))\alpha_0 \tag{3-61}$$

$$\hat{\alpha}_{2sh_3} = B(1,m)\hat{\alpha}_{2ub} + (1-B(1,m))\alpha_0 \tag{3-62}$$

$$\hat{\alpha}_{3sh_3} = B(1,\mathrm{w})\hat{\alpha}_{3ub} + (1-B(1,\mathrm{w}))\alpha_0 \tag{3-63}$$

By substituting equations (3-61) and (3-62) in equation (2-32) , we get the beta shrinkage estimator of the reliability in (S-S) model for modified exponentiated Lomax distribution which is contain one component as below

$$\hat{R}_{sh3} = \frac{\hat{\alpha}_{1sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3}} \tag{3-64}$$

The beta shrinkage estimator for reliability system R_S for series component of system for modified exponentiated Lomax distribution, and by substituting equations (3-61), (3-62) and (3-63) in equation (2-57) as follows

$$\hat{R}_{s(\text{sh}3)} = \frac{\hat{\alpha}_{1sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{3sh3}} + \frac{\hat{\alpha}_{2sh3}}{\hat{\alpha}_{2sh3} + \hat{\alpha}_{3sh3}} - \frac{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3} + \hat{\alpha}_{3sh3}} \tag{3-65}$$

(3-4) <u>Estimation Methods the Reliability System of Power Lomax Distribution</u>

(3-4-1) <u>Maximum Likelihood Method (MLE)[29]</u>

Let $x_1, x_2 \ldots\ldots x_n$ be a random variable sample of size n and probability density function

$$f(x, \alpha_1) = \alpha_1 (1 + x)^{-(\alpha_1 + 1)} \qquad 0<x<\infty$$

And let $y_1, y_2 \ldots\ldots y_m$ be a random variable sample of size m and the probability density function

$$f(\text{y}, \alpha_2) = \alpha_2 (1 + y)^{-(\alpha_2 + 1)} \qquad 0<y<\infty$$

And let $z_1, z_2 \ldots\ldots z_w$ be a random variable sample of size m and the probability density function

$$f(\text{z}, \alpha_3) = \alpha_3 (1 + z)^{-(\alpha_3 + 1)} \qquad 0<z<\infty$$

Then

$$Lf\ (x, y, z, \alpha_1, \alpha_2, \alpha_3) = \alpha_1^n \prod_{i=1}^{n} (1+x_i)^{-(\alpha_1+1)} \alpha_2^m \prod_{j=1}^{m} (1+y_j)^{-(\alpha_2+1)} \alpha_3^w \prod_{q=1}^{w} (1+z_q)^{-(\alpha_3+1)}$$

Take ln for both sides

$$LnL = nLn\alpha_1 - (\alpha_1+1)\sum_{i=1}^{n} Ln(1+x_i) + mLn\alpha_2 - (\alpha_2+1)\sum_{j=1}^{m} Ln(1+y_j)$$

$$+wLn\alpha_3 - (\alpha_3+1)\sum_{q=1}^{w} Ln(1+z_q)$$

The partial derivative for Ln-function with respect to parameter α_1

$$\frac{\partial LnL}{\partial \alpha_1} = \frac{n}{\alpha_1} - \sum_{i=1}^{n} Ln(1+x_i)$$

We equate partial derivative with zero

$$\frac{\partial LnL}{\partial \alpha_1} = 0$$

$$\frac{n}{\alpha_1} - \sum_{i=1}^{n} Ln(1+x_i) = 0$$

$$\hat{\alpha}_{1mle} = \frac{n}{\sum_{i=1}^{n} Ln(1+x_i)} \tag{3-66}$$

By the same way

$$\hat{\alpha}_{2mle} = \frac{m}{\sum_{j=1}^{m} Ln(1+y_j)} \tag{3-67}$$

$$\hat{\alpha}_{3mle} = \frac{w}{\sum_{q=1}^{w} Ln(1+z_q)} \tag{3-68}$$

By substituting equations (3-66) and (3-67) in equation (2-37) , we get the maximum likelihood estimator of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{2mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle}} \quad (3\text{-}69)$$

The maximum likelihood estimator for reliability system R_S for series component of system for power Lomax distribution ,and by substituting equations (3-66), (3-67) and (3-68) in equation (2-67) as follows

$$\hat{R}_{s(mle)} = \frac{\hat{\alpha}_{3mle}}{\hat{\alpha}_{1mle} + \hat{\alpha}_{2mle} + \hat{\alpha}_{3mle}} \quad (3\text{-}70)$$

(3-4-2) <u>Moment Method (MOM)[2][37]</u>

Let$x_1, x_2 \ldots\ldots x_n$ be a random variable sample of size n by the properties, the first moment for the power Lomax distribution of x

$$E(x) = \frac{1}{\alpha_1 - 1} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$= \alpha_1 - 1 = \frac{n}{\sum_{i=1}^{n} x_i}$$

$$\hat{\alpha}_{1mom} = \frac{n}{\sum_{i=1}^{n} x_i} + 1 \quad (3\text{-}71)$$

By the same way, can find

$$\hat{\alpha}_{2mom} = \frac{m}{\sum_{j=1}^{m} y_j} + 1 \qquad (3\text{-}72)$$

$$\hat{\alpha}_{3mom} = \frac{w}{\sum_{q=1}^{w} z_q} + 1 \qquad (3\text{-}73)$$

By substituting equations (3-71) and (3-72) in equation (2-37) , we get the moment estimator of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{2mom}}{\hat{\alpha}_{1mom} + \hat{\alpha}_{2mom}} \qquad (3\text{-}74)$$

The moment estimator for reliability system R_S for series component of system for power Lomax distribution , and by substituting equations (3-71), (3-72) and (3-73) in equation (2-67) as follows

$$\hat{R}_{s\,(\mathrm{mom})} = \frac{\hat{\alpha}_{3mom}}{\hat{\alpha}_{1mom} + \hat{\alpha}_{2mom} + \hat{\alpha}_{3mom}} \qquad (3\text{-}75)$$

(3-4-3) <u>Least Square Method (LS)[6][29][33]</u>

Let $x_1, x_2 \ldots\ldots. x_n$ a random variable follows the polo distribution $f(x, \alpha_1) = \alpha_1 (1+x)^{-(\alpha_1+1)}$ $\qquad 0<x<\infty$

Now, to estimate parameter (α_1)

$$F(x_i) = 1-(1+x_i)^{-\alpha_1}$$

$$(1+x_i)^{-\alpha_1} = 1-F(\mathrm{x})$$

$$1+x_i = (1-F(x))^{\frac{-1}{\alpha_1}}$$

Take Logarithm for both sides

$$-Ln(1+x_i)=\frac{1}{\alpha_1}Ln(1-F(x))$$

$$ax_i+b=y_i$$

$$a=\frac{1}{\alpha_1}$$
$$b=0$$
$$x_i=Ln(1-F(x))$$
$$y_i=-Ln(1+x_i)$$

$$a=\frac{\sum_{i=1}^{n}x_i y_i-\frac{\sum_{i=1}^{n}x_i y_i}{n}}{\sum_{i=1}^{n}x_i^{\ 2}-\frac{\left(\sum_{i=1}^{n}x_i\right)^2}{n}}$$

$$\frac{1}{\alpha_1}=\frac{\sum_{i=1}^{n}Ln(1-F\left(x_i\right))[-Ln(1+x_i)]-\frac{\sum_{i=1}^{n}Ln(1-F\left(x_i\right))\sum_{i=1}^{n}[-Ln(1+x_i)]}{n}}{\sum_{i=1}^{n}\left[Ln(1-F\left(x_i\right))\right]^2-\frac{\left[\sum_{i=1}^{n}Ln(1-F\left(x_i\right))\right]^2}{n}}$$

$$\hat{\alpha}_{1Ls}=\frac{\sum_{i=1}^{n}\left[Ln(1-F\left(x_i\right))\right]^2-\frac{\left[\sum_{i=1}^{n}Ln(1-F\left(x_i\right))\right]^2}{n}}{\sum_{i=1}^{n}Ln(1-F\left(x_i\right))[-Ln(1+x_i)]-\frac{\sum_{i=1}^{n}Ln(1-F\left(x_i\right))\sum_{i=1}^{n}[-Ln(1+x_i)}{n}} \qquad (3\text{-}76)$$

By the same way

$$\hat{\alpha}_{2ls} = \frac{\sum_{j=1}^{m}\left[Ln(1-F\left(y_j\right))\right]^2 - \frac{\left[\sum_{j=1}^{m} Ln(1-F\left(y_j\right))\right]^2}{m}}{\sum_{j=1}^{m} Ln(1-F\left(y_j\right))[-Ln(1+y_j)] - \frac{\sum_{j=1}^{m} Ln(1-F\left(y_i\right))\sum_{j=1}^{m}[-Ln(1+y_j)]}{m}} \qquad (3\text{-}77)$$

$$\hat{\alpha}_{3ls} = \frac{\sum_{q=1}^{w}\left[Ln(1-F\left(z_q\right))\right]^2 - \frac{\left[\sum_{q=1}^{w} Ln(1-F\left(z_q\right))\right]^2}{w}}{\sum_{q=1}^{w} Ln(1-F\left(z_q\right))[-Ln(1+z_q)] - \frac{\sum_{q=1}^{w} Ln(1-F\left(z_q\right))\sum_{q=1}^{w}[-Ln(1+z_q)]}{w}} \qquad (3\text{-}78)$$

By substituting equations (3-76) and (3-77) in equation (2-37) , we get the least square estimator of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{Ls} = \frac{\hat{\alpha}_{2Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls}} \qquad (3\text{-}79)$$

The least square estimator for reliability system R_S for series component of system for modified power distribution ,and by substituting equations (3-93), (3-94) and (3-95) in equation (2-67) as follows

$$\hat{R}_{s\,(\text{Ls})} = \frac{\hat{\alpha}_{3Ls}}{\hat{\alpha}_{1Ls} + \hat{\alpha}_{2Ls} + \hat{\alpha}_{3LS}} \qquad (3\text{-}80)$$

(3-4-4) <u>Shrinkage Estimation Method (sh)[1][29][51]</u>

In this section, we will study estimation of parameter for the polo distribution by using equation

$$\hat{\alpha}_{sh} = \phi\left(\alpha\right)\alpha_{ub} + \left[1-\phi\left(\alpha\right)\right]\alpha_0 \qquad (3\text{-}81)$$

Noted that: $\hat{\alpha}_{mle} = \frac{n}{\sum_{i=1}^{n} Ln(1+x_i)}$ is biased

$$E(\hat{\alpha}_{mle}) = \frac{n}{n-1}\alpha$$

To find α_{ub}

$$\hat{\alpha}_{ub} = \frac{n-1}{n}\times\hat{\alpha}_{mle}$$

$$= \frac{n-1}{n}\times\frac{n}{\sum_{i=1}^{n} Ln(1+x_i)}$$

$$\hat{\alpha}_{1ub} = \frac{(n-1)}{\sum_{i=1}^{n} Ln(1+x_i)} \qquad (3\text{-}82)$$

By the same way can find $\hat{\alpha}_{2ub}, \hat{\alpha}_{3ub}$

We can be introduced three estimation method as follow

(3-4-4-1) <u>Shrinkage Estimator Function(sh_1)</u>

In this case, the Shrinkage weight factor as function $n, \phi_1(\hat{\alpha}) = \left|\frac{\sin\ n}{n}\right|$, $\phi_2(\hat{\alpha}) = \left|\frac{\sin\ m}{m}\right|$ and $\phi_3(\hat{\alpha}) = \left|\frac{\sin\ w}{w}\right|$ it was implemented in the equation (3-81)

$$\hat{\alpha}_{1sh_1} = \left|\frac{\sin\ n}{n}\right|\hat{\alpha}_{1ub} + \left(1-\left|\frac{\sin\ n}{n}\right|\right)\alpha_0 \qquad (3\text{-}83)$$

$$\hat{\alpha}_{2sh_1} = \left|\frac{\sin\ m}{m}\right|\hat{\alpha}_{2ub} + \left(1 - \left|\frac{\sin\ m}{m}\right|\right)\alpha_0 \quad (3\text{-}84)$$

$$\hat{\alpha}_{3sh_1} = \left|\frac{\sin\ w}{w}\right|\hat{\alpha}_{3ub} + \left(1 - \left|\frac{\sin\ w}{w}\right|\right)\alpha_0 \quad (3\text{-}85)$$

By substituting equations (3-83) and (3-84) in equation (2-37) , we get the shrinkage estimator function of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{sh_1} = \frac{\hat{\alpha}_{2sh_1}}{\hat{\alpha}_{1sh_1} + \hat{\alpha}_{2sh_1}} \quad (3\text{-}86)$$

The shrinkage estimator function for reliability system R_S for series component of system for power Lomax distribution ,and by substituting equations (3-83), (3-84) and (3-85) in equation (2-67) as follows

$$\hat{R}_{s\,(\mathrm{sh1})} = \frac{\hat{\alpha}_{3sh1}}{\hat{\alpha}_{1sh1} + \hat{\alpha}_{2sh1} + \hat{\alpha}_{3sh1}} \quad (3\text{-}87)$$

(3-4-4-2) <u>Constant Shrinkage Estimator (sh_2)</u>

In this case, using to suggest study of constant shrinkage weight factor will be assumed $\phi(\alpha) = k$, when $k = 0.001$and use the equation (3-81)

$$\hat{\alpha}_{1sh_2} = k_1\hat{\alpha}_{1ub} + (1 - k_1)\alpha_0 \quad (3\text{-}88)$$

$$\hat{\alpha}_{2sh_2} = k_2\hat{\alpha}_{2ub} + (1 - k_2)\alpha_0 \quad (3\text{-}89)$$

$$\hat{\alpha}_{3sh_2} = k_3\hat{\alpha}_{3ub} + (1 - k_3)\alpha_0 \quad (3\text{-}90)$$

By substituting equations (3-88) and (3-89) in equation (2-37) , we get the constant shrinkage estimator of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{sh_2} = \frac{\hat{\alpha}_{2sh_2}}{\hat{\alpha}_{1sh_2} + \hat{\alpha}_{2sh_2}} \tag{3-91}$$

The constant shrinkage estimator for reliability system R_S for series component of system for power Lomax distribution, and by substituting equations (3-88),(3-89) and (3-90) in equation (2-67) as follows

$$\hat{R}_{s(\text{sh}2)} = \frac{\hat{\alpha}_{3sh2}}{\hat{\alpha}_{1sh2} + \hat{\alpha}_{2sh2} + \hat{\alpha}_{3sh2}} \tag{3-92}$$

(3-4-4-3) <u>Beta Shrinkage Estimator (sh_3)</u>

In this case, using the Beta shrinkage weight factor will be assumed $\phi_1(\hat{\alpha}) = B(1,n)$, $\phi_2(\hat{\alpha}) = B(1,\text{m})$ and $\phi_3(\hat{\alpha}) = B(1,\text{w})$,and subsisted in the equation (3-81)

$$\hat{\alpha}_{1sh_3} = B(1,n)\hat{\alpha}_{ub} + \left(1 - B(1,n)\right)\alpha_0 \tag{3-93}$$

$$\hat{\alpha}_{2sh_3} = B(1,m)\hat{\alpha}_{2ub} + \left(1 - B(1,m)\right)\alpha_0 \tag{3-94}$$

$$\hat{\alpha}_{3sh_3} = B(1,\text{w})\hat{\alpha}_{3ub} + \left(1 - B(1,\text{w})\right)\alpha_0 \tag{3-95}$$

By substituting equations (3-93) and (3-94) in equation (2-37) , we get the beta estimator of the reliability in (S-S) model for power Lomax distribution which is contain one component as below

$$\hat{R}_{sh3} = \frac{\hat{\alpha}_{2sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3}} \tag{3-96}$$

The beta shrinkage estimator for reliability system R_S for series component of system for power Lomax distribution, and by substituting equations (3-93), (3-94) and (3-95) in equation (2-67) as follows

$$\hat{R}_{s\,(\mathrm{sh}3)} = \frac{\hat{\alpha}_{3sh3}}{\hat{\alpha}_{1sh3} + \hat{\alpha}_{2sh3} + \hat{\alpha}_{3sh3}} \tag{3-97}$$

Chapter Four

Experimental Aspect

Chapter Four
Experimental Aspect

(4-1)Introduction

In this chapter the Monte Carlo simulation used to comparison between methods to estimate the reliability system for stress- strength model (S-S) which is contain one component and series system , for the studied distributions in this thesis (power distribution, modified exponentiated Lomax distribution and the power Lomax distribution).

The simulation process were done using unlike sample size (30 , 50 and 100) and built on 1000 replications and find the best method using mean square error (MSE)

$$MSE = \frac{\sum_{i=1}^{L}(\hat{R}_i - R)^2}{L}$$

Based on MATLAB software.

(4-2)The Simulation Study[26][27]

The simulation one of most important applications, it is used in many scientific fields including applied mathematics, financial, economics and many applications, simulation is considered one of the applications of computer science, there are a lot of types of simulation such as ordinary simulation, constraint simulation and contaminated simulation, from the statistical point of view, the simulation is an artificial data generation process that simulation reality and driven by parameter settings and model design, we use simulations to replicate the real world which cannot be done or accomplished, on the other hand, simulation are contain defects that lead to poor performance and low performance.

In this section, we will study estimation of reliability $R=P(y<x)$ for one component and R_S for two series components system for the three distributions using different estimation methods studied in chapter three, and comparing the results of these methods depending on mean square error criteria.

(4-2-1)<u>The Simulation for Estimate the System Reliability Based on Power Function Distribution</u>

Step 1: the random sample generated for X according to the uniform distribution over the interval (0, 1) as $r_1, r_2 \ldots\ldots\ldots\ldots r_n$.

Step 2: the random sample generated for Y according to the uniform distribution over the interval (0, 1), as $s_1, s_2, \ldots\ldots\ldots\ldots s_m$.

Step 3: transforming the above power distribution with using (c.d.f)as in the following

$$F(x_j) = r_j$$

$$x_j^{\alpha_1} = r_j$$

$$x_i = (\mathrm{r}_i)^{\frac{1}{\alpha_1}}$$

and by the same way, from step 2, we get

$$y_j = (s_j)^{\frac{1}{\alpha_2}}$$

Step 4: calculating $\hat{\alpha}_{1_{mle}}$ and $\hat{\alpha}_{2mle}$ using equations (3-1) and (3-2).

Step 5: calculating$\hat{\alpha}_{1_{mom}}$ and $\hat{\alpha}_{2_{mom}}$ using equations (3-6) and (3-7).

Step 6: calculating$\hat{\alpha}_{1_{Ls}}$ and $\hat{\alpha}_{2_{Ls}}$ using equations (3-11) and (3-12).

Step 7: calculating $\hat{\alpha}_{1_{sh_i}}$and $\hat{\alpha}_{2_{sh_i}}$ when *i*=1, 2, 3 using equations (3-19) , (3-20), (3-24), (3-25), (3-29), (3-30) respectively.

Step 8: calculating$\hat{R}_{mle}$, $\hat{R}_{mom}$, $\hat{R}_{Ls}$ $\hat{R}_{sh_1}$, $\hat{R}_{sh_2}$, *and* $\hat{R}_{sh_3}$ using equations (3-4),(3-9), (3-14), (3-22), (3-27) and (3-32) respectively.

Step 9: calculating$\hat{R}_{s(mle)}$, $\hat{R}_{s(mom)}$, $\hat{R}_{s(Ls)}$, $\hat{R}_{s(sh_1)}$, $\hat{R}_{s(sh_2)}$, and $\hat{R}_{s(sh_3)}$ using equations (3-5), (3-10), (3-15), (3-23), (3-28) and (3-33) when $\hat{\alpha}_3$=2 for all methods and cases.

We using random sample for x_i and y_j for size (n,m)= (30,30),(30,50),(30,100), (50,30),(50,50),(50,100), (100,30),(100,50), (100,100)

The result were as show in the tables

Table (4-1):Estimation for power function distribution when R=0.5, alpha1= 1, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.490139	0.511770	0.500105	0.500003	0.500106	0.491397
	50	0.502377	0.501424	0.485568	0.500000	0.492930	0.498530
	100	0.500411	0.497019	0.485535	0.500002	0.487835	0.495937
50	30	0.497673	0.499256	0.514426	0.500001	0.507071	0.501789
	50	0.501290	0.502633	0.499992	0.499999	0.499969	0.501975
	100	0.501921	0.501610	0.499903	0.499999	0.494837	0.500422
100	30	0.496428	0.499589	0.514526	0.500001	0.512236	0.502114
	50	0.497654	0.498059	0.500099	0.500001	0.505168	0.501268
	100	0.500210	0.500975	0.499999	0.500000	0.499997	0.498823

Table(4-2): MSE values for power function distribution when R = 0.5, alpha1= 1, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00051901058	0.00040384956	0.0000000578	0.000000000048	0.00000005936	0.00186749265	sh_2
	50	0.00333549875	0.00404372155	0.0002110012	0.00000000367	0.00005342323	0.00596515484	sh_2
	100	0.00275059025	0.00358047587	0.0002120389	0.00000000287	0.00015090743	0.00535912842	sh_2
50	30	0.00338030723	0.00439697723	0.0002109310	0.00000000358	0.00005343251	0.00621983678	sh_2
	50	0.00264717587	0.00349108397	0.0000000776	0.00000000277	0.00000120029	0.00505326237	sh_2
	100	0.00186086110	0.00248158856	0.0000000631	0.00000000196	0.00002730713	0.00387593133	sh_2
100	30	0.00268746093	0.00362852707	0.0002140383	0.00000000301	0.00015287915	0.00474061679	sh_2
	50	0.00191579777	0.00247303858	0.0000000641	0.00000000198	0.00002736419	0.00382297898	sh_2
	100	0.00130008952	0.00169115906	0.0000000350	0.00000000134	0.00000013940	0.00230190166	sh_2

Table (4-3): Estimation for power function distribution when R =0.333333, alpha1= 1, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.333767	0.334263	0.333414	0.333336	0.333415	0.336289
	50	0.337382	0.334902	0.320640	0.333333	0.327086	0.336407
	100	0.338877	0.336147	0.320533	0.333332	0.322533	0.335001
50	30	0.336594	0.338421	0.346229	0.333331	0.339564	0.342261
	50	0.333468	0.332847	0.333339	0.333334	0.333357	0.334368
	100	0.335005	0.333733	0.333255	0.333334	0.328789	0.335376
100	30	0.328662	0.329458	0.346444	0.333337	0.344379	0.334704
	50	0.332007	0.332608	0.333424	0.333335	0.337963	0.334676
	100	0.335370	0.335221	0.333327	0.333332	0.333320	0.335713

Table (4-4) : MSE values for power function distribution when R = 0.333333, alpha1= 1, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00333045942	0.00407349011	0.00000456676	0.00000000366	0.00000468648	0.0060234867	sh_2
	50	0.00272675676	0.00317425319	0.00016326720	0.00000000289	0.00004172719	0.0049113501	sh_2
	100	0.00235315660	0.00304112894	0.00016601546	0.00000000241	0.00011895110	0.0041766848	sh_2
50	30	0.00253964500	0.00309479408	0.00016860628	0.00000000272	0.00004151959	0.0048615657	sh_2
	50	0.0020245050	0.00236857438	0.00000005918	0.00000000211	0.00000091460	0.0036771595	sh_2
	100	0.00155424555	0.00194082119	0.00000004951	0.00000000158	0.00002113966	0.0029309887	sh_2
100	30	0.00224635487	0.00265176602	0.00017436372	0.00000000250	0.00012458560	0.0042434439	sh_2
	50	0.00149852589	0.00180930296	0.00000005236	0.00000000161	0.00002197596	0.0029250396	sh_2
	100	0.00105247266	0.00125820431	0.00000002811	0.00000000107	0.00000011190	0.0020364450	sh_2

Table (4-5): Estimation for power function distribution when R =0.666666, alpha1= 2, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.662728	0.663078	0.666702	0.666668	0.666702	0.664002
	50	0.663332	0.663325	0.653800	0.666670	0.660460	0.658256
	100	0.667976	0.666353	0.653665	0.666666	0.655736	0.664065
50	30	0.662408	0.663825	0.679385	0.666668	0.672938	0.661386
	50	0.666429	0.665317	0.666658	0.666665	0.666634	0.664104
	100	0.666855	0.665989	0.666582	0.666666	0.662053	0.663561
100	30	0.660560	0.663617	0.679517	0.666669	0.677516	0.667085
	50	0.665754	0.666582	0.666742	0.666665	0.671203	0.667305
	100	0.66579	0.665672	0.666668	0.666667	0.666668	0.665879

Table(4-6): MSE values for power function distribution when R = 0.666666, alpha1= 2, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00328250122	0.00373222061	0.0000042569	0.00000000342	0.00000436837	0.00606980612	sh_2
	50	0.00259447656	0.00324478044	0.0001676739	0.00000000266	0.00004106095	0.00517968821	sh_2
	100	0.00206071212	0.00242868274	0.0001712677	0.00000000222	0.00012180771	0.00389842610	sh_2
50	30	0.00273481415	0.00325502994	0.0001639921	0.00000000291	0.00004210945	0.00532038893	sh_2
	50	0.00185242541	0.00225935108	0.0000000546	0.00000000194	0.00000084415	0.00353063805	sh_2
	100	0.00157365989	0.00185528680	0.0000000529	0.00000000166	0.00002184280	0.00302255985	sh_2
100	30	0.00239094911	0.00285689399	0.0001676246	0.00000000253	0.00012031708	0.00390968491	sh_2
	50	0.00138303657	0.00166057068	0.0000000442	0.00000000141	0.00002102378	0.00268810353	sh_2
	100	0.00096451894	0.00119679615	0.0000000258	0.00000000099	0.00000010287	0.00200269936	sh_2

Table (4-7): Estimation for power function distribution when R =0.2500000, alpha1= 1, alpha2= 3

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.251955	0.249262	0.250052	0.250001	0.250053	0.251874
	50	0.253903	0.253116	0.239376	0.250000	0.244759	0.253825
	100	0.253407	0.252539	0.239330	0.250001	0.241002	0.253077
50	30	0.251326	0.252295	0.260959	0.250000	0.255319	0.256074
	50	0.251116	0.249421	0.250005	0.250001	0.250020	0.251890
	100	0.250869	0.250132	0.249939	0.250002	0.246185	0.251854
100	30	0.248037	0.248544	0.261095	0.250002	0.259334	0.252563
	50	0.250922	0.250898	0.250069	0.250000	0.253894	0.254081
	100	0.250477	0.250285	0.250003	0.250000	0.250005	0.250916

Table (4-8): MSE values for power function distribution when R = 0.2500000, alpha1= 1, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.002387948689	0.002815755086	0.00000323926	0.000000002596	0.00000332424	0.0041100572	sh_2
	50	0.001792312433	0.002194623563	0.00011427459	0.000000001810	0.00002918976	0.0034616466	sh_2
	100	0.001546224001	0.002007092701	0.00011526072	0.000000001531	0.00008245963	0.0030737594	sh_2
50	30	0.001944482079	0.002350645135	0.00012174548	0.000000002092	0.00003029239	0.0036080767	sh_2
	50	0.001485866484	0.001741949682	0.00000004300	0.000000001534	0.00000066465	0.0027353326	sh_2
	100	0.001077488216	0.001332699669	0.00000003359	0.000000001097	0.00001488988	0.0021941248	sh_2
100	30	0.001528344543	0.001698567219	0.00012497442	0.000000001735	0.00008905011	0.0028684420	sh_2
	50	0.001121747127	0.001257705394	0.00000003779	0.000000001204	0.00001556871	0.0021571368	sh_2
	100	0.000722280044	0.000834738472	0.00000001933	0.000000000741	0.00000007694	0.0014559525	sh_2

Table (4-9): Estimation for power function distribution when R =0.5000000, alpha1= 2, alpha2= 2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.500752	0.500283	0.499984	0.500000	0.499984	0.498106
	50	0.501153	0.500582	0.485629	0.500000	0.492974	0.497582
	100	0.502017	0.500986	0.485552	0.500001	0.487843	0.498328
50	30	0.497610	0.498436	0.514437	0.500001	0.507076	0.501432
	50	0.498133	0.498306	0.500010	0.500002	0.500040	0.495669
	100	0.502684	0.501383	0.499900	0.499999	0.494824	0.500963
100	30	0.497646	0.499452	0.514483	0.499999	0.512190	0.500630
	50	0.499560	0.500307	0.500088	0.499999	0.505127	0.502454
	100	0.500073	0.500067	0.500000	0.500000	0.499999	0.501504

Table (4-10): MSE values for power function distribution when R = 0.5000000, alpha1= 2, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.004128967671	0.00460449271	0.00000567290	0.000000004557	0.00000582153	0.0074662485	sh_2
	50	0.003355245833	0.00378472871	0.00020938358	0.000000003620	0.00005286337	0.0061857540	sh_2
	100	0.002539072228	0.00286249390	0.00021132211	0.000000002679	0.00015048273	0.0046498588	sh_2
50	30	0.003222287571	0.00361639031	0.00021127527	0.000000003490	0.00005344580	0.0062506657	sh_2
	50	0.002543357768	0.00295166248	0.00000007765	0.000000002770	0.00000120064	0.0050539378	sh_2
	100	0.001813708137	0.00197189492	0.00000006178	0.000000001892	0.00002740909	0.0034941292	sh_2
100	30	0.002623859568	0.00303969536	0.00021239274	0.000000002745	0.00015135488	0.0046707692	sh_2
	50	0.001796372291	0.00197502008	0.00000005768	0.000000001826	0.00002687547	0.0031709383	sh_2
	100	0.001240136539	0.00138857281	0.00000003349	0.000000001284	0.00000013329	0.0024475615	sh_2

Table (4-11): Estimation for power function distribution when R =0.750000, alpha1= 3, alpha2= 1

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.747445	0.747676	0.749984	0.750000	0.749984	0.742832
	50	0.748323	0.748968	0.739063	0.750000	0.744691	0.743426
	100	0.749614	0.749635	0.739003	0.750001	0.740768	0.745187
50	30	0.745730	0.747290	0.760672	0.750000	0.755268	0.746282
	50	0.746656	0.747272	0.750008	0.750001	0.750029	0.742852
	100	0.750657	0.749959	0.749925	0.749999	0.746097	0.748081
100	30	0.746228	0.748209	0.760705	0.750000	0.759030	0.746915
	50	0.748316	0.749575	0.750066	0.749999	0.753825	0.749445
	100	0.748535	0.748006	0.750002	0.750000	0.750005	0.749047

Table (4-12): MSE values for power function distribution when R = 0.750000, alpha1= 3, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00238445948	0.00285162889	0.00000319234	0.00000000256	0.00000327599	0.004521397784	sh_2
	50	0.00194898516	0.00224424898	0.00012134037	0.00000000203	0.00003021915	0.003732969877	sh_2
	100	0.00143114987	0.00160672829	0.00012247210	0.00000000150	0.00008682394	0.002762765254	sh_2
50	30	0.00192077201	0.00230907188	0.00011539193	0.00000000196	0.00002958562	0.003739290580	sh_2
	50	0.00149700726	0.00181428213	0.00000004367	0.00000000155	0.00000067500	0.003051262077	sh_2
	100	0.00102005230	0.00116997860	0.00000003477	0.00000000106	0.00001558602	0.002017961664	sh_2
100	30	0.00154712043	0.00199628632	0.00011599990	0.00000000154	0.00008302358	0.002746512587	sh_2
	50	0.00102687262	0.00122997959	0.00000003243	0.00000000102	0.00001495680	0.001827470288	sh_2
	100	0.000702148814	0.000841172097	0.000000018293	0.000000000701	0.000000072800	0.001371989596	sh_2

Table (4-13): Estimation for power function distribution when R =0.400000, alpha1= 2, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.404506	0.403784	0.399881	0.399997	0.399879	0.403496
	50	0.404397	0.402729	0.386249	0.399998	0.393222	0.399588
	100	0.402856	0.400909	0.386181	0.400001	0.388360	0.398094
50	30	0.398661	0.399451	0.413926	0.400001	0.406817	0.406414
	50	0.403676	0.403770	0.399987	0.399997	0.399947	0.407139
	100	0.401190	0.400025	0.399914	0.400001	0.395066	0.403155
100	30	0.397284	0.397751	0.414066	0.400001	0.411838	0.402213
	50	0.401166	0.401576	0.400081	0.399998	0.404928	0.402906
	100	0.401270	0.401548	0.399996	0.399999	0.399992	0.403123

Table (4-14): MSE values for power function distribution when R = 0.400000, alpha1= 2, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00396871983	0.00433924409	0.000005400944	0.00000000433	0.000005542471	0.007343458817	sh_2
	50	0.00287105463	0.00316849106	0.000191532791	0.00000000300	0.000048857539	0.005249976869	sh_2
	100	0.00256005925	0.00275943158	0.000193533534	0.00000000272	0.000138181811	0.004430761642	sh_2
50	30	0.00305726604	0.00336888467	0.000196646030	0.00000000329	0.000049708350	0.005687576189	sh_2
	50	0.00235482700	0.00257213006	0.000000068179	0.00000000243	0.000001053332	0.004695426152	sh_2
	100	0.00175858331	0.00185157299	0.000000057177	0.00000000182	0.000024931773	0.003154647116	sh_2
100	30	0.00250920425	0.00263416744	0.000200568619	0.00000000275	0.000142971140	0.004464657451	sh_2
	50	0.00177628561	0.00193943902	0.000000056642	0.00000000183	0.000024872335	0.003451614797	sh_2
	100	0.00110348574	0.00118595780	0.000000029215	0.00000000112	0.000000116266	0.002211734727	sh_2

Table (4-15): Estimation for power function distribution when R =0.600000, alpha1= 3, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.598078	0.599079	0.600008	0.600000	0.600008	0.597839
	50	0.599919	0.598806	0.586143	0.600000	0.593246	0.594149
	100	0.602922	0.601586	0.585963	0.599999	0.588188	0.597739
50	30	0.597608	0.597803	0.613749	0.600000	0.606745	0.594849
	50	0.598873	0.599349	0.600002	0.600000	0.600008	0.598037
	100	0.599726	0.598941	0.599914	0.600001	0.595049	0.596674
100	30	0.595986	0.597631	0.613846	0.600000	0.611669	0.598814
	50	0.600409	0.601163	0.600076	0.599998	0.604884	0.602419
	100	0.598798	0.598789	0.600004	0.600001	0.600008	0.599235

Table (4-16): MSE values for power function distribution when R = 0.600000, alpha1= 3, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00410272180	0.00452252787	0.00000553985	0.00000000445	0.000005685018	0.006919868113	sh_2
	50	0.00299182399	0.00329254043	0.00019459238	0.00000000315	0.000048701124	0.005449357116	sh_2
	100	0.00231417418	0.00243198895	0.00019945071	0.00000000248	0.000142040001	0.004537099349	sh_2
50	30	0.00319384452	0.00341948833	0.00019180652	0.00000000345	0.000048845005	0.005550811660	sh_2
	50	0.00228735369	0.00245594491	0.00000006662	0.00000000237	0.000001029306	0.004372047489	sh_2
	100	0.00180543427	0.00196530762	0.00000005964	0.00000000191	0.000025136702	0.003323110300	sh_2
100	30	0.00259058207	0.00291703876	0.00019425914	0.00000000272	0.000138841425	0.004619688057	sh_2
	50	0.00163762502	0.00173354982	0.00000005160	0.00000000168	0.000024385304	0.003120969456	sh_2
	100	0.00115799063	0.00127127192	0.00000003114	0.00000000119	0.000000123958	0.002418378384	sh_2

Table (4-17):Estimation for power function distribution when R =0.500000, alpha1= 3, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.497664	0.497663	0.500090	0.500003	0.500091	0.499114
	50	0.501338	0.501712	0.485595	0.500001	0.492961	0.498385
	100	0.502409	0.501254	0.485551	0.500000	0.487837	0.495447
50	30	0.498054	0.498944	0.514417	0.500000	0.507064	0.501263
	50	0.501699	0.501955	0.499991	0.499998	0.499964	0.501553
	100	0.499867	0.498715	0.499915	0.500001	0.494868	0.498597
100	30	0.496367	0.497546	0.514469	0.500001	0.512188	0.499887
	50	0.500860	0.501257	0.500083	0.499998	0.505135	0.502239
	100	0.499098	0.499134	0.500005	0.500001	0.500010	0.498614

Table (4-18): MSE values for power distribution function when R = 0.500000, alpha1= 3, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00406058313	0.00432786438	0.00000559972	0.00000000449	0.00000574646	0.007686836923	sh_2
	50	0.00321589901	0.00347952789	0.00021030428	0.00000000354	0.00005292387	0.006368191686	sh_2
	100	0.00269169202	0.00282220037	0.00021167418	0.00000000288	0.00015096520	0.004745640879	sh_2
50	30	0.00310688972	0.00337207020	0.00021055525	0.00000000335	0.00005318508	0.005623109954	sh_2
	50	0.00249607823	0.00267972262	0.00000007414	0.00000000264	0.00000114584	0.004620145844	sh_2
	100	0.00183679786	0.00194480966	0.00000005935	0.00000000190	0.00002694376	0.003330357287	sh_2
100	30	0.00290003025	0.00301517985	0.00021228991	0.00000000307	0.00015162001	0.004919492350	sh_2
	50	0.00200308754	0.00210672945	0.00000006624	0.00000000216	0.00002708434	0.003655290108	sh_2
	100	0.00120786950	0.00127345251	0.00000003250	0.00000000124	0.00000012935	0.002575201627	sh_2

Table (4-19): Estimation for power function distribution when R_s = 0.1666666, alpha1= 1, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.1706601	0.1677637	0.1666783	0.1666671	0.1666784	0.1677672
	50	0.1706213	0.1686402	0.1666928	0.1666678	0.1666979	0.1686371
	100	0.1686706	0.1665765	0.1666554	0.1666661	0.1666542	0.1675851
50	30	0.1700426	0.1676888	0.1666702	0.1666672	0.1666772	0.1663190
	50	0.1694333	0.1679456	0.1666700	0.1666673	0.1666789	0.1681784
	100	0.1678778	0.1667309	0.1666646	0.1666662	0.1666637	0.1672496
100	30	0.1684820	0.1666102	0.1666514	0.1666660	0.1666501	0.1659335
	50	0.1684570	0.1670482	0.1666672	0.1666667	0.1666716	0.1674407
	100	0.1687852	0.1672705	0.1666717	0.1666676	0.1666766	0.1676850

Table (4- 20): MSE values for power function distribution when R_s = 0.1666666, alpha1= 1, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00065084832	0.00081290894	0.00000071454	0.00000000066	0.00000073192	0.00114020659	sh_2
	50	0.00052583759	0.00066250437	0.00000037307	0.00000000053	0.00000044961	0.00094767028	sh_2
	100	0.00040331800	0.00052167407	0.00000033342	0.00000000040	0.00000034945	0.00072893670	sh_2
50	30	0.00059791517	0.00077089287	0.00000041484	0.00000000060	0.00000051319	0.00100502781	sh_2
	50	0.00040374392	0.00053695215	0.00000001101	0.00000000040	0.00000015983	0.00074890777	sh_2
	100	0.00027391324	0.00036477267	0.00000000747	0.00000000027	0.00000008434	0.00049883009	sh_2
100	30	0.00039001223	0.00052499723	0.00000032628	0.00000000039	0.00000034028	0.00068918756	sh_2
	50	0.00028118578	0.00036597032	0.00000000758	0.00000000028	0.00000008581	0.00053706345	sh_2
	100	0.00018575611	0.00024282175	0.00000000470	0.00000000018	0.00000001835	0.00034504331	sh_2

Table (4-21): Estimation for power function distribution when R_s = 0.233333, alpha1= 1, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2375308	0.2342140	0.2333485	0.2333339	0.2333486	0.2314412
	50	0.2373029	0.2352214	0.2333309	0.2333340	0.2333422	0.2336377
	100	0.2364957	0.2342592	0.2333334	0.2333335	0.2333341	0.2339414
50	30	0.2350612	0.2328826	0.2333132	0.2333325	0.2333087	0.2322343
	50	0.2357965	0.2336688	0.2333344	0.2333335	0.2333370	0.2331177
	100	0.2347198	0.2326558	0.2333298	0.2333326	0.2333246	0.2331256
100	30	0.2336461	0.2314013	0.2332899	0.2333316	0.2332866	0.2300585
	50	0.2347625	0.2333497	0.2333337	0.2333334	0.2333328	0.2319424
	100	0.2334218	0.2328530	0.2333277	0.2333322	0.2333223	0.2323900

Table (4-22): MSE values for power function distribution when R_s = 0.233333, alpha1= 1, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00102361394	0.00130094800	0.00000115277	0.00000000106	0.00000118079	0.00178112254	sh_2
	50	0.00085423856	0.00108543033	0.00000078458	0.00000000088	0.00000085972	0.00145745811	sh_2
	100	0.00081129366	0.00099103410	0.00000082813	0.00000000084	0.00000085418	0.00142111429	sh_2
50	30	0.00066346184	0.00080515911	0.00000031707	0.00000000069	0.00000047263	0.00126179779	sh_2
	50	0.00055926253	0.00069006845	0.00000001577	0.00000000057	0.00000022877	0.00104652581	sh_2
	100	0.00049691193	0.00064005659	0.00000001375	0.00000000050	0.00000017424	0.00089442807	sh_2
100	30	0.00047582275	0.00056691939	0.00000028704	0.00000000048	0.00000031291	0.00086131702	sh_2
	50	0.00035597928	0.00041680730	0.00000000955	0.00000000036	0.00000008559	0.00067141533	sh_2
	100	0.00026748197	0.00034751610	0.00000000689	0.00000000026	0.00000002689	0.00050643986	sh_2

Table(4-23): Estimation for power function distribution when R_s = 0.233333, alpha1= 2, alpha2= 1, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.2378048	0.2345817	0.2333555	0.2333341	0.2333557	0.2338962
	50	0.2371178	0.2352135	0.2333577	0.2333345	0.2333651	0.2345116
	100	0.2347496	0.2330547	0.2333216	0.2333327	0.2333199	0.2333189
50	30	0.2356931	0.2328730	0.2332956	0.2333322	0.2332939	0.2308408
	50	0.2360767	0.2340679	0.2333358	0.2333338	0.2333423	0.2323557
	100	0.2340114	0.2326190	0.2333299	0.2333327	0.2333202	0.2332568
100	30	0.2377335	0.2352703	0.2333832	0.2333348	0.2333831	0.2351196
	50	0.2357034	0.2340652	0.2333350	0.2333336	0.2333423	0.2341502
	100	0.2356341	0.2341696	0.2333390	0.2333344	0.2333445	0.2343772

Table(4-24): MSE values for power function distribution when R_s = 0.233333, alpha1= 2, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00097302978	0.00119376345	0.00000108735	0.00000000100	0.00000111379	0.00169856657	sh_2
	50	0.00069123325	0.00083099961	0.00000032463	0.00000000070	0.00000048389	0.00126258453	sh_2
	100	0.00046603730	0.00055198225	0.00000028882	0.00000000047	0.00000031223	0.00084885028	sh_2
50	30	0.00085591938	0.00112040265	0.00000072287	0.00000000086	0.00000082061	0.00157180213	sh_2
	50	0.00056043247	0.00071984477	0.00000001566	0.00000000056	0.00000022722	0.00107437451	sh_2
	100	0.00036426940	0.00046514259	0.00000000982	0.00000000037	0.00000008667	0.00072273406	sh_2
100	30	0.00079363289	0.00095311482	0.00000083541	0.00000000083	0.00000085755	0.00139151270	sh_2
	50	0.00048409014	0.00060364833	0.00000001330	0.00000000048	0.00000017221	0.00091821968	sh_2
	100	0.00027646590	0.00035216149	0.00000000706	**0.**00000000027	0.00000002754	0.00051956783	sh_2

Table (4-25): Estimation for power function distribution when R_s = 0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3375103	0.3343821	0.3333610	0.3333343	0.3333612	0.3291963
	50	0.3362164	0.3338660	0.3333415	0.3333332	0.3333330	0.3299738
	100	0.3335762	0.3316017	0.3332977	0.3333314	0.3332918	0.3296960
50	30	0.3360507	0.3337276	0.3333304	0.3333333	0.3333290	0.3319653
	50	0.3358057	0.3340300	0.3333360	0.3333338	0.3333426	0.3308986
	100	0.3330266	0.3317832	0.3333244	0.3333316	0.3332994	0.3304864
100	30	0.3347630	0.3328073	0.3333055	0.3333326	0.3333056	0.3296771
	50	0.3345436	0.3333208	0.3333313	0.3333329	0.3333299	0.3321027
	100	0.3358270	0.3345373	0.3333405	0.3333347	0.3333475	0.3340069

Table (4- 26): MSE values for power function distribution when R_s = 0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00135033482	0.00152459642	0.00000156987	0.00000000145	0.00000160799	0.00240937500	sh_2
	50	0.00095516932	0.00104672816	0.00000068759	0.00000000098	0.00000083523	0.00187775793	sh_2
	100	0.00083649194	0.00094614879	0.00000074965	0.00000000089	0.00000078399	0.00147920780	sh_2
50	30	0.00096450174	0.00111948952	0.00000073826	0.00000000100	**0.**00000086919	0.00186667950	sh_2
	50	0.00071530220	0.00081914071	0.00000002039	0.00000000074	0.00000029580	0.00144746177	sh_2
	100	0.00056937024	0.00065281426	0.00000001574	0.00000000058	0.00000017812	0.00110548378	sh_2
100	30	0.00084528422	0.00096751562	0.00000072717	0.00000000086	0.00000075788	0.00147846572	sh_2
	50	0.00057854486	0.00063859526	0.00000001574	0.00000000058	0.00000017648	0.00109520007	sh_2
	100	0.00037140693	0.00041400840	0.00000000962	**0.**00000000037	0.00000003753	0.00072579603	sh_2

Table (4-27): Estimation for power function distribution when R_s = 0.266666, alpha1= 1, alpha2= 3, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.2703137	0.2678911	0.2666559	0.2666664	0.2666557	0.2654148
	50	0.2707123	0.2691354	0.2666800	0.2666670	0.2666776	0.2654509
	100	0.2693205	0.2667505	0.2666439	0.2666660	0.2666437	0.2668609
50	30	0.2695943	0.2671637	0.2666854	0.2666673	0.2666855	0.2661619
	50	0.2677376	0.2652385	0.2666597	0.2666653	0.2666397	0.2638882
	100	0.2683396	0.2661571	0.2666637	0.2666660	0.2666590	0.2665718
100	30	0.2683085	0.2666069	0.2666641	0.2666667	0.2666646	0.2643266
	50	0.2677063	0.2662139	0.2666653	0.2666664	0.2666602	0.2649322
	100	0.2665088	0.2660142	0.2666597	0.2666652	0.2666528	0.2652756

Table (4- 28): MSE values for power function distribution when R_s = 0.266666, alpha1= 1, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00110439969	0.00139066966	0.00000126283	0.00000000117	0.00000129352	0.00195598108	sh_2
	50	0.00111614429	0.00137772841	0.00000110553	0.00000000115	0.00000118593	0.00182301507	sh_2
	100	0.00100958473	0.00130803126	0.00000114673	0.00000000109	0.00000117346	0.00187617854	sh_2
50	30	0.00079197169	0.00094964249	0.00000024512	0.00000000083	0.00000048126	0.00137332688	sh_2
	50	0.00066402528	0.00079968369	0.00000001857	0.00000000067	0.00000026957	0.00123354060	sh_2
	100	0.00062520906	0.00080651944	0.00000001755	0.00000000064	0.00000023635	0.00109555531	sh_2
100	30	0.00045108298	0.00053408731	0.00000020715	0.00000000046	0.00000023376	0.00089823953	sh_2
	50	0.00037980246	0.00046276846	0.00000001003	0.00000000038	0.00000006875	0.00072497729	sh_2
	100	0.00031479245	0.00041228925	0.00000000809	**0.**00000000031	0.00000003155	0.00060340375	sh_2

Table (4-29): Estimation for power function distribution when R_s = 0.266666, alpha1= 3, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2714154	0.2680929	0.2666962	0.2666677	0.2666965	0.2673120
	50	0.2702236	0.2684241	0.2666870	0.2666678	0.2666956	0.2676088
	100	0.2677080	0.2663186	0.2666562	0.2666661	0.2666543	0.2663847
50	30	0.2691217	0.2661901	0.2666226	0.2666653	0.2666210	0.2638078
	50	0.2694794	0.2674366	0.2666694	0.2666672	0.2666764	0.2653542
	100	0.2673116	0.2659544	0.2666639	0.2666661	0.2666558	0.2667589
100	30	0.2717887	0.2691576	0.2667255	0.2666684	0.2667256	0.2688086
	50	0.2693919	0.2676617	0.2666692	0.2666671	0.2666783	0.2676253
	100	0.2689729	0.2675533	0.2666723	0.2666677	0.2666777	0.2678056

Table (4-30): MSE values for power function distribution when R_s = 0.266666, alpha1= 3, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00114827153	0.00143560351	0.00000128944	0.00000000119	0.00000132079	0.00199849105	sh_2
	50	0.00075729520	0.00092626726	0.00000022258	0.00000000077	0.00000044089	0.00137682692	sh_2
	100	0.00045789991	0.00055136858	0.00000019564	0.00000000046	0.00000022251	0.00083646861	sh_2
50	30	0.00103585097	0.00137596550	0.00000098732	0.00000000105	**0.**00000106846	0.00187987331	sh_2
	50	0.00067424886	0.00087217339	0.00000001888	0.00000000068	0.00000027406	0.00126932957	sh_2
	100	0.00038952729	0.00050274865	0.00000001034	0.00000000039	0.00000007192	0.00078231713	sh_2
100	30	0.00104500404	0.00126367387	0.00000114782	0.00000000109	0.00000117575	0.00180248830	sh_2
	50	0.00060977651	0.00076242729	0.00000001687	0.00000000061	0.00000023107	0.00115312104	sh_2
	100	0.00032344753	0.00042090097	0.00000000829	0.00000000032	0.00000003234	0.00061285465	sh_2

Table (4-31): Estimation for power function distribution when R_s = 0.385714, alpha1= 2, alpha2= 3, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.38933169	0.38645957	0.38573347	0.38571509	0.38573361	0.38051594
	50	0.38878608	0.38581745	0.38571885	0.38571443	0.38571645	0.38254271
	100	0.38755945	0.38581870	0.38568499	0.38571342	0.38568407	0.38411423
50	30	0.38903730	0.38658827	0.38571908	0.38571517	0.38572962	0.38256583
	50	0.38576577	0.38409789	0.38570544	0.38571261	0.38567936	0.38203491
	100	0.38783798	0.38616721	0.38571650	0.38571471	0.38572497	0.38494776
100	30	0.38693876	0.38524815	0.38568687	0.38571389	0.38568842	0.38244507
	50	0.38810116	0.38689331	0.38572106	0.38571562	0.38573020	0.38482129
	100	0.38563633	0.38481569	0.38570849	0.38571315	0.38570276	0.38407681

Table (4-32): MSE values for power function distribution when R_s = 0.385714, alpha1= 2, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00146243808	0.00164548166	0.00000171298	0.00000000159	0.00000175457	0.00264303086	sh_2
	50	0.00128083883	0.00142359497	0.00000110138	0.00000000136	0.00000127230	0.00225691123	sh_2
	100	0.00102194799	0.00112648752	0.00000101252	0.00000000107	0.00000104769	0.00191134747	sh_2
50	30	0.00104458874	0.00111906151	0.00000053870	0.00000000109	**0.**00000078331	0.00190142453	sh_2
	50	0.00087358506	0.00100682587	0.00000002509	0.00000000091	0.00000036403	0.00170076370	sh_2
	100	0.00071638187	0.00078620125	0.00000002008	0.00000000074	0.00000024932	0.00135842461	sh_2
100	30	0.00074779412	0.00082612858	0.00000051247	0.00000000077	0.00000054826	0.00146165181	sh_2
	50	0.00057942924	0.00061155871	0.00000001600	0.00000000060	0.00000015038	0.00109015089	sh_2
	100	0.00038919638	0.00043584074	0.00000001014	0.00000000039	0.00000003956	0.00077650360	sh_2

Table (4-33) : Estimation for power function distribution when R_s = 0.385714, alpha1= 3, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3877173	0.3846726	0.3856818	0.3857135	0.3856814	0.3803887
	50	0.3863363	0.3845392	0.3856815	0.3857126	0.3856680	0.3814726
	100	0.3878064	0.3861624	0.3857216	0.3857147	0.3857226	0.3843360
50	30	0.3890062	0.3867384	0.3857222	0.3857148	0.3857246	0.3813358
	50	0.3866784	0.3854299	0.3857105	0.3857135	0.3856988	0.3835064
	100	0.3857353	0.3848661	0.3857078	0.3857130	0.3856985	0.3837071
100	30	0.3882423	0.3862706	0.3857147	0.3857142	0.3857136	0.3826754
	50	0.3867653	0.3856629	0.3857116	0.3857137	0.3857058	0.3843628
	100	0.3866285	0.3857865	0.3857143	0.3857143	0.3857143	0.3852230

Table (4-34): MSE values for power function distribution when R_s = 0.385714, alpha1= 3, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00146292612	0.00164749853	0.00000170176	0.00000000157	0.00000174310	0.00276344573	sh_2
	50	0.00103310804	0.00112220506	0.00000054285	0.00000000106	0.00000077112	0.00202527962	sh_2
	100	0.00072433529	0.00079358894	0.00000052555	0.00000000076	0.00000055811	0.00148649792	sh_2
50	30	0.00122799345	0.00135442634	0.00000110542	0.00000000131	**0.**00000124561	0.00217529382	sh_2
	50	0.00086559857	0.00094367181	0.00000002472	0.00000000089	0.00000035869	0.00163957924	sh_2
	100	0.00056211831	0.00061429157	0.00000001549	0.00000000058	0.00000014692	0.00103179831	sh_2
100	30	0.00105911213	0.00119503633	0.00000102732	0.00000000109	0.00000106283	0.00203466382	sh_2
	50	0.00067218180	0.00075183594	0.00000001897	0.00000000069	0.00000024354	0.00131154786	sh_2
	100	0.00042308059	0.00048434544	0.00000001120	0.00000000043	0.00000004370	0.00081840374	sh_2

Table (4-35): Estimation for power function distribution when R_s = 0.4500000, alpha1= 3, alpha2= 3, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.4530178	0.4508969	0.4500141	0.4500006	0.4500142	0.4462972
	50	0.4535394	0.4519616	0.4500376	0.4500017	0.4500449	0.4485924
	100	0.4511750	0.4494844	0.4499815	0.4499992	0.4499797	0.4479870
50	30	0.4509373	0.4490713	0.4499592	0.4499988	0.4499567	0.4436012
	50	0.4520917	0.4506904	0.4500029	0.4500005	0.4500100	0.4465516
	100	0.4499315	0.4487988	0.4499945	0.4499989	0.4499787	0.4476203
100	30	0.4528012	0.4511685	0.4500488	0.4500014	0.4500483	0.4485440
	50	0.4513988	0.4503277	0.4500008	0.4500001	0.4500070	0.4486737
	100	0.4523255	0.4512733	0.4500076	0.4500015	0.4500149	0.4498893

Table (4-36) : MSE values for power function distribution when R_s = 0.4500000, alpha1= 3, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00139879826	0.00149342476	0.00000161274	0.00000000149	0.00000165190	0.00257204718	sh_2
	50	0.00111943022	0.00118363305	0.00000084158	0.00000000120	0.00000101458	0.00213249968	sh_2
	100	0.00088433986	0.00094306596	0.00000075288	0.00000000092	0.00000078908	0.00165325797	sh_2
50	30	0.00117636662	0.00126498498	0.00000077377	0.00000000121	**0.**00000099117	0.00228411319	sh_2
	50	0.00078728084	0.00085062587	0.00000002266	0.00000000082	0.00000032862	0.00162174738	sh_2
	100	0.00063551979	0.00069637533	0.00000001757	0.00000000065	0.00000019788	0.00124519229	sh_2
100	30	0.00089823800	0.00093580131	0.00000088065	0.00000000099	0.00000091084	0.00172932412	sh_2
	50	0.00062007442	0.00065856446	0.00000001716	0.00000000063	0.00000019403	0.00122234239	sh_2
	100	0.00040733704	0.00043172892	0.00000001064	0.00000000041	0.00000004151	0.00077455318	sh_2

(4-2-2)<u>The Simulation for Estimate the System Reliability Based on Modified Exponentiated Lomax Distribution</u>

Step 1: the random sample generated for X according to the uniform distribution over the interval (0, 1) as$r_1, r_2 \ldots\ldots\ldots\ldots r_n$.

Step 2: the random sample generated for Y according to the uniform distribution over the interval (0, 1), as $s_1, s_2, \ldots\ldots\ldots\ldots s_m$.

Step 3: transforming the above modified exponentiated Lomax distribution with using (c.d.f)as in the following, $F(\mathrm{x}_j) = r_j$

$$r_j = [1-(1+x_j)^{-2}]^{\alpha}$$

$$x_i = [1-(r_i^{\frac{1}{\alpha}})]^{\frac{-1}{2}} - 1$$

and by the same way, from step 2 ,we get

$$y_j = [1-(\mathrm{s}_j^{\frac{1}{\alpha}})]^{\frac{-1}{2}} - 1$$

Step 4: calculating$\hat{\alpha}_{1_{mle}}$ and $\hat{\alpha}_{2mle}$ using equations (3-34) and (3-35).

Step 5: calculating$\hat{\alpha}_{1_{mom}}$ and $\hat{\alpha}_{2_{mom}}$ using equations (3-39) and (3-40).

Step 6: calculating$\hat{\alpha}_{1_{Ls}}$ and $\hat{\alpha}_{2_{Ls}}$ using equations (3-44) and (3-45).

Step 7: calculating $\hat{\alpha}_{1_{sh_i}}$and $\hat{\alpha}_{2_{sh_i}}$ when i=1, 2, 3 using equations (3-51) , (3-52), (3-56), (3-57), (3-61), (3-62) respectively.

Step 8: calculating$\hat{R}_{mle}$, $\hat{R}_{mom}$, $\hat{R}_{Ls}$, $\hat{R}_{sh_1}$, $\hat{R}_{sh_2}$, and $\hat{R}_{sh_3}$ using equations (3-37), (3-42), (3-47), (3-54), (3-59) and (3-64) respectively.

Step 9: calculating $\hat{R}_{s(mle)}$, $\hat{R}_{s(mom)}$, $\hat{R}_{s(Ls)}$, $\hat{R}_{s(sh_1)}$, $\hat{R}_{s(sh_2)}$, and $\hat{R}_{s(sh_3)}$ using equations (3-38), (3-43), (3-48), (3-55), (3-60) and (3-65) when $\hat{\alpha}_3$=2 for all methods and cases.

We using random sample for x_i and y_j for size (n,m)= (30,30),(30,50),(30,100), (50,30),(50,50),(50,100), (100,30),(100,50), (100,100)

The result were as show in the tables

Table (4-37) : Estimation for modified exponentiated Lomax distribution when R = 0.50000, alpha1= 1, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.500475	0.492403	0.500026	0.500000	0.500026	0.496673
	50	0.496051	0.491338	0.500002	0.500001	0.500026	0.497260
	100	0.492299	0.492665	0.500042	0.500001	0.500044	0.500258
50	30	0.503786	0.515426	0.499938	0.499998	0.499952	0.502514
	50	0.504399	0.494969	0.500020	0.500003	0.500080	0.502310
	100	0.496909	0.494117	0.500004	0.500000	0.500002	0.499171
100	30	0.510170	0.502464	0.500119	0.500000	0.500105	0.502460
	50	0.499696	0.499996	0.499978	0.499995	0.499919	0.495541
	100	0.500389	0.504133	0.500001	0.500000	0.500002	0.497391

Table (4-38): MSE values for modified exponentiated Lomax distribution when R = 0.50000, alpha1= 1, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.006002668	0.016374362	0.000006214	0.000000005	0.000006365	0.006930884	sh_2
	50	0.001903856	0.011618368	0.000000500	0.0000000005	0.000000464	0.001501114	sh_2
	100	0.079360605	0.072824615	0.000019843	0.000000021	0.000020631	0.020862330	sh_2
50	30	0.004959409	0.051855377	0.000002261	0.000000003	0.000002800	0.007504992	sh_2
	50	0.034849498	0.038894682	0.000000859	0.000000031	0.000012506	0.027662765	sh_2
	100	0.002636328	0.011033664	0.000000056	0.000000002	0.000000729	0.001977947	sh_2
100	30	0.011340509	0.002820631	0.000003670	0.000000001	0.000003175	0.003873784	sh_2
	50	0.004530144	0.042642993	0.000000582	0.000000021	0.000007662	0.028396066	sh_2
	100	0.0079074274	0.048826481	0.000000191	0.000000007	0.000000746	0.01995959	sh_2

Table (4-39) : Estimation for modified exponentiated Lomax distribution when R = 0.3333, alpha1= 1, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.331588	0.323635	0.333354	0.333334	0.333354	0.326357
	50	0.327932	0.322604	0.333334	0.333334	0.333354	0.327064
	100	0.324910	0.324680	0.333369	0.333334	0.333371	0.330783
50	30	0.335684	0.333987	0.333371	0.333332	0.333341	0.330182
	50	0.327192	0.358813	0.333309	0.333328	0.333240	0.327702
	100	0.331616	0.328491	0.333344	0.333335	0.333364	0.332552
100	30	0.342996	0.329883	0.333460	0.333336	0.333456	0.334939
	50	0.339682	0.333334	0.333352	0.333337	0.333399	0.334497
	100	0.330743	0.330730	0.333324	0.333331	0.333316	0.330020

Table (4-40): MSE values for modified exponentiated Lomax distribution when R = 0.33333, alpha1= 1, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.005352524	0.017444178	0.000004911	0.000000004	0.000005031	0.010214360	sh_2
	50	0.003178791	0.013660662	0.000000394	0.0000000004	0.000000358	0.004574635	sh_2
	100	0.087720906	0.091107614	0.000015599	0.000000016	0.000016217	0.024923802	sh_2
50	30	0.013257046	0.050083471	0.000006037	0.000000006	0.000005498	0.033543134	sh_2
	50	0.000721972	0.033158527	0.000000016	0.0000000006	0.000000241	0.000545037	sh_2
	100	0.006439134	0.034445821	0.000000212	0.000000007	0.000002328	0.027941111	sh_2
100	30	0.010413994	0.001822883	0.000002642	0.000000001	0.000002568	0.002254364	sh_2
	50	0.051445712	0.005030021	0.000000649	0.000000023	0.000008104	0.012947424	sh_2
	100	0.009859736	0.016735501	0.000000145	0.000000005	0.000000569	0.021795616	sh_2

Table (4-41):Estimation for modified exponentiated Lomax distribution when R = 0.6666, alpha1= 2, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.670644	0.673761	0.666714	0.666668	0.666714	0.671054
	50	0.663320	0.655894	0.666664	0.666666	0.666657	0.666083
	100	0.659543	0.661369	0.666602	0.666665	0.666607	0.664609
50	30	0.676389	0.665154	0.666728	0.666669	0.666746	0.673573
	50	0.665932	0.666396	0.666656	0.666664	0.666627	0.669496
	100	0.664234	0.660481	0.666665	0.666666	0.666664	0.667471
100	30	0.681372	0.672737	0.666831	0.666670	0.666829	0.681342
	50	0.670994	0.667062	0.666665	0.666666	0.666649	0.669889
	100	0.668769	0.672502	0.666673	0.666667	0.666679	0.669900

Table (4-42): MSE values for modified exponentiated Lomax distribution when R = 0.66666, alpha1= 2, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.028234581	0.233989076	0.000014778	0.000000013	0.000015142	0.036490924	sh_2
	50	0.015703297	0.130395173	0.000002538	0.000000004	0.000003293	0.012771093	sh_2
	100	0.614200092	0.461803015	0.000193604	0.000000095	0.000176753	0.234732083	sh_2
50	30	0.106603643	0.054519803	0.000012235	0.000000020	0.000017121	0.087700896	sh_2
	50	0.132166127	0.067593026	0.000004718	0.000000171	0.000068227	0.263818911	sh_2
	100	0.132828198	0.461094265	0.000001893	0.000000070	0.000019918	0.144690636	sh_2
100	30	0.220091572	0.060564985	0.000036605	0.000000021	0.000034884	0.289469399	sh_2
	50	0.257713888	0.083168605	0.000001647	0.000000061	0.000018571	0.386141764	sh_2
	100	0.013217174	0.238442841	0.000000285	0.000000011	0.000001112	0.016311314	sh_2

Table (4-43): Estimation for modified exponentiated Lomax distribution when R = 0.500000, alpha1= 2, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.500475	0.493166	0.500026	0.500000	0.500026	0.496673
	50	0.496051	0.492136	0.500002	0.500001	0.500026	0.497260
	100	0.489019	0.485763	0.499928	0.499998	0.499928	0.499872
50	30	0.504675	0.506444	0.500044	0.499999	0.500010	0.500231
	50	0.505597	0.502586	0.500027	0.500005	0.500105	0.504625
	100	0.493941	0.493004	0.499989	0.499997	0.499984	0.494804
100	30	0.506682	0.506464	0.499916	0.499997	0.499918	0.502878
	50	0.500619	0.501888	0.499984	0.499996	0.499963	0.496296
	100	0.497709	0.497540	0.499987	0.499997	0.499976	0.496810

Table (4-44): MSE values for modified exponentiated Lomax distribution when R = 0.500000, alpha1= 2, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.006002668	0.016460808	0.000006214	0.000000005	0.000006365	0.006930884	sh_2
	50	0.001903856	0.010977405	0.000000500	0.0000000005	0.000000464	0.001501114	sh_2
	100	0.012999775	0.022276827	0.000001300	0.000000001	0.000001361	0.003711212	sh_2
50	30	0.003123882	0.013362216	0.000000778	0.0000000008	0.000000699	0.002820448	sh_2
	50	0.005445073	0.002487371	0.000000137	0.000000004	0.000001996	0.004301892	sh_2
	100	0.004426971	0.005645088	0.000000027	0.000000001	0.000000240	0.005231317	sh_2
100	30	0.005700750	0.012975972	0.000001614	0.000000001	0.000001600	0.002827166	sh_2
	50	0.001053722	0.003492566	0.000000048	0.000000001	0.000000420	0.003143218	sh_2
	100	0.001226984	0.001783573	0.000000032	0.000000001	0.000000126	0.001721086	sh_2

Table (4-45): Estimation for modified exponentiated Lomax distribution when R = 0.25, alpha1= 1, alpha2= 3

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.247618	0.240610	0.250016	0.250000	0.250016	0.242459
	50	0.244659	0.239707	0.250000	0.250001	0.250017	0.243124
	100	0.236585	0.236032	0.249934	0.249995	0.249920	0.239052
50	30	0.252024	0.243497	0.250011	0.250000	0.250022	0.246713
	50	0.2509858	0.245903	0.250014	0.250002	0.250055	0.249077
	100	0.2481243	0.247139	0.250011	0.250002	0.250030	0.247603
100	30	0.2563030	0.248820	0.250063	0.250001	0.250063	0.247976
	50	0.2507572	0.253537	0.249995	0.249999	0.249983	0.248375
	100	0.2500377	0.249814	0.250004	0.250000	0.250009	0.247633

Table (4-46):MSE values for modified exponentiated Lomax distribution when R = 0.25, alpha1= 1, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.006780820	0.016201945	0.000004894	0.000000004	0.000005014	0.002607951	sh_2
	50	0.003035860	0.011801799	0.000000280	0.0000000003	0.000000252	0.005203938	sh_2
	100	0.190193986	0.202840385	0.000008592	0.000000025	0.000011336	0.138836395	sh_2
50	30	0.014525665	0.058988639	0.000010708	0.000000010	0.000011027	0.030438043	sh_2
	50	0.000774350	0.002783139	0.000000030	0.000000001	0.000000437	0.002630294	sh_2
	100	0.001100781	0.002630618	0.000000029	0.000000001	0.000000235	0.001319031	sh_2
100	30	0.004385924	0.002014990	0.000000637	0.0000000007	0.000000657	0.001335484	sh_2
	50	0.000502048	0.003278226	0.000000014	0.0000000005	0.000000174	0.000589709	sh_2
	100	0.000239192	0.000505750	0.000000008	0.0000000003	0.000000031	0.000997759	sh_2

Table (4-47): Estimation for modified exponentiated Lomax distribution when R = 0.750000, alpha1= 3, alpha2= 1

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.755734	0.748044	0.750082	0.750002	0.750083	0.754089
	50	0.747484	0.743238	0.749927	0.749998	0.749938	0.755993
	100	0.746065	0.745587	0.750015	0.750001	0.750019	0.749192
50	30	0.755921	0.756221	0.749998	0.750000	0.750001	0.752759
	50	0.750889	0.749384	0.749995	0.749999	0.749984	0.753031
	100	0.749510	0.747731	0.750004	0.750000	0.750004	0.752929
100	30	0.757137	0.757004	0.749961	0.749998	0.749959	0.758184
	50	0.754389	0.750021	0.749999	0.749999	0.749992	0.756804
	100	0.749224	0.750425	0.749991	0.749998	0.749982	0.751943

Table (4-48): MSE values for modified exponentiated Lomax distribution when R = 0.750000, alpha1= 3, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.036134627	0.020203038	0.000010858	0.000000009	0.000011130	0.029019873	sh_2
	50	0.016840759	0.071568906	0.000020848	0.000000014	0.000017238	0.069488845	sh_2
	100	0.022253943	0.02365548	0.000005650	0.000000008	0.000006119	0.004885042	sh_2
50	30	0.039787275	0.08559165	0.000005181	0.000000003	0.000003905	0.029367299	sh_2
	50	0.010599588	0.00710283	0.000000332	0.000000012	0.000004799	0.016728442	sh_2
	100	0.004699094	0.013846798	0.000000137	0.000000005	0.000001464	0.019873193	sh_2
100	30	0.076539196	0.067514491	0.000013100	0.000000021	0.000014899	0.090790818	sh_2
	50	0.021427399	0.005993704	0.000000040	0.000000001	0.000000686	0.055409633	sh_2
	100	0.003688607	0.007614364	0.000000151	0.000000005	0.000000588	0.011194837	sh_2

Table (4-49): Estimation for modified exponentiated Lomax distribution when R = 0.400000, alpha1= 2, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.399050	0.391347	0.400023	0.400000	0.400023	0.394188
	50	0.394979	0.390309	0.400001	0.400001	0.400023	0.394875
	100	0.391577	0.391983	0.400039	0.400001	0.400041	0.398444
50	30	0.402963	0.398266	0.399979	0.399999	0.399984	0.399179
	50	0.403020	0.406418	0.400019	0.400003	0.400073	0.400784
	100	0.398825	0.397170	0.400015	0.400002	0.400067	0.399961
100	30	0.410136	0.401917	0.400076	0.400002	0.400080	0.406548
	50	0.401547	0.406483	0.399993	0.399998	0.399994	0.397933
	100	0.398835	0.396113	0.399996	0.399999	0.399992	0.397019

Table (4-50): MSE values for modified exponentiated Lomax distribution when R = 0.400000, alpha1= 2, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.005834714	0.018125851	0.000005727	0.000000005	0.000005867	0.009272790	sh_2
	50	0.002831141	0.012838123	0.000000460	0.0000000005	0.000000422	0.003354160	sh_2
	100	0.009009576	0.008279508	0.000001823	0.000000001	0.000001895	0.002298330	sh_2
50	30	0.002564994	0.003637504	0.000001249	0.000000001	0.000001406	0.001841287	sh_2
	50	0.001147120	0.017054489	0.000000042	0.000000001	0.000000616	0.000751142	sh_2
	100	0.002760032	0.002675167	0.000000082	0.000000003	0.000001086	0.004816905	sh_2
100	30	0.010734650	0.002533521	0.000000917	0.000000001	0.000001010	0.005521380	sh_2
	50	0.001852906	0.030120632	0.000000047	0.000000001	0.000000431	0.002102470	sh_2
	100	0.001360866	0.003083601	0.000000034	0.000000001	0.000000133	0.004186143	sh_2

Table (4-51): Estimation for modified exponentiated Lomax distribution when R = 0.60000, alpha1= 3, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.599965	0.594228	0.599956	0.599998	0.599956	0.604850
	50	0.597252	0.593312	0.599938	0.600000	0.599984	0.598718
	100	0.591781	0.589941	0.600003	0.599999	0.600002	0.601835
50	30	0.608563	0.603401	0.600004	0.600001	0.600036	0.607182
	50	0.600595	0.597543	0.5999997	0.5999999	0.599999	0.600587
	100	0.601205	0.609301	0.600020	0.600003	0.600061	0.604753
100	30	0.611781	0.617803	0.599980	0.600001	0.599995	0.608276
	50	0.605456	0.599253	0.600005	0.600001	0.600027	0.607343
	100	0.601724	0.598618	0.600005	0.600001	0.600011	0.602782

Table (4-52): MSE values for modified exponentiated Lomax distribution when R = 0.60000, alpha1= 3, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.0290998852	0.092931908	0.000029266	0.000000026	0.000029980	0.049376244	sh_2
	50	0.0252583029	0.060236540	0.000011332	0.000000016	0.000012121	0.023441447	sh_2
	100	0.0084769863	0.012002784	0.000001937	0.000000001	0.000001963	0.001509325	sh_2
50	30	0.0116280561	0.005939451	0.000004147	0.000000005	0.000004998	0.009613544	sh_2
	50	0.0164246021	0.041184519	0.000000425	0.000000015	0.000006185	0.028322354	sh_2
	100	0.0010532501	0.013759339	0.000000066	0.000000002	0.000000564	0.008802075	sh_2
100	30	0.0179399775	0.035575536	0.000002722	0.000000005	0.000003132	0.009594983	sh_2
	50	0.0587201297	0.015041596	0.000000866	0.000000031	0.000010149	0.081666814	sh_2
	100	0.0064518820	0.017957291	0.000000117	0.000000004	0.000000459	0.013422827	sh_2

Table (4-53): Estimation for modified exponentiated Lomax distribution when R = 0.50000, alpha1= 3, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.500475	0.493457	0.500026	0.500000	0.500026	0.496673
	50	0.496051	0.492439	0.500002	0.500001	0.500026	0.497260
	100	0.492299	0.493380	0.500042	0.500001	0.500044	0.500258
50	30	0.504675	0.507075	0.500044	0.499999	0.500010	0.500231
	50	0.494800	0.546276	0.499972	0.499994	0.499896	0.496510
	100	0.495996	0.495441	0.500001	0.500000	0.500015	0.498652
100	30	0.511198	0.504557	0.500052	0.500001	0.500053	0.505113
	50	0.503559	0.503156	0.499996	0.499999	0.499987	0.498372
	100	0.502771	0.499872	0.500013	0.500002	0.500026	0.504741

Table (4-54): MSE values for modified exponentiated Lomax distribution when R = 0.50000, alpha1= 3, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.006002668	0.016517519	0.000006214	0.000000005	0.000006365	0.006930884	sh_2
	50	0.001903856	0.010776564	0.000000500	0.0000000005	0.000000464	0.001501114	sh_2
	100	0.007936060	0.006304575	0.000001984	0.000000002	0.000002063	0.002086233	sh_2
50	30	0.003123882	0.014719555	0.000000778	0.0000000008	0.000000699	0.002820448	sh_2
	50	0.000075115	0.008237986	0.000000002	0.00000000007	0.000000030	0.000046086	sh_2
	100	0.002017933	0.002696567	0.000000010	0.0000000003	0.000000082	0.000865650	sh_2
100	30	0.013315204	0.006237248	0.000000825	0.0000000009	0.000000859	0.004064256	sh_2
	50	0.001538733	0.003189330	0.000000007	0.0000000002	0.000000121	0.001970133	sh_2
	100	0.001877359	0.000401418	0.000000048	0.0000000018	0.000000188	0.004399187	sh_2

Table (4-55): Estimation for modified exponentiated Lomax distribution when R_s = 0.166666, alpha1= 1, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.1709133	0.1629435	0.1666801	0.1666671	0.1666802	0.1574149
	50	0.1699756	0.1620999	0.1666681	0.1666672	0.1666771	0.1600741
	100	0.1689781	0.1631980	0.1666618	0.1666664	0.1666610	0.1597518
50	30	0.1687529	0.1625399	0.1666501	0.1666658	0.1666427	0.1577547
	50	0.1689410	0.1634592	0.1666672	0.1666667	0.1666684	0.1602565
	100	0.1684775	0.1645325	0.1666674	0.1666668	0.1666710	0.1608036
100	30	0.1688880	0.1652187	0.1666464	0.1666666	0.1666494	0.1606635
	50	0.1682051	0.1642894	0.1666664	0.1666666	0.1666700	0.1609259
	100	0.1679375	0.1648470	0.1666674	0.1666668	0.1666682	0.1629076

Table (4-56): MSE values for modified exponentiated Lomax distribution when R_s = 0.166666, alpha1= 1, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00061744974	0.00075889532	0.00000066945	0.00000000061	0.00000068573	0.00110059583	sh_2
	50	0.00055499583	0.00062675584	0.00000041153	0.00000000056	0.00000048925	0.00095654928	sh_2
	100	0.00041754028	0.00052991362	0.00000033753	0.00000000041	0.00000035557	0.00075035587	sh_2
50	30	0.00050660258	0.00065426234	0.00000035851	0.00000000051	0.00000043553	0.00098889036	sh_2
	50	0.00039981043	0.00047280438	0.00000001104	0.00000000040	0.00000016026	0.00068893064	sh_2
	100	0.00029579087	0.00043533800	0.00000000805	0.00000000029	0.00000009060	0.00057021956	sh_2
100	30	0.00042820763	0.00058433686	0.00000036805	0.00000000043	0.00000038301	0.00074809224	sh_2
	50	0.00030140321	0.00044575527	0.00000000821	0.00000000030	0.00000009305	0.00061853114	sh_2
	100	0.0001629076	0.00032040824	0.00000000479	0.00000000018	0.00000001871	0.00036712670	sh_2

Table (4-57): Estimation for modified exponentiated Lomax distribution when R_s = 0.2333333, alpha1= 1, alpha2= 2, alph3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2375352	0.2278526	0.2333557	0.2333341	0.2333559	0.2218034
	50	0.2378302	0.2274234	0.2333690	0.2333344	0.2333687	0.2250104
	100	0.2352197	0.2289016	0.2332959	0.2333321	0.2332946	0.2230050
50	30	0.2364956	0.2281299	0.2333463	0.2333335	0.2333426	0.2237739
	50	0.2356589	0.2289907	0.2333337	0.2333334	0.2333340	0.2250858
	100	0.2360951	0.2299195	0.2333371	0.2333340	0.2333484	0.2267482
100	30	0.2345590	0.2286338	0.2333268	0.2333327	0.2333239	0.2246643
	50	0.2353054	0.2301737	0.2333363	0.2333339	0.2333378	0.2272621
	100	0.2344579	0.2306860	0.2333331	0.2333333	0.2333329	0.2279446

Table (4-58): MSE values for modified exponentiated Lomax distribution when R_s = 0.2333333, alpha1= 1, alpha2= 2, alph3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00102087356	0.00111636429	0.00000116510	0.00000000108	0.00000119340	0.00180435945	sh_2
	50	0.00088791858	0.00092840808	0.00000083117	0.00000000091	0.00000090669	0.00156554810	sh_2
	100	0.00077768704	0.00095321345	0.00000077318	0.00000000078	0.00000079773	0.00153881443	sh_2
50	30	0.00064909700	0.00087589597	0.00000030112	0.00000000065	0.00000045347	0.00124496590	sh_2
	50	0.00058537780	0.00070583543	0.00000001618	0.00000000058	0.00000023484	0.00110692304	sh_2
	100	0.00049668714	0.00069244245	0.00000001363	0.00000000050	0.00000017658	0.00096953504	sh_2
100	30	0.00044989782	0.00064736609	0.00000030849	0.00000000045	0.00000032804	0.00095497520	sh_2
	50	0.00037787545	0.00065489163	0.00000001011	0.00000000038	0.00000009023	0.00070649278	sh_2
	100	0.00028109699	0.00049511345	0.00000000723	0.00000000028	0.00000002819	0.00053707594	sh_2

Table (4-59): Estimation for modified exponentiated Lomax distribution when R_s = 0.233333, alpha1= 2, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2360132	0.2268709	0.2333005	0.2333324	0.2333000	0.2194938
	50	0.2362333	0.2274982	0.2333295	0.2333336	0.2333346	0.2235883
	100	0.2359024	0.2281697	0.2333399	0.2333340	0.2333423	0.2243378
50	30	0.2371687	0.2298518	0.2333314	0.2333336	0.2333363	0.2236364
	50	0.2355211	0.2291798	0.2333337	0.2333334	0.2333341	0.2240360
	100	0.2350879	0.2298363	0.2333349	0.2333336	0.2333391	0.2274882
100	30	0.2356348	0.2289627	0.2333005	0.2333322	0.2332992	0.2244643
	50	0.2350205	0.2289632	0.2333318	0.2333330	0.2333214	0.2254272
	100	0.2337719	0.2295340	0.2333297	0.2333326	0.2333261	0.2275228

Table (4-60): MSE values for modified exponentiated Lomax distribution when R_s = 0.233333, alpha1= 2, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00096689369	0.00115814315	0.00000108783	0.00000000100	0.00000111429	0.00175624417	sh_2
	50	0.00075739161	0.00096157760	0.00000032255	0.00000000077	0.00000051491	0.00145015309	sh_2
	100	0.00047291423	0.00059479490	0.00000029809	0.00000000048	0.00000032086	0.00095129534	sh_2
50	30	0.00090397465	0.00119981752	0.00000077776	0.00000000092	0.00000088120	0.00155799559	sh_2
	50	0.00059655573	0.00080104399	0.00000001693	0.00000000061	0.00000024570	0.00108384578	sh_2
	100	0.00035975690	0.00053902944	0.00000000986	0.00000000037	0.00000009166	0.00069777677	sh_2
100	30	0.00071010184	0.00092071322	0.00000066556	0.00000000070	0.00000069024	0.00129019552	sh_2
	50	0.00049401821	0.00059578611	0.00000001364	0.00000000050	0.00000017305	0.00093191502	sh_2
	100	0.00027970063	0.00040114420	0.00000000726	**0.**00000000028	0.00000002834	0.00056041313	sh_2

Table (4-61): Estimation for modified exponentiated Lomax distribution when R_s = 0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	$\widehat{R}_{\text{mle}}$	$\widehat{R}_{\text{mom}}$	$\widehat{R}_{\text{sh1}}$	$\widehat{R}_{\text{sh2}}$	$\widehat{R}_{\text{sh3}}$	$\widehat{R}_{\text{Ls}}$
30	30	0.3351198	0.3223172	0.3332600	0.3333312	0.3332591	0.3140584
	50	0.3357032	0.3253657	0.3333037	0.3333329	0.3333100	0.3189993
	100	0.3357490	0.3272290	0.3333530	0.3333338	0.3333521	0.3218564
50	30	0.3377634	0.3271258	0.3333447	0.3333346	0.3333591	0.3201587
	50	0.3358002	0.3277818	0.3333365	0.3333339	0.3333444	0.3217712
	100	0.3359024	0.3284802	0.3333372	0.3333341	0.3333410	0.3256148
100	30	0.3365869	0.3258237	0.3333578	0.3333347	0.3333610	0.3221640
	50	0.3361147	0.3290260	0.3333406	0.3333347	0.3333618	0.3249873
	100	0.3340588	0.3287964	0.3333320	0.3333330	0.3333307	0.3263983

Table (4-62): MSE values for modified exponentiated Lomax distribution when R_s = 0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00131938582	0.00148943117	0.00000150903	0.00000000139	0.00000154573	0.00267000453	sh_2
	50	0.00105341206	0.00150965465	0.00000075050	0.00000000108	0.00000092153	0.00203238977	sh_2
	100	0.00091493527	0.00117911250	0.00000082883	0.00000000097	0.00000086545	0.00178943044	sh_2
50	30	0.00104153838	0.00124369092	0.00000072847	0.00000000107	**0.**00000090919	0.00202586004	sh_2
	50	0.00082623225	0.00110087966	0.00000002374	0.00000000086	0.00000034432	0.00160465427	sh_2
	100	0.00056694624	0.00079915257	0.00000001549	0.00000000057	0.00000016645	0.00110886528	sh_2
100	30	0.00092006341	0.00106412726	0.00000085606	0.00000000101	0.00000089543	0.00171885489	sh_2
	50	0.00061106462	0.00089952850	0.00000001729	0.00000000064	0.00000019760	0.00121041820	sh_2
	100	0.00037282641	0.00065298900	0.00000000982	**0.**00000000038	0.00000003830	0.00075926492	sh_2

Table (4-63): Estimation for modified exponentiated Lomax distribution when R_s = 0.266666, alpha1= 1, alpha2= 3, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2695024	0.2596028	0.2666196	0.2666653	0.2666190	0.2522178
	50	0.2695581	0.2617844	0.2666310	0.2666658	0.2666339	0.2552976
	100	0.2707394	0.2621354	0.2666928	0.2666674	0.2666925	0.2572039
50	30	0.2708188	0.2626032	0.2666757	0.2666680	0.2666932	0.2567795
	50	0.2697336	0.2627158	0.2666713	0.2666675	0.2666838	0.2580591
	100	0.2694850	0.2632168	0.2666689	0.2666671	0.2666714	0.2599541
100	30	0.2690446	0.2612466	0.2666810	0.2666678	0.2666847	0.2586351
	50	0.2684953	0.2632605	0.2666700	0.2666673	0.2666806	0.2599023
	100	0.2679390	0.2632469	0.2666674	0.2666668	0.2666682	0.2615770

Table (4-64): MSE values for modified exponentiated Lomax distribution when R_s = 0.266666, alpha1= 1, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00116222833	0.00133253130	0.00000130525	0.00000000120	0.00000133701	0.00220196353	sh_2
	50	0.00108061824	0.00143865583	0.00000106791	0.00000000110	0.00000113959	0.00188697301	sh_2
	100	0.00111970613	0.00120208136	0.00000118901	0.00000000116	0.00000122321	0.00195672049	sh_2
50	30	0.00074034974	0.00087594611	0.00000021788	0.00000000074	0.00000043484	0.00144330667	sh_2
	50	0.00073406483	0.00092487391	0.00000002068	0.00000000075	0.00000030011	0.00131416157	sh_2
	100	0.00058022860	0.00076723507	0.00000001589	0.00000000058	0.00000021263	0.00109193232	sh_2
100	30	0.00049300038	0.00064923356	0.00000024544	0.00000000052	0.00000027510	0.00093507984	sh_2
	50	0.00039508825	0.00063398175	0.00000001059	0.00000000040	0.00000007769	0.00080222467	sh_2
	100	0.00033299304	0.00049526326	0.00000000870	**0.**00000000033	0.00000003395	0.00064263001	sh_2

Table (4-65): Estimation for modified exponentiated Lomax distribution when R_s = 0.266666, alpha1= 3, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2704460	0.2616656	0.2666677	0.2666668	0.2666676	0.2537304
	50	0.2702456	0.2600031	0.2666848	0.2666677	0.2666932	0.2565210
	100	0.2693363	0.2627137	0.2666900	0.2666678	0.2666926	0.2598956
50	30	0.2718917	0.2617456	0.2667136	0.2666681	0.2667138	0.2574223
	50	0.2687642	0.2614121	0.2666653	0.2666664	0.2666607	0.2570601
	100	0.2691836	0.2635345	0.2666727	0.2666678	0.2666839	0.2615142
100	30	0.2682387	0.2620224	0.2666038	0.2666648	0.2666029	0.2562008
	50	0.2703575	0.2638081	0.2666743	0.2666681	0.2666928	0.2612692
	100	0.2674110	0.2639475	0.2666651	0.2666663	0.2666636	0.2605872

Table (4-66): MSE values for modified exponentiated Lomax distribution when R_s = 0.266666, alpha1= 3, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00114036094	0.00145811246	0.00000128303	0.00000000118	0.00000131423	0.00209991614	sh_2
	50	0.00075314079	0.00095308463	0.00000021753	0.00000000077	0.00000044009	0.00134678786	sh_2
	100	0.00044313789	0.00063691512	0.00000021701	0.00000000045	0.00000024114	0.00089102201	sh_2
50	30	0.00110950226	0.00127294378	0.00000107392	0.00000000112	0.00000115356	0.00197753005	sh_2
	50	0.00067183667	0.00091886913	0.00000001904	0.00000000069	0.00000027634	0.00137196809	sh_2
	100	0.00038854587	0.00061136486	0.00000001025	0.00000000039	0.00000007295	0.00076048646	sh_2
100	30	0.00101444570	0.00136164745	0.00000108080	0.00000000105	0.00000111154	0.00177071770	sh_2
	50	0.00060949106	0.00090283743	0.00000001706	0.00000000062	0.00000023453	0.00111921322	sh_2
	100	0.00035534082	0.00056434661	0.00000000919	0.00000000035	0.00000003587	0.00070362486	sh_2

Table (4-67): Estimation for modified exponentiated Lomax distribution when R_s = 0.385714, alpha1= 2, alpha2= 3, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.3883645	0.3737153	0.3856899	0.3857137	0.3856895	0.3669795
	50	0.3885667	0.3766683	0.3857224	0.3857151	0.3857296	0.3707204
	100	0.3877822	0.3771733	0.3856981	0.3857139	0.3856979	0.3720899
50	30	0.3867319	0.3766373	0.3856915	0.3857130	0.3856786	0.3693732
	50	0.3873906	0.3780910	0.3857135	0.3857141	0.3857103	0.3734879
	100	0.3876910	0.3803119	0.3857161	0.3857146	0.3857226	0.3748057
100	30	0.3873459	0.3811033	0.3856901	0.3857146	0.3856953	0.3747095
	50	0.3864987	0.3798669	0.3857128	0.3857139	0.3857155	0.3749957
	100	0.3869367	0.3814466	0.3857154	0.3857145	0.3857165	0.3788336

Table (4-68): MSE values for modified exponentiated Lomax distribution when R_s = 0.385714, alpha1= 2, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00140166300	0.00169476790	0.00000160647	0.00000000148	0.00000164552	0.00257400792	sh_2
	50	0.00130199282	0.00156429154	0.00000128289	0.00000000141	0.00000138854	0.00245864435	sh_2
	100	0.00106279005	0.00140664773	0.00000109326	0.00000000113	0.00000112752	0.00206746788	sh_2
50	30	0.00099896335	0.00151451667	0.00000053042	0.00000000103	0.00000074952	0.00220638169	sh_2
	50	0.00085869665	0.00112964343	0.00000002443	0.00000000088	0.00000035441	0.00164918330	sh_2
	100	0.00071544604	0.00107497819	0.00000002037	0.00000000075	0.00000025616	0.00147197386	sh_2
100	30	0.00075520793	0.00117897309	0.00000053966	0.00000000080	0.00000057277	0.00149598634	sh_2
	50	0.00057405764	0.00094556544	0.00000001579	0.00000000059	0.00000015072	0.00131118983	sh_2
	100	0.00040189334	0.00075291555	0.00000001049	0.00000000040	0.00000004091	0.00085080681	sh_2

Table (4-69): Estimation for modified exponentiated Lomax distribution when R_s = 0.385714, alpha1= 3, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3894220	0.3771645	0.3857462	0.3857154	0.3857465	0.3690335
	50	0.3888210	0.3773033	0.3857418	0.3857152	0.3857409	0.3731605
	100	0.3860783	0.3783220	0.3856808	0.3857131	0.3856788	0.3714141
50	30	0.3895788	0.3765243	0.3857402	0.3857150	0.3857371	0.3719043
	50	0.3879171	0.3778274	0.3857163	0.3857147	0.3857210	0.3742013
	100	0.3874769	0.3804256	0.3857167	0.3857147	0.3857251	0.3762786
100	30	0.3876969	0.3782171	0.3857057	0.3857137	0.3857032	0.3725003
	50	0.3873066	0.3799181	0.3857146	0.3857143	0.3857165	0.3764758
	100	0.3866871	0.3812027	0.3857144	0.3857143	0.3857144	0.3788872

Table (4-70): MSE values for modified exponentiated Lomax distribution when R_s = 0.385714, alpha1= 3, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00147463193	0.00181824588	0.00000173715	0.00000000161	0.00000177931	0.00282299006	sh_2
	50	0.00101507049	0.00139297614	0.00000057668	0.00000000106	0.00000078905	0.00206790781	sh_2
	100	0.00073855218	0.00104158449	0.00000053262	0.00000000075	0.00000056549	0.00173799905	sh_2
50	30	0.00115802377	0.00139895409	0.00000102289	0.00000000121	0.00000114900	0.00225595910	sh_2
	50	0.00087876597	0.00110046785	0.00000002517	0.00000000091	0.00000036505	0.00168574798	sh_2
	100	0.00054106871	0.00095534775	0.00000001483	0.00000000055	0.00000014238	0.00119959352	sh_2
100	30	0.00106663947	0.00140519594	0.00000106960	0.00000000109	0.00000110276	0.00220486395	sh_2
	50	0.00076245782	0.00114885407	0.00000002126	0.00000000078	0.00000025748	0.00143847029	sh_2
	100	0.00044215624	0.00067169350	0.00000001143	0.00000000044	0.00000004460	0.00090944433	sh_2

Table (4-71): Estimation for modified exponentiated Lomax distribution when R_s =0.45, alpha1= 3, alpha2= 3, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.4510536	0.4357576	0.4499349	0.4499982	0.4499340	0.4278106
	50	0.4521776	0.4399197	0.4500071	0.4500009	0.4500161	0.4338503
	100	0.4512730	0.4406390	0.4499852	0.4499997	0.4499851	0.4358392
50	30	0.4508115	0.4392741	0.4499734	0.4499987	0.4499620	0.4320120
	50	0.4516080	0.4417896	0.4500008	0.4500001	0.4500019	0.4370464
	100	0.4513899	0.4438589	0.4500011	0.4500002	0.4500062	0.4384864
100	30	0.4513500	0.4441903	0.4499679	0.4500000	0.4499722	0.4373235
	50	0.4509262	0.4433906	0.4499997	0.4499999	0.4500046	0.4385127
	100	0.4510430	0.4450717	0.4500011	0.4500002	0.4500022	0.4425772

Table (4-72): MSE values for modified exponentiated Lomax distribution when R_s =0.45, alpha1= 3, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00129447623	0.00168668070	0.00000152602	0.00000000141	0.00000156310	0.00300128587	sh_2
	50	0.00117877285	0.00154129735	0.00000096885	0.00000000128	0.00000112396	0.00235541784	sh_2
	100	0.00089824488	0.00126702375	0.00000082585	0.00000000096	0.00000085905	0.00186017566	sh_2
50	30	0.00110481437	0.00162074259	0.00000080946	0.00000000115	**0.**00000098361	0.00249430172	sh_2
	50	0.00086731917	0.00119493076	0.00000002498	0.00000000090	0.00000036225	0.00168565184	sh_2
	100	0.00064677638	0.00103166042	0.00000001823	0.00000000067	0.00000020480	0.00139367696	sh_2
100	30	0.00092558572	0.00138418480	0.00000083098	0.00000000099	0.00000086474	0.00186440050	sh_2
	50	0.00066381550	0.00106023309	0.00000001859	0.00000000069	0.00000021038	0.00151476586	sh_2
	100	0.00041554278	0.00076871817	0.00000001085	0.00000000042	0.00000004233	0.00087943917	sh_2

(4-2-3)The Simulation for Estimate the System Reliability Based on Power Lomax Distribution

Step 1: the random sample generated for X according to the uniform distribution over the interval (0, 1) as$r_1, r_2 \ldots\ldots\ldots\ldots r_n$.

Step 2: the random sample generated for Y according to the uniform distribution over the interval (0, 1), as $s_1, s_2, \ldots\ldots\ldots\ldots s_m$.

Step 3: transforming the above power Lomax distribution with using (c.d.f)as in the following, $F(x_j) = r_j$

$$r_j = 1 - (1 + x_i)^{-\alpha}$$

$$x_i = (1 - r_i)^{\frac{-1}{\alpha}} - 1$$

and by the same way, from step 2 ,we get

$$y_j = (1 - s_j)^{\frac{-1}{\alpha}} - 1$$

Step 4: calculating$\hat{\alpha}_{1_{mle}}$ and $\hat{\alpha}_{2mle}$ using equations (3-66) and (3-67).

Step 5: calculating$\hat{\alpha}_{1_{mom}}$ and $\hat{\alpha}_{2_{mom}}$ using equations (3-71) and (3-72).

Step 6: calculating$\hat{\alpha}_{1_{Ls}}$ and $\hat{\alpha}_{2_{Ls}}$ using equations (3-76) and (3-77).

Step 7: calculating $\hat{\alpha}_{1_{sh_i}}$and $\hat{\alpha}_{2_{sh_i}}$ when *i*=1, 2, 3 using equations (3-83) , (3-84), (3-88), (3-89), (3-93), (3-94) respectively.

Step 8: calculating$\hat{R}_{mle}$, $\hat{R}_{mom}$, $\hat{R}_{Ls}$, $\hat{R}_{sh_1}$, $\hat{R}_{sh_2}$, and $\hat{R}_{sh_3}$ using equations (3-69),(3-74), (3-79), (3-86), (3-91) and (3-96) respectively.

Step 9: calculating $\hat{R}_{s(mle)}$, $\hat{R}_{s(mom)}$, $\hat{R}_{s(Ls)}$, $\hat{R}_{s(sh_1)}$, $\hat{R}_{s(sh_2)}$, $and \hat{R}_{s(sh_3)}$ using equations (3-70), (3-75), (3-80), (3-87), (3-92) and (3-97) when $\hat{\alpha}_3$ =2 for all methods and cases.

We using random sample for x_i and y_j for size (n,m)= (30,30),(30,50),(30,100), (50,30),(50,50),(50,100), (100,30),(100,50),(100,100)

The result were as show in the tables

Table (4-73): Estimation for power Lomax distribution when R = 0.50000, alpha1= 1, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.4990691	0.4995320	0.4999636	0.4999988	0.4999631	0.4933728
	50	0.4915832	0.4925759	0.4999419	0.4999964	0.4999094	0.4914627
	100	0.4885727	0.4852478	0.4999040	0.4999974	0.4999044	0.4912454
50	30	0.5053804	0.5075598	0.4999918	0.5000000	0.4999980	0.5021542
	50	0.5001760	0.5006124	0.5000011	0.5000002	0.5000044	0.5005587
	100	0.4941137	0.4919213	0.4999904	0.4999981	0.4999794	0.4946685
100	30	0.5097196	0.5135999	0.4999964	0.5000005	0.5000009	0.5086943
	50	0.5056655	0.5082287	0.5000103	0.5000019	0.5000340	0.5033106
	100	0.5003044	0.4993608	0.5000016	0.5000003	0.5000031	0.4980302

Table (4-74): MSE values for power Lomax distribution when R = 0.50000, alpha1= 1, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00001391217	0.00209737853	0.00000001305	0.00000000001	0.00000001337	0.00003305240	sh_2
	50	0.00000695913	0.00175473338	0.00000000243	0.00000000002	0.00000000936	0.00003373632	sh_2
	100	0.00013195531	0.00166060196	0.00000001348	0.00000000002	0.00000001573	0.00007948377	sh_2
50	30	0.00004677992	0.00172865260	0.00000000589	0.00000000001	0.00000000982	0.00006716573	sh_2
	50	0.00000829406	0.00128969733	0.00000000021	0.00000000000	0.00000000313	0.00001142325	sh_2
	100	0.00001173653	0.00118971356	0.00000000008	0.00000000000	0.00000000110	0.00000643071	sh_2
100	30	0.00007439509	0.00148970220	0.00000002012	0.00000000001	0.00000002051	0.00006267325	sh_2
	50	0.00001639385	0.00111662614	0.00000000092	0.00000000003	0.00000001174	0.00004014611	sh_2
	100	0.00000815920	0.00085866957	0.00000000018	0.00000000000	0.00000000072	0.00000497111	sh_2

Table (4-75): Estimation for power Lomax distribution when R = 0.66666, alpha1= 1, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.6697736	0.6270314	0.6667093	0.6666678	0.6667099	0.6706177
	50	0.6637399	0.6211789	0.6666640	0.6666664	0.6666604	0.6624070
	100	0.6575665	0.6172032	0.6666065	0.6666641	0.6666015	0.6632227
50	30	0.6740815	0.6347745	0.6667081	0.6666676	0.6667056	0.6736018
	50	0.6711390	0.6315067	0.6666830	0.6666697	0.6667293	0.6741909
	100	0.6647063	0.6242857	0.6666688	0.6666670	0.6666816	0.6677485
100	30	0.6769933	0.6397353	0.6666809	0.6666661	0.6666770	0.6729704
	50	0.6741539	0.6367498	0.6666799	0.6666692	0.6667088	0.6726842
	100	0.6655813	0.6310720	0.6666575	0.6666648	0.6666486	0.6667459

Table (4-76): MSE values for power Lomax distribution when R = 0.66666, alpha1= 1, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00003473260	0.00429854681	0.00000002916	0.00000000002	0.00000002987	0.00006578488	sh_2
	50	0.00002161066	0.00420715593	0.00000000899	0.00000000001	0.00000001218	0.00003780462	sh_2
	100	0.00005283151	0.00396702779	0.00000001037	0.00000000000	0.00000000924	0.00001730238	sh_2
50	30	0.00008258476	0.00326205651	0.00000000823	0.00000000001	0.00000001194	0.00005814712	sh_2
	50	0.00000604757	0.00324952453	0.00000000009	0.00000000001	0.00000000142	0.00003108260	sh_2
	100	0.00000964434	0.00292340542	0.00000000023	0.00000000001	0.00000000256	0.00001578951	sh_2
100	30	0.00008832101	0.00300269374	0.00000001174	0.00000000002	0.00000001244	0.00009928189	sh_2
	50	0.00001366603	0.00258721883	0.00000000027	0.00000000001	0.00000000330	0.00003035958	sh_2
	100	0.00000410418	0.00258228427	0.00000000010	0.00000000001	0.00000000042	0.00001123746	sh_2

Table (4-77): Estimation for power Lomax distribution when R = 0.33333, alpha1= 2, alpha2= 1

n	m	$\widehat{R}_{\text{mle}}$	$\widehat{R}_{\text{mom}}$	$\widehat{R}_{\text{sh1}}$	$\widehat{R}_{\text{sh2}}$	$\widehat{R}_{\text{sh3}}$	$\widehat{R}_{\text{Ls}}$
30	30	0.3300586	0.372508	0.3332982	0.3333323	0.3332978	0.3229578
	50	0.3241037	0.3652725	0.333280	0.3333302	0.3332512	0.3220745
	100	0.3217389	0.3586051	0.3332466	0.3333310	0.3332469	0.3226039
50	30	0.3332275	0.3750927	0.3332767	0.3333302	0.3332520	0.3244266
	50	0.3294948	0.3695886	0.3333206	0.3333309	0.3332844	0.3293246
	100	0.3298600	0.3677060	0.3333398	0.3333345	0.3333721	0.3312327
100	30	0.3393926	0.3819004	0.3333333	0.333332	0.3333299	0.3348501
	50	0.3351528	0.3750733	0.3333310	0.3333329	0.3333404	0.3343544
	100	0.3292615	0.3645674	0.3333165	0.3333295	0.3332994	0.3295427

Table (4-78): Shown MSE values for power Lomax distribution when R = 0.33333, alpha1= 2, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00002336815	0.00454971944	0.00000003077	0.00000000002	0.00000003151	0.00007496964	sh_2
	50	0.00001022162	0.00355431211	0.00000003930	0.00000000003	0.00000003683	0.00001671073	sh_2
	100	0.00010588123	0.00278803441	0.00000001072	0.00000000001	0.00000001055	0.00005784728	sh_2
50	30	0.00004058348	0.00425999399	0.00000000574	0.00000000002	0.00000001212	0.00005953914	sh_2
	50	0.00000981596	0.00322203337	0.00000000029	0.00000000001	0.00000000429	0.00001539347	sh_2
	100	0.00002861410	0.00253799101	0.00000000028	0.00000000001	0.00000000243	0.00002864544	sh_2
100	30	0.00008349547	0.00385758961	0.00000001102	0.00000000001	0.00000001124	0.00001392615	sh_2
	50	0.00001450263	0.00311167645	0.00000000033	0.00000000001	0.00000000327	0.00003157511	sh_2
	100	0.00001525106	0.00225186524	0.00000000038	0.00000000001	0.00000000150	0.00002271479	sh_2

Table (4-79): Estimation for power Lomax distribution when R = 0.500000, alpha1= 2, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.4990691	0.4994854	0.4999636	0.49999889	0.49996317	0.4933728
	50	0.4915832	0.4936654	0.4999419	0.4999964	0.4999094	0.4914627
	100	0.4885727	0.4899624	0.4999040	0.4999974	0.4999044	0.4912454
50	30	0.5021328	0.5025395	0.4999379	0.4999964	0.4999105	0.4942368
	50	0.4973005	0.4984363	0.4999857	0.4999972	0.4999456	0.4982315
	100	0.4974011	0.4995413	0.5000073	0.5000013	0.5000440	0.4999699
100	30	0.5086968	0.5100602	0.5000016	0.4999992	0.4999979	0.5048152
	50	0.5033262	0.5049378	0.4999978	0.4999995	0.5000084	0.5037560
	100	0.4962179	0.4957485	0.4999807	0.4999961	0.4999619	0.4972384

Table (4-80): MSE values for power Lomax distribution when R = 0.500000, alpha1= 2, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00001335278	0.00437838392	0.00000001210	0.00000000001	0.00000001240	0.00003360687	sh_2
	50	0.00001622629	0.00322220885	0.00000006299	0.00000000005	0.00000006112	0.00013398874	sh_2
	100	0.00011061016	0.00275124770	0.00000000842	0.00000000001	0.00000000896	0.00003763656	sh_2
50	30	0.00008979404	0.00386381367	0.00000002977	0.00000000005	0.00000004278	0.00012220819	sh_2
	50	0.00002354842	0.00277294930	0.00000000063	0.00000000002	0.00000000916	0.00003357154	sh_2
	100	0.00002220056	0.00218787626	0.00000000013	0.00000000001	0.00000000203	0.00001615632	sh_2
100	30	0.00011609210	0.00303619215	0.00000002237	0.00000000003	0.00000002449	0.00002851570	sh_2
	50	0.00003568027	0.00223763908	0.00000000019	0.00000000001	0.00000000249	0.00001665454	sh_2
	100	0.00000904488	0.00168220481	0.00000000024	0.00000000001	0.00000000093	0.00001522764	sh_2

Table (4-81): Estimation for power Lomax distribution when R = 0.750000, alpha1= 1, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.75358550	0.71118421	0.75001575	0.75000037	0.75001598	0.75292370
	50	0.7515927	0.70968383	0.75001065	0.75000307	0.75005895	0.75546213
	100	0.7441725	0.7063645	0.7499707	0.7499988	0.7499687	0.7480085
50	30	0.7567540	0.7160345	0.7499968	0.7500008	0.7500153	0.7589898
	50	0.7513602	0.71424191	0.74999860	0.74999972	0.74999518	0.7537772
	100	0.74916844	0.7114088	0.7500037	0.7500007	0.7500150	0.7523684
100	30	0.7573445	0.7209771	0.7499756	0.7499995	0.7499779	0.7563732
	50	0.75055183	0.71979720	0.74998200	0.7499965	0.7499389	0.7497687
	100	0.75254007	0.71760062	0.75000749	0.75000147	0.75001485	0.75147096

Table (4-82): MSE values for power Lomax distribution when R = 0.750000, alpha1= 1, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00001344768	0.00352886720	0.00000000855	0.00000000001	0.00000000875	0.00001740833	sh_2
	50	0.00001926001	0.00345817620	0.00000000548	0.00000000001	0.00000000779	0.00001682967	sh_2
	100	0.00005511045	0.00310337317	0.00000001435	0.00000000001	0.00000001481	0.00001242772	sh_2
50	30	0.00003631341	0.00312246514	0.00000001296	0.00000000002	0.00000001926	0.00002666844	sh_2
	50	0.00000363446	0.00276378107	0.00000000020	0.00000000001	0.00000000286	0.00001120660	sh_2
	100	0.00000356643	0.00229643521	0.00000000010	0.00000000001	0.00000000204	0.00001234462	sh_2
100	30	0.00009028493	0.00245360284	0.00000001312	0.00000000001	0.00000001346	0.00006781488	sh_2
	50	0.00002042027	0.00203198729	0.00000000011	0.00000000001	0.00000000135	0.00003849444	sh_2
	100	0.00000525549	0.00201152918	0.00000000024	0.00000000001	0.00000000096	0.00000814140	sh_2

Table (4-83): Estimation for power Lomax distribution when R = 0.250000, alpha1= 3, alpha2= 1

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.24663834	0.29131690	0.25000606	0.25000030	0.25000609	0.25000609
	50	0.24383882	0.28305826	0.24997483	0.25000003	0.24998621	0.24507096
	100	0.24101272	0.27929397	0.25003631	0.24999966	0.25002758	0.24159965
50	30	0.25291857	0.29261902	0.25001866	0.25000123	0.25003058	0.24813287
	50	0.2482922	0.2858582	0.25000097	0.25000019	0.25000316	0.24768146
	100	0.24618959	0.28184160	0.25000356	0.25000068	0.2500150	0.24520349
100	30	0.25314006	0.2937443	0.25001035	0.24999861	0.25000087	0.24839344
	50	0.2502366	0.287619	0.2499937	0.24999881	0.24997512	0.24821459
	100	0.2490360	0.28112102	0.25000043	0.25000009	0.25000081	0.24928422

Table (4-84): MSE values for power Lomax distribution when R = 0.250000, alpha1= 3, alpha2= 1

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00006422096	0.00363015151	0.00000003755	0.00000000003	0.00000003848	0.00026891648	sh_2
	50	0.00006734742	0.00302412655	0.00000000532	0.00000000001	0.00000000828	0.00011241404	sh_2
	100	0.00008081886	0.00261427274	0.00000001715	0.00000000003	0.00000002019	0.00006747079	sh_2
50	30	0.00001499249	0.00339145051	0.00000001392	0.00000000001	0.00000001666	0.00003664742	sh_2
	50	0.00000531757	0.00283038422	0.00000000005	0.00000000001	0.00000000077	0.00002253765	sh_2
	100	0.00000747590	0.00213452387	0.00000000014	0.00000000001	0.00000000218	0.00000610952	sh_2
100	30	0.00003573768	0.00332201770	0.00000002216	0.00000000002	0.00000002230	0.00004458118	sh_2
	50	0.00001341778	0.00261319352	0.00000000017	0.00000000001	0.00000000122	0.00000768861	sh_2
	100	0.00000754483	0.00194852759	0.00000000031	0.00000000001	0.00000000122	0.00000519950	sh_2

Table (4-85): Estimation for power Lomax distribution when R = 0.600000, alpha1= 2, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.6006989	0.5922099	0.5999668	0.5999989	0.5999664	0.5966105
	50	0.5930852	0.5879602	0.5999450	0.5999966	0.5999140	0.5942524
	100	0.5950025	0.5871299	0.6000670	0.6000023	0.6000694	0.6018506
50	30	0.6025707	0.5963460	0.5999010	0.5999962	0.5998882	0.6012623
	50	0.6020145	0.5968586	0.6000049	0.6000009	0.6000192	0.6012780
	100	0.5980965	0.5934635	0.6000059	0.6000011	0.6000212	0.5990634
100	30	0.6101153	0.6022995	0.6000378	0.6000004	0.6000350	0.6065977
	50	0.6022280	0.5974171	0.5999899	0.5999980	0.5999674	0.6007204
	100	0.6013152	0.5961165	0.6000041	0.6000008	0.6000081	0.6020934

Table (4-86): Shown MSE values for power Lomax distribution when R = 0.600000, alpha1= 2, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00000359745	0.00385023284	0.00000000333	0.00000000001	0.00000000341	0.00003641390	sh_2
	50	0.00005081064	0.00336787939	0.00000002234	0.00000000007	0.00000004683	0.00006977848	sh_2
	100	0.00009798745	0.00283256153	0.00000000878	0.00000000001	0.00000000994	0.00005018269	sh_2
50	30	0.00009960230	0.00336888485	0.00000001737	0.00000000002	0.00000001952	0.00011042842	sh_2
	50	0.00001557751	0.00247405893	0.00000000032	0.00000000001	0.00000000479	0.00000780103	sh_2
	100	0.00002563885	0.00198664193	0.00000000049	0.00000000001	0.00000000410	0.00003720428	sh_2
100	30	0.00004372190	0.00269094226	0.00000001906	0.00000000001	0.00000001897	0.00003364561	sh_2
	50	0.00002406768	0.00192998341	0.00000000008	0.00000000001	0.00000000077	0.00001197590	sh_2
	100	0.00000821399	0.00139738873	0.00000000021	0.00000000001	0.00000000083	0.00001122839	sh_2

Table (4-87): Estimation for power Lomax distribution when R = 0.40000, alpha1= 3, alpha2= 2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3975174	0.4070787	0.3999633	0.3999989	0.3999628	0.3907105
	50	0.3907783	0.4005944	0.3999434	0.3999966	0.3999120	0.3894054
	100	0.3880890	0.3973748	0.3999070	0.3999975	0.3999073	0.3896690
50	30	0.40075181	0.4083201	0.3999395	0.3999966	0.3999129	0.3920002
	50	0.39647341	0.4040998	0.3999862	0.3999973	0.3999474	0.3967195
	100	0.3967493	0.4050166	0.4000070	0.4000012	0.4000420	0.3986258
100	30	0.4072729	0.4136307	0.4000006	0.3999992	0.3999970	0.4028370
	50	0.4024579	0.4080811	0.3999979	0.3999995	0.4000078	0.4021069
	100	0.3986232	0.4015373	0.3999963	0.3999992	0.3999926	0.3980189

Table (4-88): MSE values for power Lomax distribution when R = 0.40000, alpha1= 3, alpha2= 2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00001553755	0.00397788660	0.00000002784	0.00000000002	0.00000002851	0.00003432214	sh_2
	50	0.00021595470	0.00302995314	0.00000002960	0.00000000008	0.00000005807	0.00016834411	sh_2
	100	0.00002998667	0.00277296830	0.00000002652	0.00000000003	0.00000002797	0.00002078291	sh_2
50	30	0.00004258075	0.00323475128	0.00000000734	0.00000000001	0.00000000988	0.00006309316	sh_2
	50	0.00003137783	0.00252179808	0.00000000057	0.00000000002	0.00000000842	0.00010673619	sh_2
	100	0.00000446468	0.00192531735	0.00000000025	0.00000000001	0.00000000300	0.00002697780	sh_2
100	30	0.00011554942	0.00288274767	0.00000001194	0.00000000002	0.00000001371	0.00006935571	sh_2
	50	0.00001414576	0.00200798828	0.00000000029	0.00000000001	0.00000000280	0.00002769249	sh_2
	100	0.00000253123	0.00145134394	0.00000000004	0.00000000001	0.00000000016	0.00000909262	sh_2

Table (4-89): Estimation for power Lomax distribution when R = 0.50000, alpha1= 3, alpha2= 3

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.5007274	0.5011880	0.5000164	0.5000004	0.5000166	0.4970868
	50	0.4989371	0.4995261	0.5000118	0.5000040	0.5000758	0.5014647
	100	0.4895422	0.4946006	0.4999586	0.4999984	0.4999559	0.4925044
50	30	0.5057418	0.5043972	0.4999936	0.5000011	0.5000176	0.5060212
	50	0.4994213	0.5006940	0.4999980	0.4999996	0.4999926	0.5003678
	100	0.4969247	0.4972736	0.5000049	0.5000009	0.5000194	0.4994113
100	30	0.5076331	0.5046840	0.4999656	0.4999994	0.4999686	0.5037925
	50	0.4988983	0.5023905	0.4999759	0.4999953	0.4999181	0.4962469
	100	0.5021276	0.5000310	0.5000099	0.5000019	0.5000196	0.4994681

Table (4-90): MSE values for power Lomax distribution when R = 0.50000, alpha1= 3, alpha2= 3

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00001425754	0.00390309285	0.00000001699	0.00000000001	0.00000001740	0.00006192660	sh_2
	50	0.00004126795	0.00356426954	0.00000008231	0.00000000004	0.00000005863	0.00002834248	sh_2
	100	0.00010134193	0.00307754757	0.00000001428	0.00000000001	0.00000001394	0.00009137197	sh_2
50	30	0.00002743012	0.00342018915	0.00000000786	0.00000000002	0.00000001725	0.00004074093	sh_2
	50	0.00002873084	0.00286644433	0.00000000079	0.00000000002	0.00000001155	0.00004435667	sh_2
	100	0.00001871954	0.00226357804	0.00000000007	0.00000000001	0.00000000083	0.00002412542	sh_2
100	30	0.00002431900	0.00397133469	0.00000002694	0.00000000002	0.00000002760	0.00005092735	sh_2
	50	0.00005346458	0.00334460632	0.00000001318	0.00000000001	0.00000001036	0.00003687884	sh_2
	100	0.00012131176	0.00283464889	0.00000002139	0.00000000002	0.00000002258	0.00005939419	sh_2

Table (4-91): Estimation for power Lomax distribution when R_s = 0.50000, alpha1= 1, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.4938522	0.4419574	0.5000012	0.4999999	0.5000013	0.5080957
	50	0.4942072	0.4453597	0.4999887	0.4999993	0.4999834	0.5055995
	100	0.4949548	0.4479628	0.4999826	0.4999990	0.4999802	0.5040654
50	30	0.4950959	0.4458413	0.5000032	0.5000000	0.5000034	0.5079952
	50	0.4963182	0.4491912	0.5000002	0.5000000	0.5000013	0.5063889
	100	0.4968795	0.4525893	0.5000017	0.5000003	0.5000056	0.5065596
100	30	0.4961553	0.4494072	0.5000218	0.5000002	0.5000199	0.5070390
	50	0.4973962	0.4526431	0.5000010	0.5000002	0.5000011	0.5051279
	100	0.4994824	0.4570855	0.5000069	0.5000013	0.5000137	0.5055272

Table (4-92): MSE values for power Lomax distribution when R_s = 0.50000, alpha1= 1, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00118646457	0.00389745624	0.00000128639	0.00000000118	0.00000131771	0.00212704812	sh_2
	50	0.00083907048	0.00342100056	0.00000059949	0.00000000083	0.00000071246	0.00155840685	sh_2
	100	0.00070994810	0.00307221109	0.00000058764	0.00000000070	0.00000061369	0.00131455413	sh_2
50	30	0.00090452631	0.00337477475	0.00000067450	0.00000000091	0.00000079838	0.00162472999	sh_2
	50	0.00067069598	0.00293975982	0.00000001824	0.00000000066	0.00000026491	0.00122491821	sh_2
	100	0.00050620948	0.00244663070	0.00000001353	0.00000000050	0.00000014994	0.00091389736	sh_2
100	30	0.00070735398	0.00294766774	0.00000058228	0.00000000070	0.00000061060	0.00123667426	sh_2
	50	0.00048312472	0.00253957750	0.00000001316	0.00000000048	0.00000014684	0.00095989302	sh_2
	100	0.00030797621	0.00204053281	0.00000000788	0.00000000030	0.00000003076	0.00058095800	sh_2

Table (4-93): Estimation for power Lomax distribution when R_s = 0.4, alpha1= 1, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3919092	0.3682939	0.3999132	0.3999972	0.3999122	0.4054704
	50	0.3960659	0.3740307	0.3999988	0.4000002	0.4000051	0.4059646
	100	0.3964499	0.3750944	0.3999757	0.3999995	0.3999770	0.4050657
50	30	0.3955487	0.3730209	0.4000012	0.3999999	0.4000007	0.4081328
	50	0.3957920	0.3747782	0.3999946	0.3999989	0.3999801	0.4047473
	100	0.3976484	0.3787377	0.3999994	0.3999998	0.3999992	0.4046413
100	30	0.3972138	0.3767924	0.4000336	0.4000009	0.4000344	0.4086149
	50	0.3968256	0.3776810	0.3999971	0.3999994	0.3999876	0.4062191
	100	0.3978691	0.3803852	0.3999971	0.3999994	0.3999943	0.4030203

Table (4-94): MSE values for power Lomax distribution when R_s = 0.4, alpha1= 1, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00110404639	0.00183742509	0.00000122589	0.00000000113	0.00000125570	0.00186943908	sh_2
	50	0.00068808831	0.00127002377	0.00000025857	0.00000000069	0.00000043464	0.00140591747	sh_2
	100	0.00051232934	0.00105425331	0.00000027724	0.00000000051	0.00000030333	0.00096005120	sh_2
50	30	0.00104124570	0.00161430747	0.00000103289	0.00000000108	0.00000110778	0.00186648507	sh_2
	50	0.00066606129	0.00121686564	0.00000001853	0.00000000067	0.00000026896	0.00124175406	sh_2
	100	0.00039124969	0.00083868891	0.00000001033	0.00000000039	0.00000007797	0.00075570737	sh_2
100	30	0.00095035377	0.00142434398	0.00000098678	0.00000000098	0.00000101709	0.00175190158	sh_2
	50	0.00056121413	0.00098215740	0.00000001566	0.00000000057	0.00000020854	0.00110891143	sh_2
	100	0.00034120322	0.00074021015	0.00000000883	0.00000000034	0.00000003446	0.00066115720	sh_2

Table (4-95): Estimation for power Lomax distribution when R_s = 0.4, alpha1= 2, alpha2= 1, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.3952109	0.3716214	0.4000235	0.4000006	0.4000238	0.4089827
	50	0.3966209	0.3738687	0.4000492	0.4000011	0.4000458	0.4079938
	100	0.3967149	0.3754788	0.4000198	0.4000005	0.4000204	0.4080131
50	30	0.3952304	0.3732979	0.3999856	0.3999992	0.3999809	0.4069227
	50	0.3965706	0.3761148	0.3999986	0.3999997	0.3999951	0.4069707
	100	0.3971952	0.3784106	0.3999987	0.3999997	0.3999997	0.4054416
100	30	0.3967703	0.3760865	0.3999894	0.4000000	0.3999914	0.4049823
	50	0.3969383	0.3782046	0.3999958	0.3999992	0.3999867	0.4041701
	100	0.3979168	0.3806735	0.3999977	0.3999995	0.3999956	0.4034340

Table (4-96): MSE values for power Lomax distribution when R_s = 0.4, alpha1= 2, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00107492289	0.00170417529	0.00000117791	0.00000000108	0.00000120658	0.00208725387	sh_2
	50	0.00095863718	0.00141964093	0.00000090710	0.00000000099	0.00000099278	0.00190380979	sh_2
	100	0.00092930323	0.00142423877	0.00000096064	0.00000000094	0.00000098863	0.00169853404	sh_2
50	30	0.00076630764	0.00138927238	0.00000028235	0.00000000078	0.00000048677	0.00154377795	sh_2
	50	0.00066980510	0.00121490718	0.00000001884	0.00000000068	0.00000027346	0.00126793354	sh_2
	100	0.00058996803	0.00101078906	0.00000001645	0.00000000060	0.00000021895	0.00113858305	sh_2
100	30	0.00044093223	0.00094082802	0.00000022047	0.00000000043	0.00000024180	0.00086561703	sh_2
	50	0.00038721396	0.00085376864	0.00000001022	0.00000000038	0.00000007971	0.00076853650	sh_2
	100	0.30696883897	0.69738605369	0.00000800449	**0.**00000031223	0.00003121124	0.60077728974	sh_2

Table (4-97): Estimation for power Lomax distribution when R_s =0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.32934967	0.31923743	0.33336991	0.33333436	0.33337039	0.34446335
	50	0.33072015	0.32203947	0.33335765	0.33333458	0.33336792	0.34182452
	100	0.33003567	0.32231741	0.33333290	0.33333324	0.33333275	0.34020556
50	30	0.32985196	0.32067287	0.33335720	0.33333392	0.33335613	0.34095961
	50	0.32832872	0.32138348	0.33332208	0.33333118	0.33329094	0.33701054
	100	0.33157094	0.32512606	0.33333604	0.33333382	0.33334979	0.33918066
100	30	0.33028173	0.32271541	0.33334120	0.33333353	0.33334134	0.33900738
	50	0.33067963	0.32501157	0.33333097	0.33333286	0.33332835	0.33699522
	100	0.33215991	0.32819248	0.33333494	0.33333364	0.33333656	0.33734619

Table (4-98): MSE values for power Lomax distribution when R_s =0.333333, alpha1= 2, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00082986921	0.00100392174	0.00000090450	0.00000000083	0.00000092653	0.00153933150	sh_2
	50	0.00070828765	0.00086120601	0.00000046098	0.00000000071	0.00000059049	0.00138025029	sh_2
	100	0.00055036917	0.00066987681	0.00000048812	0.00000000056	0.00000050863	0.00113102446	sh_2
50	30	0.00063832484	0.00080859162	0.00000045972	0.00000000064	0.00000054887	0.00131356734	sh_2
	50	0.00053830521	0.00069685520	0.00000001540	0.00000000055	0.00000022343	0.00102024113	sh_2
	100	0.00036201373	0.00050040215	0.00000000993	0.00000000036	0.00000011108	0.00074136933	sh_2
100	30	0.00054435750	0.00065882818	0.00000045372	0.00000000054	0.00000047346	0.00103254659	sh_2
	50	0.00038702246	0.00050903987	0.00000001066	0.00000000039	0.00000011671	0.00074425132	sh_2
	100	0.00024704960	0.00035769766	0.00000000635	0.00000000024	0.00000002479	0.00051108096	sh_2

Table (4-99): Estimation for power Lomax distribution when R_s = 0.333333, alpha1= 1, alpha2= 3, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.32666240	0.31152785	0.33325661	0.33333089	0.33325573	0.34095318
	50	0.33032506	0.31620744	0.33333324	0.33333345	0.33333626	0.34093463
	100	0.33126236	0.31770416	0.33332659	0.33333366	0.33332972	0.33785343
50	30	0.32992637	0.31599020	0.33334170	0.33333333	0.33333932	0.34267787
	50	0.32992763	0.31693249	0.33332858	0.33333241	0.33331583	0.33972402
	100	0.33089815	0.31879312	0.33332982	0.33333264	0.33332333	0.33707158
100	30	0.33014082	0.31720698	0.33333880	0.33333329	0.33333828	0.34206448
	50	0.33251944	0.32029051	0.33334050	0.33333470	0.33335734	0.34070302
	100	0.33156811	0.32064427	0.33333085	0.33333283	0.33332848	0.33777166

Table (4-100): MSE values for power Lomax distribution when R_s = 0.333333, alpha1= 1, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00107243004	0.00138598945	0.00000122770	0.00000000113	0.00000125754	0.00203272530	sh_2
	50	0.00069907488	0.00089519059	0.00000013767	0.00000000071	0.00000036854	0.00133905081	sh_2
	100	0.00038830405	0.00059059093	0.00000012420	0.00000000039	0.00000014946	0.00073978151	sh_2
50	30	0.00106085114	0.00125394405	0.00000119037	0.00000000118	0.00000125013	0.00195653938	sh_2
	50	0.00064908443	0.00086049951	0.00000001884	0.00000000068	0.00000027332	0.00120454343	sh_2
	100	0.00034713050	0.00053037356	0.00000000924	0.00000000035	0.00000005645	0.00072205964	sh_2
100	30	0.00098950593	0.00114227107	0.00000109405	0.00000000103	0.00000112161	0.00182435293	sh_2
	50	0.00057011559	0.00071085004	0.00000001590	0.00000000057	0.00000022353	0.00114458554	sh_2
	100	0.00031691845	0.00047926684	0.00000000832	**0.**00000000032	0.00000003245	0.00063655873	sh_2

Table (4-101): Estimation for power Lomax distribution when R_s = 0.333333, alpha1= 3, alpha2= 1, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.32863556	0.31366902	0.33331377	0.33333262	0.33331359	0.34097727
	50	0.32987263	0.31568706	0.33334284	0.33333357	0.33334391	0.34009338
	100	0.32957482	0.31691729	0.33330985	0.33333271	0.33331070	0.33991633
50	30	0.33039914	0.31544444	0.33333587	0.33333365	0.33334094	0.34020501
	50	0.32994544	0.31692477	0.33332871	0.33333244	0.33331634	0.33888115
	100	0.33152418	0.31993715	0.33333430	0.33333351	0.33333527	0.34090780
100	30	0.33121456	0.31736180	0.33334821	0.33333367	0.33334785	0.33882827
	50	0.33181627	0.31969546	0.33333503	0.33333365	0.33333867	0.33878307
	100	0.33174837	0.32117767	0.33333230	0.33333312	0.33333133	0.33732526

Table (4-102): MSE values for power Lomax distribution when R_s = 0.333333, alpha1= 3, alpha2= 1, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00113245451	0.00136126102	0.00000132505	0.00000000122	0.00000135726	0.00204026380	sh_2
	50	0.00099554344	0.00118355833	0.00000105667	0.00000000103	0.00000110352	0.00189176201	sh_2
	100	0.00099519717	0.00115962038	0.00000108599	0.00000000105	0.00000111809	0.00171426276	sh_2
50	30	0.00065877815	0.00085692500	0.00000014455	0.00000000067	**0.**00000035486	0.00122506657	sh_2
	50	0.00064825847	0.00084843466	0.00000001870	0.00000000067	0.00000027130	0.00127868898	sh_2
	100	0.00062284094	0.00079139513	0.00000001815	0.00000000066	0.00000025609	0.00120412159	sh_2
100	30	0.00036412273	0.00057705656	0.0000001212	0.00000000036	0.00000014323	0.00075550357	sh_2
	50	0.00031770579	0.00048683610	0.00000000826	0.00000000031	0.00000005068	0.00068490300	sh_2
	100	0.00028393399	0.00043991348	0.00000000735	0.00000000028	0.00000002866	0.00056395302	sh_2

Table (4-103): Estimation for power Lomax distribution when R_s = 0.285714, alpha1= 2, alpha2= 3, alpha3=2

n	m	$\widehat{R}_{mle}$	$\widehat{R}_{mom}$	$\widehat{R}_{sh1}$	$\widehat{R}_{sh2}$	$\widehat{R}_{sh3}$	$\widehat{R}_{Ls}$
30	30	0.28034692	0.27437878	0.28568227	0.28571323	0.28568192	0.29186588
	50	0.28362883	0.27827082	0.28573329	0.28571544	0.28574459	0.29364409
	100	0.28297309	0.27801820	0.28571375	0.28571420	0.28571359	0.29160303
50	30	0.28278762	0.27716826	0.28574122	0.28571497	0.28574053	0.29366808
	50	0.28132123	0.27717031	0.28570425	0.28571236	0.28567656	0.28945509
	100	0.28394895	0.27997587	0.28571510	0.28571441	0.28572404	0.29053429
100	30	0.28296562	0.27859088	0.28572338	0.28571450	0.28572353	0.29209692
	50	0.28339244	0.28018903	0.28571244	0.28571391	0.28571036	0.28950900
	100	0.28478219	0.28260735	0.28571610	0.28571464	0.28571792	0.28960712

Table (4-104): MSE values for power Lomax distribution when R_s = 0.285714, alpha1= 2, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00074280893	0.00083333090	0.00000080277	0.00000000074	0.00000082231	0.00141532264	sh_2
	50	0.00056506284	0.00064628893	0.00000025591	0.00000000056	0.00000039592	0.00111489191	sh_2
	100	0.00037918001	0.00043774051	0.00000026562	0.00000000038	0.00000028262	0.00080495740	sh_2
50	30	0.00060854889	0.00068979059	0.00000055544	0.00000000062	0.00000060706	0.00128367980	sh_2
	50	0.00046311451	0.00054366650	0.00000001326	0.00000000048	0.00000019229	0.00089637783	sh_2
	100	0.00027689049	0.00034793220	0.00000000749	0.00000000028	0.00000006813	0.00056704557	sh_2
100	30	0.00056620127	0.00062237038	0.00000054884	0.00000000057	0.00000056662	0.00109224872	sh_2
	50	0.00037514179	0.00043482740	0.00000001054	0.00000000038	0.00000013187	0.00071536939	sh_2
	100	0.00021636939	0.00027287932	0.00000000559	0.00000000021	0.00000002179	0.00045560494	sh_2

Table (4-105): Estimation for power Lomax distribution when R_s = 0.285714, alpha1= 3, alpha2= 2, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.27972593	0.27385577	0.28565473	0.28571238	0.28565405	0.29257055
	50	0.28229741	0.27732032	0.28571377	0.28571427	0.28571516	0.2931989
	100	0.28240110	0.27804374	0.28569497	0.28571390	0.28569633	0.29076474
50	30	0.28226518	0.27727831	0.28571606	0.28571398	0.28571079	0.29211708
	50	0.28229779	0.27812981	0.28570998	0.28571346	0.28569836	0.29055638
	100	0.28288801	0.27979209	0.28570962	0.28571338	0.28569847	0.28966782
100	30	0.28285893	0.27832166	0.28571517	0.28571404	0.28571385	0.29079187
	50	0.28519239	0.28148196	0.28572136	0.28571566	0.28573299	0.29123487
	100	0.28423440	0.28165784	0.28571330	0.28571408	0.28571239	0.28956787

Table(4-106): MSE values for power Lomax distribution when R_s = 0.285714, alpha1= 3, alpha2= 2, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00079161966	0.00090870998	0.00000089221	0.00000000082	0.00000091391	0.00151114516	sh_2
	50	0.00066869742	0.00072162277	0.00000057894	0.00000000069	0.00000065661	0.00123896909	sh_2
	100	0.00057229585	0.00063024875	0.00000056201	0.00000000059	0.00000058259	0.00105397154	sh_2
50	30	0.00058185786	0.00065857056	0.00000029680	0.00000000061	0.00000043569	0.00104199692	sh_2
	50	0.00043073539	0.00049986639	0.00000001223	0.00000000044	0.00000017756	0.00082408378	sh_2
	100	0.00038312782	0.00046090752	0.00000001091	0.00000000040	0.00000013833	0.00075721947	sh_2
100	30	0.00039674388	0.00045557946	0.00000026558	0.00000000039	0.00000028223	0.00070889149	sh_2
	50	0.00027857127	0.00032175304	0.00000000739	0.00000000027	0.00000006711	0.00057125845	sh_2
	100	0.00021771600	0.00027193461	0.00000000564	0.00000000022	0.00000002202	0.00044837815	sh_2

Table (4-107): Estimation for power Lomax distribution when R_s = 0.25, alpha1= 3, alpha2= 3, alpha3=2

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh1}$	$\hat{R}_{sh2}$	$\hat{R}_{sh3}$	$\hat{R}_{Ls}$
30	30	0.2457117	0.2419035	0.2499909	0.2499996	0.2499908	0.2572338
	50	0.2463495	0.2428919	0.2499950	0.2499995	0.2499904	0.2551592
	100	0.2471666	0.2446018	0.2499954	0.2499997	0.2499951	0.2552504
50	30	0.2469229	0.2434706	0.2500012	0.2499998	0.2499995	0.2572968
	50	0.2479115	0.2454354	0.2500020	0.2500003	0.2500082	0.2554736
	100	0.2485291	0.2464866	0.2500017	0.2500003	0.2500067	0.2543909
100	30	0.2472270	0.2441227	0.2499947	0.2499997	0.2499945	0.2549863
	50	0.2491514	0.2468356	0.2500044	0.2500008	0.2500148	0.2550739
	100	0.2484712	0.2469817	0.2499982	0.2499996	0.2499961	0.2531264

Table (4-108): MSE values for power Lomax distribution when R_s = 0.25, alpha1= 3, alpha2= 3, alpha3=2

n	m	mse_{mle}	mse_{mom}	mse_{sh1}	mse_{sh2}	mse_{sh3}	mse_{Ls}	Best
30	30	0.00059222783	0.00063788611	0.00000066069	0.00000000061	0.00000067676	0.00125073341	sh_2
	50	0.00046502312	0.00052020131	0.00000034106	0.00000000047	0.00000040552	0.00092255034	sh_2
	100	0.00037118585	0.00042132610	0.00000033707	0.00000000038	0.00000034852	0.00071475400	sh_2
50	30	0.00052266291	0.00057194282	0.00000035802	0.00000000053	0.00000044778	0.00103671696	sh_2
	50	0.32198898164	0.35188794463	0.00000907708	0.00000032972	0.00013174916	0.68971704206	sh_2
	100	0.00025517617	0.00029118319	0.00000000687	0.00000000025	0.00000007609	0.00054743457	sh_2
100	30	0.00039123761	0.00042185014	0.00000035087	0.00000000040	0.00000036387	0.00078219610	sh_2
	50	0.00027893362	0.00032275310	0.00000000757	0.00000000028	0.00000008156	0.00057104605	sh_2
	100	0.00017405214	0.00019610179	0.00000000447	0.00000000017	0.00000001746	0.00035636706	sh_2

Chapter five

Conclusions

and

Recommendation

Chapter five

Conclusion and Recommendations

(5-1) Conclusion

It can be mentioned from the obtained simulation numerical result which have been presented in previous chapter for three employed distributions studied (power function distribution, modified exponentiated Lomax distribution , power Lomax distribution) the results can be summarized as follows

1-From the result in tables (4-1) to (4-18) estimation of the reliability one component in stress-strength model R the power function distribution the result refer to the constant shrinkage estimator Sh_2 is the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and often followed by (Sh_1, Sh_3, MLE, MOM and Ls) .

2- From the result in tables (4-19) to (4-36) estimation of the reliability system of two series components in stress-strength model R_s the power function distribution the result refer to the constant shrinkage estimator Sh_2 the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and then followed by (Sh_1, Sh_3, MLE, MOM and Ls)

3- From the result in tables (4-37) to (4-54) estimation the reliability one component in stress-strength model R the modified exponentiated Lomax distribution the result refer to the constant shrinkage estimator Sh_2 is the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and often followed by (Sh_1, Sh_3, MLE, Ls and MOM) .

4- From the result in tables (4-54) to (4-72) estimation of the reliability system of two series components in stress-strength model R_s the modified exponentiated Lomax distribution the result refer to the constant shrinkage estimator Sh_2 is the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and then followed by (Sh_1, Sh_3, MLE, MOM and Ls).

5- From the result in tables (4-73) to (4-90) estimation the reliability one component in stress-strength model R the power Lomax distribution the result refer to the constant shrinkage estimator Sh_2 is the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and then followed by (Sh_1, Sh_3, MLE, Ls and MOM).

6- From the result in tables (4-91) to (4-108) estimation of the reliability system of two series components in stress-strength model R_s the power Lomax distribution the result refer to the constant shrinkage estimator Sh_2 is the best performance for all sample size n =(30,50,100) and m=(30,50,100) for all cases and then followed by (Sh_1, Sh_3, MLE, MOM and Ls).

(5-2) Recommendation

From the aforementioned conclusion , the main recommendations can be summarized as follows

1- Recommends using the special case from shrinkage method (constant shrinkage estimator Sh_2) to estimate system reliability R in stress-strength model for one component and estimate the reliability for two series component system R_s because it is the best method and owns less mean square error (MSE).

2-It can be applied the real data in different medical fields and engineering systems through and using power function distribution, modified exponentiated Lomax distribution and power Lomax distribution.

3- Recommend using other estimation methods such as Bayes estimation methods for the studied distributions.

4- May be applied it is possible to find the formula of reliability and estimate reliability system r-out of –k for multi component and parallel system.

5- One may have to use estimate methods to estimate two unknown parameter for power function distribution, modified exponentiated Lomax distribution and power Lomax distribution.

6- Finally, it can used the proposed estimation methods to estimate the reliability when the stress and strength not identical (different failure distributions).

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Appendix

We will show samples of the programs that have been worked out, as

Programs forms

Program 1:

```
%Power distribution
%modification to 2_1_1
clc; clear;
%n=30,50,100; alpha=1,2,3;
%pi=(i-0.2)/(n+0.3); %median rank
%pi=i/(n+1); %mean rank
L=1000; n=30; m=30; alpha1=1; alpha2=2; alpha3=2;
nm={'n=',n,'m=',m}; disp(nm);
R=alpha1/(alpha1+alpha3)+alpha2/(alpha2+alpha3)...
    -(alpha1+alpha2)/(alpha1+alpha2+alpha3);
format long
r={'R=',R,'alpha1=',alpha1,'alpha2=',alpha2,'alpha3=',alpha3}; disp(r);
%rand('seed',n);
for j=1:L
for i=1:n u1=rand(1,n); end
for i=1:m u2=rand(1,m); end
%S2_1=alpha1/((alpha1+2)*(alpha1+1)^2);
%S2_2=alpha1/((alpha2+2)*(alpha2+1)^2);
for i=1:n
   x(i)=(u1(i))^(1/alpha1);
   logx(i)=log(x(i)); logx2(i)=log(x(i))^2;
   pi=(i-0.6)/(n+0.6); %pi=(i-0.5)/(n+0.5); %symmetrical
   log1_pi(i)=log(pi); log1_pi2(i)=log(pi)^2;
   log_pi_x(i)=log(pi)*log(x(i));
end
for i=1:m
   y(i)=(u2(i))^(1/alpha2);
   logy(i)=log(y(i)); logy2(i)=log(y(i))^2;
   pi=(i-0.6)/(m+0.6); %symmetrical
   log2_pi(i)=log(pi); log2_pi2(i)=log(pi)^2;
   log_pi_y(i)=log(pi)*log(y(i));
end
sumx=sum(x); sumy=sum(y);
%xbar=mean(x); ybar=mean(y);
lnx=sum(logx); lny=sum(logy);
%*************** Mle *************
alpha1_mle(j)=-n/sum(lnx);
alpha2_mle(j)=-m/sum(lny);
R_mle(j)=alpha1_mle(j)/(alpha1_mle(j)+alpha3)+alpha2_mle(j)/(alpha2_mle(j)+alph
a3)...
           -(alpha1_mle(j)+alpha2_mle(j))/(alpha1_mle(j)+alpha2_mle(j)+alpha3);
mseRmle(j)=(R_mle(j)-R)^2;
mse_a1_mle(j)=(alpha1_mle(j)-alpha1)^2;
%****************************
```

```
alpha1_mom(j)=sumx/(n-sumx);
alpha2_mom(j)=sumy/(m-sumy);
R_mom(j)=alpha1_mom(j)/(alpha1_mom(j)+alpha3)+alpha2_mom(j)/(alpha2_mom(j
)+alpha3)...
            -
(alpha1_mom(j)+alpha2_mom(j))/(alpha1_mom(j)+alpha2_mom(j)+alpha3);
mseRmom(j)=(R_mom(j)-R)^2;
mse_a1_mom(j)=(alpha1_mom(j)-alpha1)^2;
%******************************
alpha1_ub=-(n-1)/sum(lnx); alpha2_ub=-(m-1)/sum(lny);
alpha1_sh1(j)=abs(sin(n)/n)*alpha1_ub+(1-abs(sin(n)/n))*alpha1;
alpha2_sh1(j)=abs(sin(m)/m)*alpha2_ub+(1-abs(sin(m)/m))*alpha2;
R_sh1(j)=alpha1_sh1(j)/(alpha1_sh1(j)+alpha3)+alpha2_sh1(j)/(alpha2_sh1(j)+alpha
3)...
            -(alpha1_sh1(j)+alpha2_sh1(j))/(alpha1_sh1(j)+alpha2_sh1(j)+alpha3);
mseRsh1(j)=(R_sh1(j)-R)^2;
mse_a1_sh1(j)=(alpha1_sh1(j)-alpha1)^2;
%******************************
k=0.001;
alpha1_sh2(j)=k*alpha1_ub+(1-k)*alpha1;
alpha2_sh2(j)=k*alpha2_ub+(1-k)*alpha2;
R_sh2(j)=alpha1_sh2(j)/(alpha1_sh2(j)+alpha3)+alpha2_sh2(j)/(alpha2_sh2(j)+alpha
3)...
            -(alpha1_sh2(j)+alpha2_sh2(j))/(alpha1_sh2(j)+alpha2_sh2(j)+alpha3);
mseRsh2(j)=(R_sh2(j)-R)^2;
mse_a1_sh2(j)=(alpha1_sh2(j)-alpha1)^2;
%******************************
alpha1_sh3(j)=beta(n,1)*alpha1_ub+(1-beta(n,1))*alpha1;
alpha2_sh3(j)=beta(m,1)*alpha2_ub+(1-beta(m,1))*alpha2;
R_sh3(j)=alpha1_sh3(j)/(alpha1_sh3(j)+alpha3)+alpha2_sh3(j)/(alpha2_sh3(j)+alpha
3)...
            -(alpha1_sh3(j)+alpha2_sh3(j))/(alpha1_sh3(j)+alpha2_sh3(j)+alpha3);
mseRsh3(j)=(R_sh3(j)-R)^2;
mse_a1_sh3(j)=(alpha1_sh3(j)-alpha1)^2;
%******************************
aa1=sum(log1_pi2);aa2=(sum(log1_pi))^2/n;
bb1=sum(log2_pi2);bb2=(sum(log2_pi))^2/m;
a1=aa1-aa2; a2=sum(log_pi_x)-sum(log_pi_x)/n;
b1=bb1-bb2; b2=sum(log_pi_y)-sum(log_pi_y)/m;
alpha1_Ls(j)=a1/a2;
alpha2_Ls(j)=b1/b2;
R_Ls(j)=alpha1_Ls(j)/(alpha1_Ls(j)+alpha3)+alpha2_Ls(j)/(alpha2_Ls(j)+alpha3)...
            -(alpha1_Ls(j)+alpha2_Ls(j))/(alpha1_Ls(j)+alpha2_Ls(j)+alpha3);
mseRLs(j)=(R_Ls(j)-R)^2;
mse_a1_Ls(j)=(alpha1_Ls(j)-alpha1)^2;
end
mean_R_mle=mean(R_mle); m_R_mle1=num2str(mean_R_mle,6);
m_R_mle=str2num(m_R_mle1);
mean_R_mom=mean(R_mom); m_R_mom1=num2str(mean_R_mom,6);
m_R_mom=str2num(m_R_mom1);
```

```
mean_R_sh1=mean(R_sh1); m1_R_sh1=num2str(mean_R_sh1,6);
m_R_sh1=str2num(m1_R_sh1);
mean_R_sh2=mean(R_sh2); m1_R_sh2=num2str(mean_R_sh2,6);
m_R_sh2=str2num(m1_R_sh2);
mean_R_sh3=mean(R_sh3); m1_R_sh3=num2str(mean_R_sh3,6);
m_R_sh3=str2num(m1_R_sh3);
mean_R_Ls=mean(R_Ls); m1_R_Ls=num2str(mean_R_Ls,6);
m_R_Ls=str2num(m1_R_Ls);
%*************** mean alpha 1,2 ***************
mean_a1_mle=mean(alpha1_mle); mean_a2_mle=mean(alpha2_mle);
mean_a1_mom=mean(alpha1_mom); mean_a2_mom=mean(alpha2_mom);
mean_a1_sh1=mean(alpha1_sh1); mean_a2_sh1=mean(alpha2_sh1);
mean_a1_sh2=mean(alpha1_sh2); mean_a2_sh2=mean(alpha2_sh2);
mean_a1_sh3=mean(alpha1_sh3); mean_a2_sh3=mean(alpha2_sh3);
mean_a1_Ls=mean(alpha1_Ls); mean_a2_Ls=mean(alpha2_Ls);
show=['1-alpha1_ml','   ','2-alpha1_mo','   ','3-alpha1_sh1']; disp(show);
show= ['4-alpha1_sh2','  ','5-alpha1_sh3','   ','6-alpha1_Ls']; disp(show);
show=vertcat(mean_a1_mle,mean_a1_mom,mean_a1_sh1,mean_a1_sh2,...
                  mean_a1_sh3,mean_a1_Ls); disp(show);
show=['1-alpha2_ml','   ','2-alpha2_mo','   ','3-alpha2_sh1']; disp(show);
show= ['4-alpha2_sh2','  ','5-alpha2_sh3','   ','6-alpha2_Ls']; disp(show);
show=vertcat(mean_a2_mle,mean_a2_mom,mean_a2_sh1,mean_a2_sh2,...
                   mean_a2_sh3,mean_a2_Ls); disp(show);
 %**************************************************
 meanMseA1mle=mean(mse_a1_mle);
 meanMseA1mom=mean(mse_a1_mom);
 meanMseA1sh1=mean(mse_a1_sh1);
 meanMseA1sh2=mean(mse_a1_sh2);
 meanMseA1sh3=mean(mse_a1_sh3);
 meanMseA1Ls=mean(mse_a1_Ls);
 %mse of alpha1 for methods
 %show=vertcat(meanMseA1mle,meanMseA1mom,meanMseA1sh1,...
         %meanMseA1sh2,meanMseA1sh3,meanMseA1Ls); disp(show);
 %**************************************************
show=['1-R_ml','   ','2-R_mo','   ','3-R_sh1','   ','4-R_sh2','  ','5-R_sh3',...
        '   ','6-R_Ls']; disp(show);
show=[mean_R_mle;mean_R_mom;mean_R_sh1;mean_R_sh2;...
   mean_R_sh3;mean_R_Ls]; disp(show);
show=vertcat(m_R_mle,m_R_mom,m_R_sh1,m_R_sh2,...
   m_R_sh3,m_R_Ls); %disp(show);
%*****************************
mse_R_mle=mean(mseRmle); mse_R_mom=mean(mseRmom);
mse_R_sh1=mean(mseRsh1); mse_R_sh2=mean(mseRsh2);
mse_R_sh3=mean(mseRsh3); mse_R_Ls=mean(mseRLs);
show=['1. mseMle','  ','2. msemom','  ','3. mseSh1']; disp(show);
show=['4. mseSh2','  ','5. mseSh3','  ','6-mseLs']; disp(show);
show=vertcat(mse_R_mle,mse_R_mom,mse_R_sh1,mse_R_sh2,mse_R_sh3,mse_R_
Ls);
disp(show);
```

program 2:

```
%The Power Lomax Dist.(24/4/19)(1/5/19)
clc;clear
%alphas(2,3),(3,2)(2.4,3),(3,2.4) %n=30,50,100; m=30,50,100;
alpha1=2; alpha2=3; n=100; m=100;
nm={'n=',n,'m=',m}; disp(nm);
beta_n=beta(1,n); beta_m=beta(1,m);
k1=abs(sin(n)/n);  k2=abs(sin(m)/m);
format bank
R=alpha2/(alpha1+alpha2);
r={'R=',R,'alpha1=',alpha1,'alpha2=',alpha2}; disp(r);
format long
for t=1:1000
   for i=1:n Rx=rand(1,i); end
   for j=1:m Ry=rand(1,j); end
   for i=1:n x(i)=(1-Rx(i))^(-1/alpha1)-1; end
   for i=1:m y(i)=(1-Ry(i))^(-1/alpha2)-1; end
   for i=1:n Ln_sumx(i)=log(1+x(i)); end
   for j=1:m Ln_sumy(j)=log(1+y(j)); end
Ln1=sum(Ln_sumx); Ln2=sum(Ln_sumy);
sumx1=sum(x); sumy1=sum(y);
%*****************MLE******************************
alpha1_ml(t)=n/Ln1; alpha2_ml(t)=m/Ln2;
R_ml(t)=alpha2_ml/(alpha1_ml+alpha2_ml);
mse_R_ml(t)=(R_ml(t)-R)^2;
%*****************Mom*****************************
alpha1_mom(t)=n/sumx1+1; alpha2_mom(t)=m/sumy1+1;
R_mom(t)=alpha2_mom(t)/(alpha1_mom(t)+alpha2_mom(t));
mse_R_mom(t)=(R_mom(t)-R)^2;
%*****************Unbiased************************
alpha1_unb(t)=(n-1)/Ln1; alpha2_unb(t)=(m-1)/Ln2;
%******************SH1****************************
alpha1_sh1(t)=k1*alpha1_unb(t)+(1-k1)*alpha1;
alpha2_sh1(t)=k2*alpha2_unb(t)+(1-k2)*alpha2;
R_sh1(t)=alpha2_sh1/(alpha1_sh1+alpha2_sh1);
mse_R_sh1(t)=(R_sh1(t)-R)^2;
%*******************SH2***************************
k=0.001;
alpha1_sh2(t)=k*alpha1_unb(t)+(1-k)*alpha1;
alpha2_sh2(t)=k*alpha2_unb(t)+(1-k)*alpha2;
R_sh2(t)=alpha2_sh2/(alpha1_sh2+alpha2_sh2);
mse_R_sh2(t)=(R_sh2(t)-R)^2;
%*****************SH3******************************
alpha1_sh3(t)=beta_n*alpha1_unb(t)+(1-beta_n)*alpha1;
alpha2_sh3(t)=beta_m*alpha2_unb(t)+(1-beta_m)*alpha2;
R_sh3(t)=alpha2_sh3/(alpha1_sh3+alpha2_sh3);
mse_R_sh3(t)=(R_sh3(t)-R)^2;
```

```
%*******************Ls****************************
%pi(i)=i/(n+1); pi(i)=(i-0.5)/(n+0.5);
for i=1:n
   pi(i)=(i-0.2)/(n+0.3);
end
for j=1:m
   qi(j)=(j-0.2)/(m+0.3);
end
for i=1:n
   p(i)=log(1-pi(i))^2; p1(i)=log(1-pi(i));
   p2(i)=log(1-pi(i))*(-log(1+x(i)));
   p3(i)=(log(1-pi(i))*(-log(1+x(i))))/n;
end
p1_2=(sum(p1))^2/n; lp=sum(p); lp2=sum(p2); lp3=sum(p3);
alpha1_Ls(t)=(lp-p1_2)/(lp2-lp3);
for j=1:m
   q(j)=log(1-qi(j))^2; q1(j)=log(1-qi(j));
   q2(j)=log(1-qi(j))*(-log(1+y(j)));
   q3(j)=(log(1-qi(j))*(-log(1+y(j))))/m;
end
q1_2=(sum(q1))^2/m; lq=sum(q); lq2=sum(q2); lq3=sum(q3);
alpha2_Ls(t)=(lq-q1_2)/(lq2-lq3);
R_Ls(t)=alpha2_Ls/(alpha1_Ls+alpha2_Ls);
mse_R_Ls(t)=(R_Ls(t)-R)^2;
end
m_alpha1_ml=mean(alpha1_ml); m_alpha2_ml=mean(alpha2_ml);
m_alpha1_mom=mean(alpha1_mom); m_alpha2_mom=mean(alpha2_mom);
m_alpha1_sh1=mean(alpha1_sh1); m_alpha2_sh1=mean(alpha2_sh1);
m_alpha1_sh2=mean(alpha1_sh2); m_alpha2_sh2=mean(alpha2_sh2);
m_alpha1_sh3=mean(alpha1_sh3); m_alpha2_sh3=mean(alpha2_sh3);
m_alph1_Ls=mean(alpha1_Ls); m_alph2_Ls=mean(alpha2_Ls);
show=['1-th1_ml','   ','2-th1_mmom','   ','3-th1_sh1']; disp(show);
show=['4-th1_sh2','   ','5-th1_sh3','   ','6-th1_Ls']; disp(show);
show=[m_alpha1_ml;m_alpha1_mom;m_alpha1_sh1;m_alpha1_sh2;
         m_alpha1_sh3;m_alph1_Ls]; disp(show);
show=['1-th2_ml','   ','2-th2_mom','   ','3-th2_sh1']; disp(show);
show=['4-th2_sh2','   ','5-th2_sh3','   ','6-th2_Ls']; disp(show);
show=[m_alpha2_ml;m_alpha2_mom;m_alpha2_sh1;m_alpha2_sh2;
        m_alpha2_sh3;m_alph2_Ls]; disp(show);
show=['1-R_ml','   ','2-R_mom','   ','3-R_sh1']; disp(show);
show=['4-R_sh2','  ','5-R_sh3','  ','6-R_Ls']; disp(show);
m_R_ml=mean(R_ml); m_R_mom=mean(R_mom);
m_R_sh1=mean(R_sh1); m_R_sh2=mean(R_sh2);
m_R_sh3=mean(R_sh3);m_R_Ls=mean(R_Ls);
show=[m_R_ml;m_R_mom;m_R_sh1;m_R_sh2;m_R_sh3;m_R_Ls];
disp(show);
show=['1. mseMle','  ','2. msemom','  ','3. mseSh1']; disp(show);
show=['4. mseSh2','  ','5. mseSh3','  ','6-mseLs']; disp(show);
m_mse_R_ml=mean(mse_R_ml); m_mse_R_mom=mean(mse_R_mom);
m_mse_R_sh1=mean(mse_R_sh1); m_mse_R_sh2=mean(mse_R_sh2);
```

```
m_mse_R_sh3=mean(mse_R_sh3); m_mse_R_Ls=mean(mse_R_Ls);
show=[m_mse_R_ml;m_mse_R_mom;m_mse_R_sh1;m_mse_R_sh2;...
        m_mse_R_sh3;m_mse_R_Ls]; disp(show);
```

المستخلص

الهدف الرئيسي من الرسالة هو ايجاد الصيغ الرياضية لنموذج نظام معوليه الاجهاد والمتانة في حاله كان يحتوي على مركبه واحده يمتلك متانه X واجهاد Y $R=p(y<x)$ وفي حاله النظام يحتوي على مركبتين مربوطة على التوالي X_1, X_2 ويتعرض على اجهاد Y $R_s = P(y < \min(x_1, x_2))$ حيث تكون متغيرات المتانة والاجهاد مستقله لثلاث توزيعات مختلفة وهم توزيع القوى، توزيع لومكس المعمم والمعدل، و توزيع القوى لومكس حيث نقوم بتقدير المعلمات بالإضافة الى تقدير معوليه النظام باستخدام طرائق تقدير مختلفة مثل طريقه الامكان الاعظم (MLE)، طريقه العزوم (MOM) ، طريقه التقلص بثلاث حالات (Sh1,Sh2 and Sh3)، و طريقه المربعات الصغرى (Ls) ، ونقوم بعمل مقارنه بين الطرائق المستخدمة لجميع التوزيعات المدروسة باستخدام المحاكاة مونت-كارلو وبالاعتماد على معيار متوسط مربعات الخطأ (MSE).

ومن خلال استخدام المحاكاة ممكن ان نلاحظ ان طريقه التقلص بحالتها الثانية (Sh2) هي الطريقة الافضل في جميع التوزيعات وكل الحالات لأنها تحقق اقل متوسط مربعات الخطأ (MSE) .

جمهورية العراق
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مقارنه طرائق تقدير مختلفة لمعوليه نظام في نماذج الاجهاد-المتانة

رسالة مقدمة

إلى مجلس كلية التربية للعلوم الصرفة - ابن الهيثم - جامعة بغداد كجزء من متطلبات نيل درجة ماجستير في علوم الرياضيات

من قبل

بارق باقي سلمان

بكالوريوس رياضيات 2012

بأشراف

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