Republic of Iraq
Ministry of Higher Education and Scientific Research
University of Baghdad
College of Education for Pure Science / Ibn Al-Haitham
Department of Mathematics

Effect of Different Parameters on Peristaltic Transport for Some Types of Fluids in a Tapered and Curved Channels

A Thesis

Submitted to the College of Education for Pure Science \ Ibn Al-Haitham,

University of Baghdad

in a Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy of Science in Mathematics

By

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Supervised by

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2018 A.D. **1439 A.H.**

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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ACKNOWLEDGMENTS

All gratitude is due to Allah almighty who guided and aided me to bring for this thesis to light.

I would like to express my deepest gratitude and sincere thanks to my supervisor **Prof. Dr. Ahmed Mawlood Abdulhadi**, not only for able guidance, but for his enthusiastic encouragement while the project was in its embryonic stage, and for his great help and patience when guiding me along the other difficult stages for this work.

Also, I would like to thank the staff members of mathematics department, collage of education for pure science Ibn al Haitham, university of Baghdad, for their encouragement and support.

Finally, I would like to give my great thanks to my family.

Tamara Shihab Ahmed

2018

Publications
papers

Publications Papers

No.	Paper Title	Journal
1	Effect of magnetic field on peristaltic flow of Walters–B fluid through a porous medium in a tapered asymmetric channel	Journal of Advances in Mathematics ISSN 2347-1921 Volume 12 Number 12 Impact factor: 1.543 indexed with Google Scholar, Copernicus
2	Effect of magnetic field on peristaltic flow of Jeffery fluid through Porous medium in a tapered asymmetric channel .	Global Journal of Mathematics ISSN: 2395-4760 Vol.9, No.2, March 25, 2017 indexed with Google Scholar, Copernicus, Crossref.
3	Peristaltic transport of MHD flow and heat transfer in a tapered a symmetric channel through porous medium: effect of variable viscosity ,velocity-nonslip and temperature jump.	International Journal of Advanced Scientific and Technical Research ISSN 2249-9954 Issue 7 volume 2, March – April 2017. Impact Factor: 3.54 Indexed in THOMSON REUTERS.
4	Effect of magnetic field on peristaltic flow of Williamson fluid through a porous medium in an inclined tapered asymmetric channel.	Mathematical Theory and Modeling www.iiste.org ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online) Vol.7, No.5, 2017 Impact Factor:5.53

List of symbols and abbreviations:

ρ	Density of fluid
p	Pressure of fluid
δ	Electrically conductivity
B_0	Applied magnetic field strength
g	Acceleration due to gravity
K_1	Thermal conductivity
k_o	Porosity parameter
C_ρ	Specific heat at constant pressure
Q_0	Constant heat addition/absorption
μ_0	Constant viscosity
θ	Non- dimensional temperature
δ	Wave number
Re	Reynolds number
M	Hartmann number
k	Non-dimensional parameter of porous medium
Gr	Grashof number
Pr	Prandtl number
Br	Brinkman number
λ	wave length
Ec	Eckert number
Fr	Fraud number
E_1	Gives the rigid nature of the wall (depends upon the wall tension)
E_2	Gives the Stiffness property of the wall
E_3	Describe the dissipative feature of the wall

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ABSTRACT

In This Thesis we discussed the peristaltic flow of some viscoelastic non-Newtonian fluids namely (Walter's-B fluid, blood model, Williamson fluid and Jeffrey fluid) under the influence of constant and variable viscosity, velocity slip and non-slip conditions effects, magneto hydrodynamics field, compliant walls, porous space and heat/mass transfer for different two- dimensional channels such as straight and inclined tapered and curved channels.

The solutions for previous models of fluids have been considered and analyzed under the assumption of long wave length and low Reynolds number approximations.

The motion, temperature and concentration equations have been derived. These equations are solved analytically by means of the regular perturbation method. The salient features of pumping characteristic, friction force and trapping are analyzed through study the effects of dimensionless numbers that controlled the governing equations of flow.

Five problems have been discussed through our work which may followed by:

In the first problem, the peristaltic flow of an incompressible Walter's –B fluid through porous medium under the effects of uniform magnetic field in a straight asymmetric tapered channel is considered. It is found that the velocity of the fluid is increased near the center of the channel under the effect of short memory coefficient.

In the second problem, the peristaltic flow of blood with variable viscosity through porous medium in straight asymmetric tapered channel under the effects of uniform field and heat transfer is considered. It is found that the Reynolds parameter model of viscosity has wobbling behavior on the velocity of the fluid and we found that the temperature of the fluid enhanced under the increasing of source\sink parameter.

In the third problem, the peristaltic flow of Williamson fluid through porous medium in an inclined asymmetric tapered channel under the effects of inclined magnetic field and heat transfer are considered. It is found that the velocity of the fluid increased under the effect of inclination angle of magnetic field, also we found the temperature will be increased under the influence of Brinkman number.

In the fourth problem, the peristaltic flow of Jeffrey fluid through porous medium in an inclined symmetric tapered channel under the effects of inclined

magnetic field and heat transfer are considered. With the helping of wall properties. It is found that the velocity and temperature of the fluid will be increased under the effects of wall properties and inclination of the channel.

In the last problem, the peristaltic flow of Jeffrey fluid with variable viscosity with temperature in symmetric curved channel under the effects of radial magnetic field and heat/mass transfer are considered. It is found that cultivator coordinate parameter has oscillatory behavior on the velocity of the fluid, also it is found the curves is not symmetric in the case of curved channel and these are symmetric in the case of straight channel for large values of cultivator parameter. Also through this work the concentration of the fluid material is decreased with an increase of Schmidt number.

It is worth mentioning that the magnetic field and porous medium causes blocking to the flow of previous fluids in all above problems.

This study is done by using “MATHEMATICA” program computer to plot the figures and obtaining the numerical results.

INTRODUCTION

Peristaltic pumping has been the object of scientific and engineering research in recent years. The word peristaltic comes from a Greek word “PERISTALTIKOS” which means clasping and compressing. The phenomenon of peristalsis is defined as expansion and contraction of an extensible tube in a fluid generate progressive waves which propagate along the length of the tube, mixing and transporting the fluid in the direction of wave propagation. Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing the fluid. It is an inherent property of many tubular organs of the human body. It plays an indispensable role in transporting many physiological fluids in the body in various situations such as urine transport from the kidney to the bladder through the ureter, Vasomotion of small blood vessels, as well as mixing and transporting the contents of the gastrointestinal passage, the transport of the spermatozoa in cervical canal, transport of bile in the bile duct, transport of cilia. peristalsis play an indispensable role in transporting physiological fluids inside living bodies, and many biomechanical and engineering devices have been desighned on the basis of the principle of peristaltic pumping to transport fluids with out internal moving parts. The need for peristaltic pumping also arises in circumstances where is desirable to avoid using any internal moving parts such as pistons, in pumping process. See figs.(A-D), [96].

For example, the blood pump in dialysis is designed to prevent the transported fluid from being contaminated and peristaltic pumping mechanism have been utilized for the transport of slurries, sensitive or corrosive fluids, sanitary fluid, noxious fluids in the nuclear industry and many others. In some cases, the transport of fluids is possible without moving internal mechanical components as in the case with peristaltically operated. There are many other important applications of this principle. The study of peristalsis in the context of fluid mechanics has received considerable attention in the last three decades, mainly because of its relevance to biological systems and industrial applications, (see [98]).

Some authors have borrowed the idea and used it in applications where the material being pumped must not be contaminated (e.g. blood) or is corrosive and should not be in contact with the moving parts of ordinary pumping machinery. Also, peristaltic motion has even been found to play a role in nerve regeneration. In addition peristaltic pumping occurs in many practical applications involving biomechanical systems. For example, the heart machine and other pump instruments. (see fig.(E)), [11].

During the last 40 years researchers have extensively focused on the peristaltic flow of Newtonian fluids (see for example [23] and several references there in). Especially, peristaltic pumping occurs in many practical applications involving biomechanical systems, such as roller and finger pumps. In particular, the peristaltic pumping of corrosive fluids and slurries could be useful as it is desirable to prevent their contact with mechanical parts of the pump. In these investigations, solutions for peristaltic flow with various considerations of the nature of the fluid, the geometry of the channel and the propagating waves were obtained for various degree of approximations. Much attention has been confined to symmetric channel or tubes, but there exist also flow situations where the channel flow may not be symmetric. Mishra and Rao[69] studied the peristaltic flow of a Newtonian fluid in an asymmetric channel in a recent research. In another attempt, Rao and Mishra[83] discussed the non- linear and curvature effects on peristaltic flow of a Newtonian fluid in an a symmetric channel when the ratio of channel width to the wave length is small. Very recently, Haroun[39] extended the analysis of reference [69] for third order fluid. An example for a peristaltic type motion is the intra-uterine fluid flow due to myometrium contraction, where the myometrium contractions may occur in both symmetric and asymmetric directions. An interesting study made by Eytan and Elad[31] whose results have been used to analyze the fluid flow pattern in a non- pregnant uterus. In another paper, Eytan et al. [32] discussed the characteristic of non- pregnant women uterine contractions as they are composed of variable amplitudes and arrange of different wave lengths.

Although most prior studies of peristaltic motion have focused on Newtonian fluids, there are also studies involving non-Newtonian fluids, in which the shear stress may depend upon the shear rate (the relation between shear stress and shear rate is not linear), both shear stress and shear rate may be time dependent and the fluid may have viscous as well as elastic characteristics (sajid [93], khan [57]). Because of the different rheological properties of non- Newtonian fluids, there exists no single universal constitutive relationship between stress and rate of strain by which all the non -Newtonian fluids can be examined. Therefore, several models of non-Newtonian fluids have been suggested and considered. Complexity in non-Newtonian fluids starts due to the non-linear terms appearing in their constitutive relationships. Several researchers considered various models under different approximations and geometries by assuming the fluid content as a Newtonian fluid which is suitable in some particular cases like urine transport. (joseph [54]). However, most of the biological and industrial fluids are constituted of Newtonian and non-Newtonian fluids behaving collectively as a non-Newtonian mixture. The examples of non-Newtonian fluids includes semi-solid food called bolus in esophagus, semi-liquid food (chyme) in stomach and in testiness, blood in arteries or veins, cervical mucus in bones and semen and ovum in reproductory tracts. Where as in case of industrial fluids, waste inside the sanitary ducts, toxic materials, metal alloys, oil and grease in automobiles or mechanics, nuclear slurries inside the nuclear reactors and many others.

To investigate the non-Newtonian characteristics of the physiological fluids, different non-Newtonian fluids namely Walters-B, Jeffrey and Williamson fluids have been considered in the present study. Walter's B fluid (Walter's,[113]) is a viscolastic fluid model defines both viscous, as well as, elastic characteristic. Physically it describes the elastic nature of the physiological fluids. Walter's B fluid model has been widely studied by various researchers through different configurations. (See [19])

The non-Newtonian fluids which exhibit the characteristics of relaxation or retardation times belong to rate type fluids. Jeffrey fluid model is considered one of some important types of this kind of non-Newtonian fluids. This model shows the behavior of linearly viscoelastic fluids due to its various applications industry. (See [44], [27], [76]).

In non-Newtonian fluids, the most commonly encountered fluids are pseudo plastic fluids. The study of the boundary layer flow of pseudo plastic fluids is of great interest due to its wide range of applications in industry such as extrusion of polymer sheets, emulsion coated sheets like photographic films, solutions and melts of high molecular weight polymers, etc. The Navier stokes equations alone are insufficient to explain the rheological properties of fluids. Therefore rheological models have been proposed to overcome this deficiency. To explain the behavior of pseudo plastic fluids many models have been proposed like Williamson fluid. Williamson [116] discussed the flow of pseudo plastic materials and proposed a model equations to describe the flow of pseudo plastic fluids and experimentally verified the results. Lyubimov and Perminov [62] discussed the flow of a thin layer of a Williamson fluid over an inclined surface in the presence of gravitational field. Depra and Scarpi [24] developed the perturbation solution for Williamson fluid injected into a rock fracture. Peristaltic flow of a Williamson fluid has been discussed by Nadeem et al. [75]. Gramer et al. [37] showed that this model fits the experimental data of polymer solutions and particle suspensions better than other models. For pseudo plastic fluids the power law model predicts that the apparent /effective viscosity should decrease in definitely with increase in shear rate, which means infinite viscosity at rest and zero viscosity as the shear rate approaches infinity. A real fluid has both minimum and maximum effective viscosities depending upon the molecular structure of the fluid. In the Williamson fluid model, both the minimum (μ_∞) and maximum viscosities (μ_0) are considered, so, for pseudo plastic fluids (for which the apparent viscosity does not go to zero at infinity), it will give better results.

Consideration of porous medium is that a matter which contains a number of small holes distributed through the matter. Flows through porous medium occur in filtration of fluids and seepage of water in river beds. Movement of underground, water and oils are some important examples of flows through porous medium. An oil reservoir mostly contents of sedimentary formation such as limestone and sandstone in which oil is entrapped. Another example of flow through porous medium is the seepage under a dam which is very important. There are examples of natural porous medium such as beach sand, rye bread, wood, filter, loaf of bread, human lung, gallbladder and bile with stones, in petroleum production engineering and in many other processes as well (Fig. F,(1),(2)), [92].

The subject dealing with the motion of electrically conducting fluids in the presence of magnetic field is termed as magneto hydrodynamics (MHD) or hydro magnetics .Examples of such fluids includes plasmas, liquid metals and salt water or electrolytes. The field of MHD was initiated by Hannes Alfven [18] for which he received the Nobel Prize in physics in 1970.

Some clinical of magnetic field is applied in the medical field in the form of device called magnetic resonance imaging (MRI). Now MRI is widely used for diagnosis of brain, vascular, diseases and all the body. In actual practice a rapidly alternating current. Although these flow probes can measure blood flow through large vessels accurately and instantaneously, they have several limitations. The cross- sectional area of the vessel must be known in order to compute volume flow (flux). Also, the blood vessel must be dissected free to place the transducer around it. (Fig. (g)), [26].

Heat transfer analysis is prevalent in the study of peristaltic flows due to its large number of applications in processes like hemodialysis (method used removing waste products from blood in the case of renal failure of kidney) and oxygenation. Bio heat is currently considered as heat transfer in the human body. In view of thermotherapy (application of heat to the body for treatment, examples pain relief, increase of blood flow and others) and the human thermoregulation system (ability of living body to maintain body temperature within certain limits in case of surrounding temperature variations) as mentioned by Srinvas and Kothandapani [101], Bio heat transfer has attracted many biomedical experts. Heat transfer analysis is important especially in case of non-Newtonian peristaltic rheology as there involves many intricate processes like heat conduction in tissues, heat transfer during perfusion (process of delivery of blood to capillary bed) of arterial-venous blood, metabolic heat generation and heat transfer due to some external interactions like mobile phones and radioactive treatments. It is also helpful in the treatment of disease like removal of undesirable tissues in cancer.

Dissipative heat transfer is the most important and essential feature of peristaltic flows as suggested by Shapiro et al. [96]. In Peristaltic flows when the fluid is forced to flow due to the sinusoidal displacements of the tract boundaries, the fluid gains some velocity as well as kinetic energy. The viscosity of the fluid takes that kinetic energy and converts it into internal or thermal energy of the fluid. Consequently, the fluid is heated up and heat transfer occurs. This phenomenon is modelled by the energy equation with dissipation effects. For two dimensional flows the energy equation reduces to a second order partial differential equation that is at most parabolic in nature.

Moreover, due to the intricate nature of the bio-fluid dynamics, both heat and mass transfer occur simultaneously giving complex relations between fluxes and driving potentials as debated by srinivas and Kothandapani [102] and Eckert and Drake [28]. The mass flux caused by the temperature gradient called soret effect or thermal-diffusion is often negligible in heat and magnitude. However, for the non-Newtonian fluids with light or medium molecular weight, it is not appropriate to neglect soret effects. Therefore, in the present study, due attention has been given to the combined effects of heat and mass transfer with soret effect, also further, in the present study the Duffer effect (energy flux caused by the composition gradients) has been considered.

On the interaction of a fluid with the solid surface, the conditions where the molecules of the fluid near to the surface stick with the surface having the same velocity, is called no-slip condition. While in the case of many polymeric liquids with high molecular weight, the molecules near to the surface show slip or stick-slip on the surface. Navier [79] suggested the general slip boundary condition defining that the difference of fluid velocity and the velocity of the surface is proportional to the shear stress at that surface. The coefficient of proportionality is the slip parameter having the dimension of length. The slip condition is of great importance especially when fluids with non-Newtonian or elastic characters are considered. In such cases, the slippage may occur under a large tangential traction.

Both non-slip and slip boundary conditions have been considered in the present study.

The structure of this thesis consists of six Chapters which are:

Chapter one: Elementary concepts and basic definitions, which are used in our study are presented.

Chapter two: In this chapter, a theoretical study is presented for peristaltic flow of an incompressible Walter's-B fluid through porous medium under the effects of uniform (MHD) field and non-slip boundary conditions on the velocity in two- dimensional a symmetric tapered channel.

Chapter three This chapter concerns with the theoretical study of the incompressible conducting fluid with (MHD) field considered by the (blood)of human, (non-Newtonian fluid) through porous medium under the effects of heat transfer and variable viscosity in the form of a well known Reynolds model of viscosity in a two- dimensional a symmetric tapered channel.

Chapter four : In this chapter, we consider a mathematical model to study the peristaltic flow of Williamson fluid (theoretical tool of studying with constant viscosity and incompressible fluid under the combined effects of inclined magnetic field and heat transfer through porous medium in an inclined two-dimensional asymmetric tapered channel.

Chapter five: The aim of this chapter is to investigate the theoretical study of the mathematical model of incompressible Jeffrey fluid with constant viscosity under the combined effects of inclined magnetic field, heat transfer and wall properties. Through porous medium in an inclined tapered two-dimensional channel with flexible walls.

It is worth mentioning that the intrauterine fluid flow in a sagittal cross-section of the uterus discloses a narrow channel enclosed by two fairly parallel walls with wave trains having different amplitudes and phase difference, so with the aid of sufficient study support, a theoretical investigation on the peristaltic motion of

Walters-B fluid, Williamson fluid and Jeffrey fluid in a tapered channel or non-uniform asymmetric channel is carried out, therefore to the best of our familiarity, so far no attempt has been made to examine the peristaltic transport of above fluids in such channel which may help to imitate intra-uterine fluid motion in a sagittal cross-section of the uterus.

Chapter six: This chapter is devoted to study the peristaltic flow of Jeffrey fluid under the effect of radial magnetic field in a curved channel, the variation of viscosity to temperature will be used.

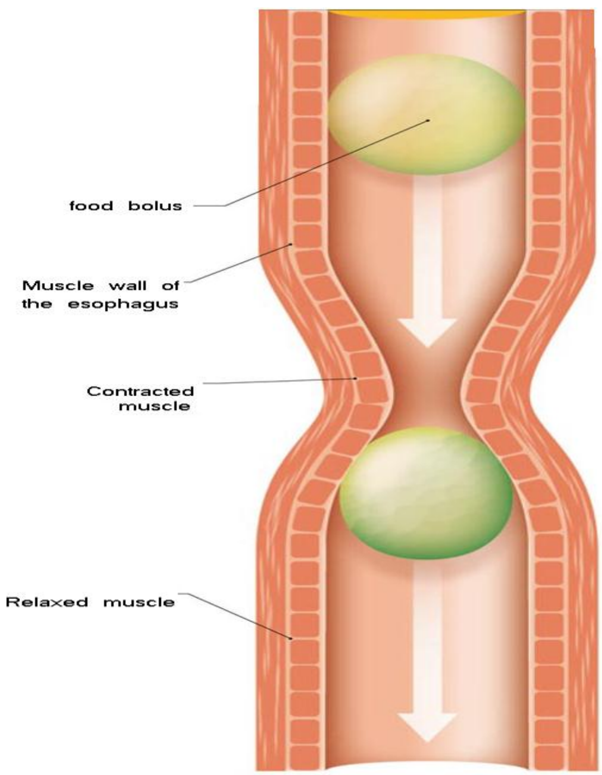
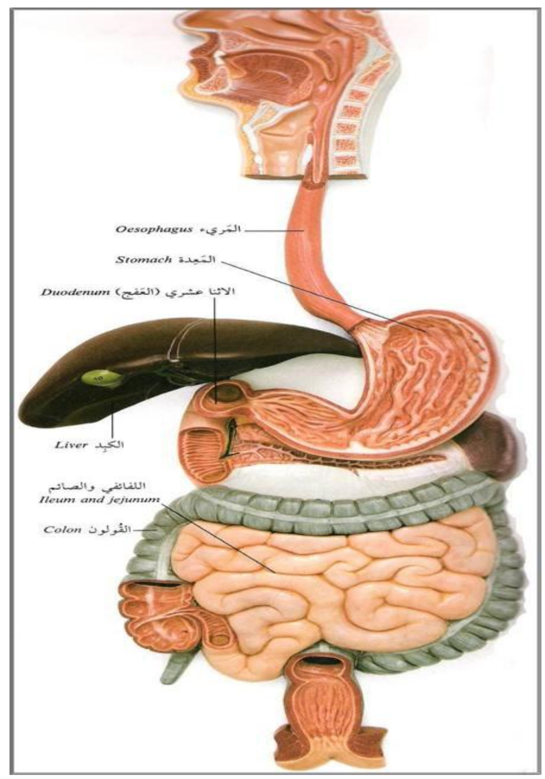


Fig.(A): The wave of involuntary muscle contractions push the food bolus



Fig(B): The human members which have peristaltic muscle.

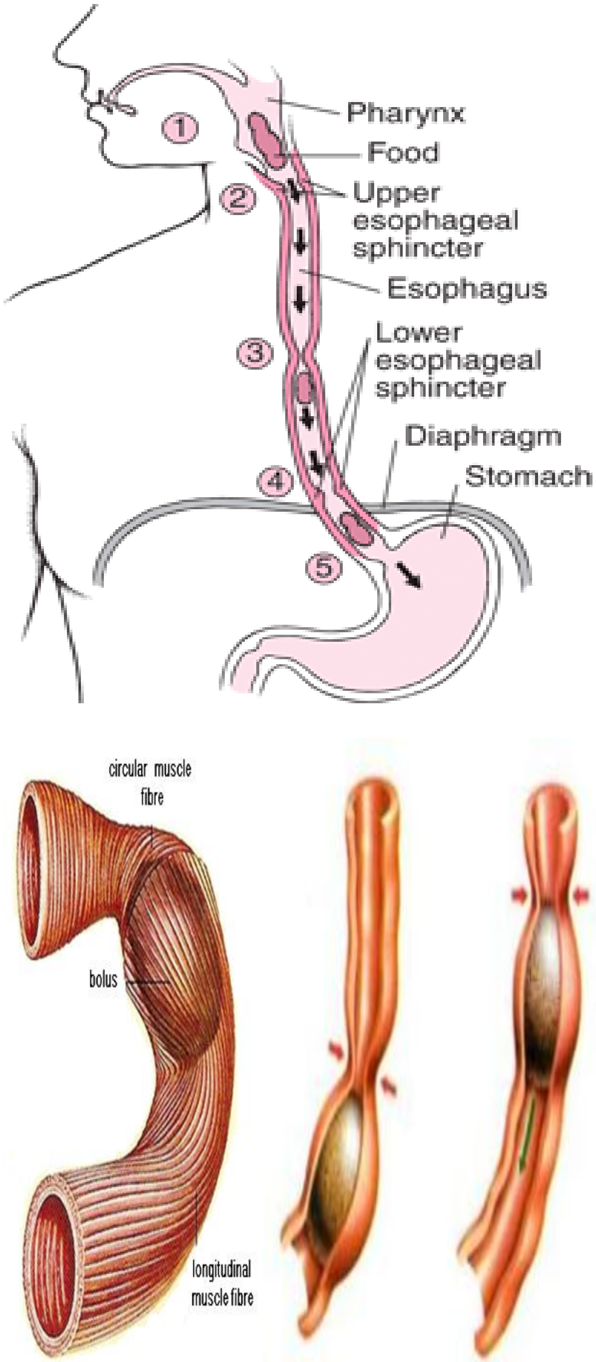
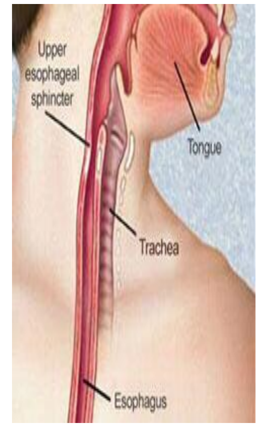


Fig.(C): The movement of circular muscles in human body

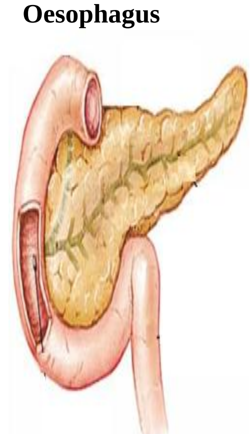
Small intestine



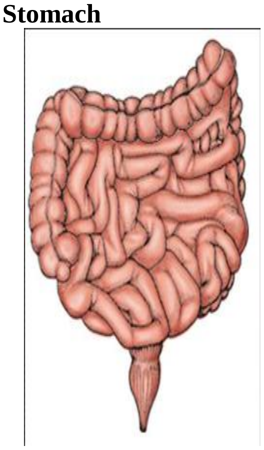
Pharynx

Oesophagus

Stomach



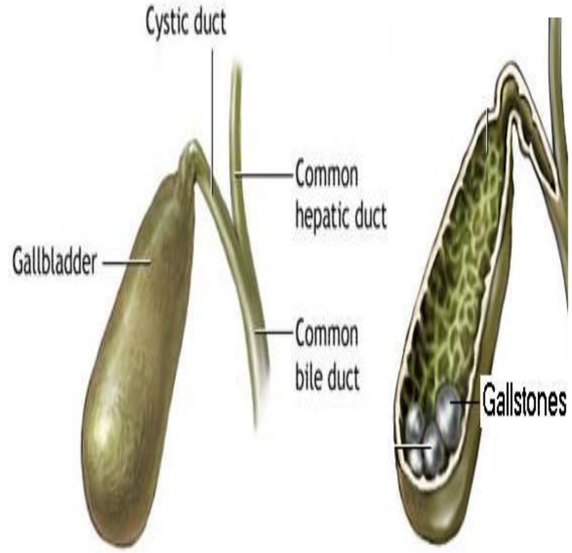
Pancreas



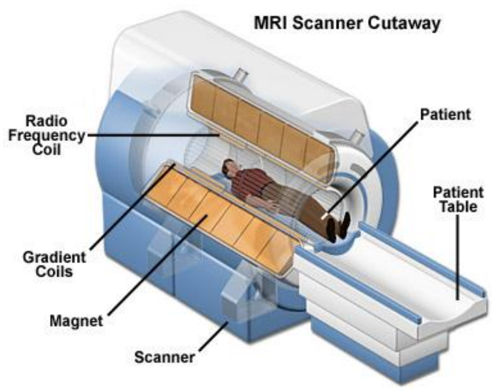
Large intestine

Fig.(D):Some peristaltic muscles of the human members

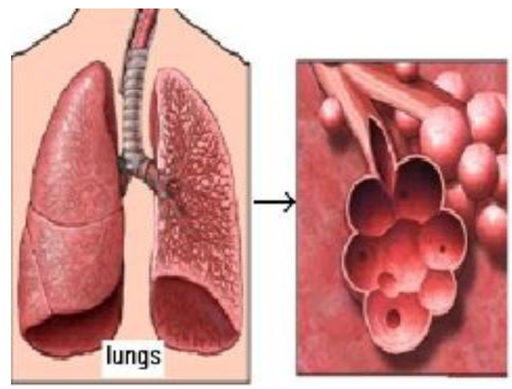
Fig(E): The blood pump machine



(F_2) Gall bladder



Fig(g) : The MRI scanner cutaway



(F_1) Human lung

Fig(F): Examples of porous medium

Chapter One

Elementary Concepts and Basic Definitions

Introduction

In this chapter, a brief history related to important contributions for understanding fluid mechanics development, is given as well as some elementary concepts and basic definitions that will be used in our work will be presented.

1.1 Basic Definitions and Fluid Properties

This section contains some basic definitions related to fluid mechanics.

1.1.1 Fluid mechanics:[90]

The word fluid means a substance having particles which readily change their relative positions. The subject of fluid mechanics deals with the behavior of fluids when subjected to a system of forces;

1- **Statics**: it deals with the fluid elements which are at rest relative to each other.
2- **Kinematics**: it deals with the effect of motion, i.e., translation, rotation and deformation on the fluid elements.
3- **Dynamics:** it deals with effect of applied forces on fluid elements.

1.1.2 Density: [90]

The density of substance is defined as the mass per unit volume and is denoted by the symbol ρ. It has dimension $\frac{M}{L^3}$, i.e.

$$\rho = \frac{m}{v} \qquad ...(1\text{-}1)$$

where m and v represents the mass and the volume, respectively.

1.1.3 Pressure: [90]

The pressure is defined as the normal compressive force per unit area and is denoted by the symbol P. It has the dimension $\frac{M}{LT^2}$, i.e.

$$P = \frac{force}{area} = \frac{F}{A} = \frac{ma}{A} \qquad ...(1\text{-}2)$$

Where a is the acceleration, F is the normal force, A is the area.

1.1.4 Shear stress: [25]

It is defined as the force per unit area and is denoted by the symbol τ. It has the dimension $\frac{M}{LT^2}$, i.e.

$$\tau = \frac{F}{A} \qquad ...(1\text{-}3)$$

where F and A is the applied force and the cross- sectional area of material with area parallel to the applied force vector, respectively.

1.1.5 **Shear strain :** [25]

It is also known as shear a deformation of solid bodies is displaced parallel places in the body, quantitatively it is the displacement of any plane relatives to second plan divided by the perpendicular distance between planes the force causing such deformation.

1.1.6 **Stream function:** [63]

It is defined as a scalar function of space and time such that it is partial derivative with respect to any direction gives the velocity in the direction perpendicular to the previous direction. It is denoted mathematically by ψ, where:

$\psi = \psi(s,t) = \psi(x,y,z,t)$. For two dimensional, unsteady flows we have:

$$u = \frac{\partial \psi}{\partial y}, \mathrm{v} = -\frac{\partial \psi}{\partial x} \qquad ...(1\text{-}4)$$

where u and v are the velocity components in the x and y directions, respectively.

1.1.7 **Stream line:** [63]

It is an imaginary line drawn through the flow field such the tangent at any point is in the direction of the velocity vector.

1.1.8 **Viscosity:** [20]

Among all the fluid properties, viscosity is the most important property, which is the resistance of a fluid to motion its internal friction. A fluid at a static state is by definition unable to resist even the slightness amount of shear stress. Application of shear force results in a continual and permanent distortion known as flow.

1.1.9 **Dynamic viscosity:** [20]

It is defined as the tangential force required per unit area to sustain a unit velocity gradient and is denoted by the symbol μ, it has dimension $\frac{M}{LT}$, i. e.

$$\mu = \frac{\tau}{du/dy} \qquad ...(1\text{-}5)$$

1.1.10 <u>Kinematic viscosity:</u> [20]

It is defined as the ratio of dynamic viscosity to density of fluid and is denoted by υ, it has dimension $\frac{L^2}{t}$, i.e.

$$\upsilon = \frac{\mu}{\rho} \qquad ...(1\text{-}6)$$

where μ and ρ is the dynamic viscosity and the density, respectively.

1.1.11 <u>Newtonian and non Newtonian fluids:</u> [20]

The Newton law of viscosity states that the shear stress τ of fluid element on a layer is directly proportional to the rate of strain, i. e. , $\tau \propto \frac{du}{dy}$

which may be written as :

$$\tau = \mu \frac{du}{dy} \qquad ...(1\text{-}7)$$

where $\frac{du}{dy}$ represents the rate of shear deformation of rate of shear and is often called the velocity gradient.

Many common fluids such as: air, water, light soils and gasoline are Newtonian fluids under normal conditions. However, there are certain fluids which exhibits non Newtonian fluids, therefore, do not follow Newton's law of viscosity. Common examples of non Newtonian fluids are: human blood, lubricating oils, clay suspension in water, sewage sludge. There is however, evidence to believe that Newtonian fluids may exhibit non Newtonian characteristics under conditions of higher shear stress, and hence the classification of fluid may change with the conditions of flow.

A general relationship between shear stress and velocity gradient (rate of shear strain) for non Newtonian fluids may be written as:

$$\tau = A\left(\frac{du}{dy}\right)^n + B \qquad \text{(power law fluids)} \qquad ...(1\text{-}8)$$

where A, B and n are constants. For Newtonian fluids $B = 0$, $A = \mu$ and $n = 1$. These relationships can be seen in the graph below for several categories.

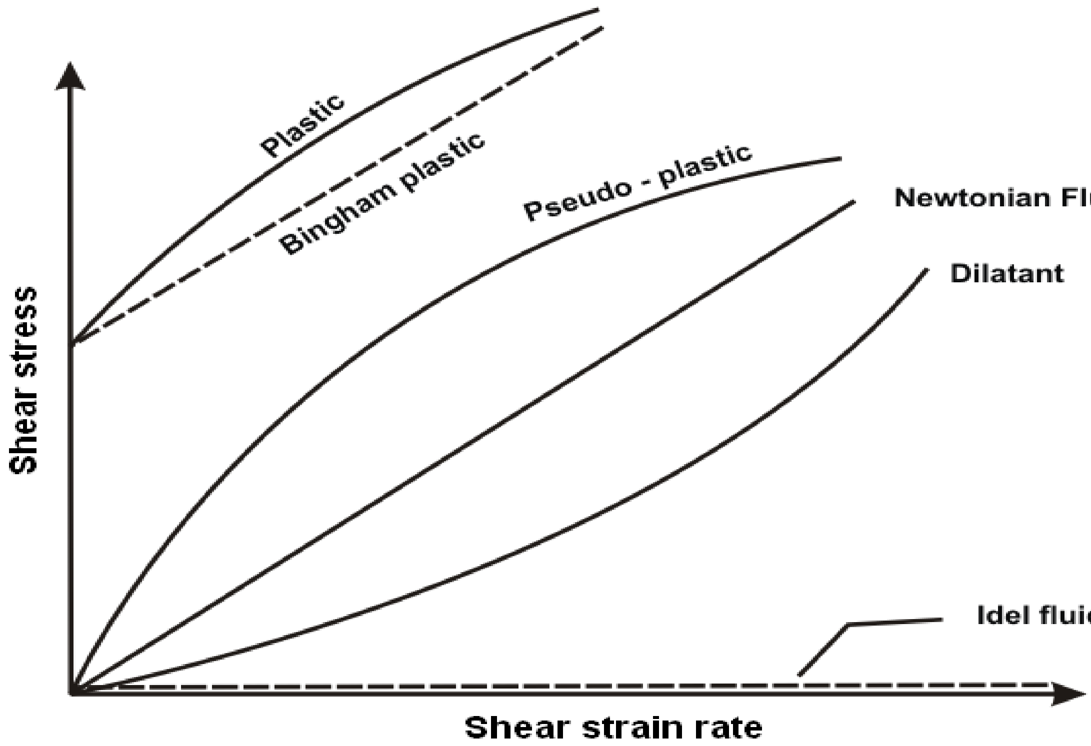


Fig (1-1): Shear stress versus rate of shear stress

Below are a brief descriptions of the physical properties of the several categories:

- Plastic: shear stress must reach a certain minimum before flow commences.
- Bingham plastic: As with the plastic above a minimum shear stress must be achieved with this classification n=1. An example is sewage sludge.
- Pseudo plastic: No minimum shear stress necessary and the viscosity decrease with the rate of shear, for example, colloidal substances like clay, milk, and cement.
- Thixotropic substance: viscosity increase with length of time shear force is applied, for example, thixotropic jelly paints.
- Rheopeetic substances: viscosity increase with length of time shear force is applied.
- Viscoelastic materials: similar to Newtonian but if there is a sudden large change in shear they behaved like plastic.

There is also one more category which is not real, it does not exist- known as the ideal fluid. This is a fluid which is assumed to have no viscosity. This is a useful concept when theoretical solutions are being considered it does help achieve some practically useful solutions.

In this thesis we used non Newtonian fluids of type pseudo plastic and viscoelastic materials.

1.1.12 Steady and unsteady flow: [59]

A flow is considered to be steady when the velocity gradient depends only upon the shear stress, is a single valued function of the latter, and means time independent fluid. Also, the properties of fluid does not depend on time, i.e., $\frac{\partial u}{\partial t} = 0, \; \frac{\partial P}{\partial t} = 0, \; \frac{\partial \rho}{\partial t} = 0$ otherwise, the flow is unsteady.

In this thesis we used study flow in our analysis.

1.1.13 Compressible and incompressible flow: [63]

Fluid mechanics deals with both incompressible and compressible fluids, that is, with fluids of constant and variable densities. Althrough there is no such thing in reality as an incompressible fluid, this term is applied where the change in density with pressure is so small as to be negligible. This is usually the case with all liquids, gases, too, may be considered as incompressible when the pressure variation is small compared with the absolute pressure.

1.1.14 Laminar and turbulent flow: [63]

Laminar flow in which fluid particles move along smooth paths in laminar or layers, with one layer gliding smoothly over an adjacent layer and it occurs for values of Reynolds number from 0 to 2000, and we say that the flow is irregular parts and when Reynolds number is greater than 4000, and we say that the flow is transition if the values of Reynolds number is between 2000 and 4000.

In this thesis our problems deal with incompressible and laminar flow

1.2 Continuity Equation: [68]

The continuity equation embodies the principle of conservation of mass according to which fluid matter can be neither created nor destroyed, which mean that, the mass per unit time entering the tube must flow out at same rate. The equation of continuity may be equivalently obtained in any appropriate coordinate system. The general equation of continuity which is applicable to any type of flow and for any fluid whether compressible or incompressible is :

$$\frac{D\rho}{Dt} + \rho \overline{\nabla} \overline{u} = 0 \qquad ...(1\text{-}9)$$

where ρ is the density, $\bar{u} = (u, v, w)$ is the velocity vector, $\overline{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ is the gradient vector, and $\frac{D}{Dt}=\frac{\partial}{\partial t}+\bar{u}.\bar{\nabla}$ is the substantial derivative. Its expansions is in the three most commonly used coordinate systems (rectangular, cylindrical, and spherical).

1.2.1 <u>Continuity equation in Cartesian coordinates:</u> [68]

The equation of continuity in three- dimensions is:

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad ...(1\text{-}10)$$

If the fluid is incompressible (ρ= constant), the continuity equation may be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad ...(1\text{-}11)$$

In two- dimensions, the continuity equation takes the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ...(1\text{-}12)$$

For one dimension, say in the x-direction:

$$\frac{\partial u}{\partial x}=0 \quad ...(1\text{-}13)$$

1.2.2 <u>Continuity equation in cylindrical coordinates:</u> [69]

The equation of continuity in cylindrical coordinates is:

$$\frac{\partial p}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad ...(1\text{-}14)$$

If the fluid is incompressible (ρ= constant), the continuity equation may be written as:

$$\frac{u}{r}+\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0 \quad ...(1\text{-}15)$$

In polar coordinates, the continuity equation takes the form:

$$\frac{u}{r}+\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \theta}=0 \quad ...(1\text{-}16)$$

1.3 <u>Navier-Stokes Equations:</u> [115]

The system of partial differential equations that describe the fluid motion is called the Navier stokes equations or the momentum equations. The general technique for obtaining the equations governing fluid motion is to consider a small control volume through which the fluid moves, and required that mass and energy are conserved, and that the rate of change of the two components of linear momentum are equal to the corresponding components of applied force.

The Navier-Stokes equations for incompressible fluid are:

$$\rho \frac{Du}{Dt} = \rho F - \nabla p + \mu \nabla^2 u \qquad ...(1\text{-}17)$$

where ∇^2 is the Laplacian operator, F is the body force.

1.3.1 Navier-Stokes equations in Cartesian coordinates

The Navier-Stokes equations in Cartesian coordinates are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon \nabla^2 \mathrm{u}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = F_y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon \nabla^2 \mathrm{v}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon \nabla^2 \mathrm{w} \qquad ...(1\text{-}18)$$

Where (u, v, w) are the components in the x, y and z directions, respectively, (F_x, F_y, F_z) are the body forces in the x, y and z directions, respectively, ν is the kinematic viscosity, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ Laplacian operator in Cartesian Coordinates.

1.3.2 Navier-Stokes equations in Cylindrical coordinates

The Navier-Stokes equations in cylindrical coordinates are:

$$\frac{Du}{Dt} - \frac{v^2}{r} = F_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \upsilon(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}$$

$$\frac{Dv}{Dt} + \frac{uv}{r} = F_\theta - \frac{1}{r\rho}\frac{\partial p}{\partial \theta} + \upsilon(\nabla^2 v + \frac{v}{r} + \frac{2}{r^2}\frac{\partial v}{\partial \theta}$$

$$\frac{Dw}{Dt} = F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon \nabla^2 w \qquad ...(1\text{-}19)$$

where (u, v, w) are the component in the r, θ and z directions, respectively, (F_r, F_θ, F_z) are the body forces in the r, θ and z directions, respectively, the operators $\frac{D}{Dt}$ and ∇^2 have the following meaning:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z} \qquad ...(1\text{-}20)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \qquad ...(1\text{-}21)$$

1.4 <u>Peristaltic Phenomenon:</u>[96]

Peristaltic pumping has been the object of scientific and engineering research in recent years. The word peristaltic comes from a Greek word “peristaltikos” which means clasping and compressing. The phenomenon of peristalsis is defined as expansion and contraction of an extensible tube in a fluid generate progressive waves which propagate along the length of the tube, mixing and transporting the fluid in the direction of wave propagation. Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible tube containing the fluid. It is an inherent property of many turbuler organs of the human body. It plays an indispensable role in transporting many physiological fluids in the body in various situations, such as urine transport from the kidney to the bladder through the ureter, vasomotion of small blood vessels, as well as, mixing and transporting the contents of the gastrointestinal passage, the transport of the spermatozoa in cervical canal, transport of bile in the bile duct, transport of cilia peristaltic play an indispensable role in transport physiological fluids inside living bodies, and many bio- mechanical and engineering devices have been designed on the basis of the principle of peristaltic pumping to transport fluids without internal moving parts. The need for peristaltic pumping also arises in circumstances where it is desirable to avoid using any internal moving parts such as pistons, in pumping process. A mathematical model to understand fluid mechanics of this phenomenon has been developed using lubrication theory, provided that the fluid inertia effects are negligible and the flow is of the low Reynolds number.

1.5 <u>Porous Medium:</u> [92]

A porous medium is a matter which contains a number of small holes distributed throughout the matter. Flows through porous medium occur in filtration of fluids and seepage of water in river beds, movement of underground water and oils are some important examples of flows through porous medium. An oil reservoir mostly contains of sedimentary formation such as limestone and sandstone in which oil is entrapped another example of flow through porous medium see page under a dam which is very problem. There are examples of natural porous medium such as beach sand, wood, filter, human hung, gall bladder and bile with stones, in petroleum production engineering and in many other processes as well. The Navier- Stokes equations with porous medium are given by:

$$\rho \frac{D\bar{u}}{Dt} = \rho F - \nabla \bar{p} + \mu \nabla^2 \bar{u} - \frac{\mu}{k_0} \bar{u} \qquad ...(1\text{-}22)$$

where k_0 is the permeability

1.6 Basic Definitions on the Electrostatic Field:[36]

Through this section, we will introduce the most important definitions, which will be used in our work later.

1.6.1 Electrostatic Field

The electrostatic field, denoted by E, is defined as the force that is exerted on a unit charge, in the field and it's vector in the same direction as the force.

$$E = \lim_{\Delta q \to 0} \frac{F}{\Delta q} \qquad ...(1\text{-}23)$$

where Δq is the charge, and F is the force.

1.6.2 Current Density

We define a current density, denoted by J as the flow of charges across a unit cross- sectional area per second.

1.6.3 Electrical Conductivity

Which is denoted by σ and it is the ratio of current density J to electrostatic field E.

$$\sigma = \frac{J}{E} \quad \text{(volamp)} \qquad ...(1.24)$$

1.6.4 Ohm's Law

It is describes the conduction current, and is given by:

$$J = \sigma(E + V \times B) \qquad ...(1\text{-}25)$$

Where σ is electrical conductivity, E is the electrostatic field, v is velocity and B is magnetic field.

1.6.5 Lorentz Force

It is denoted by F, on a charge moving q in a magnetic field B with velocity v is given by:

$$F = qv \times B$$

$$\textit{If} \ \ \mathrm{J = qv} \ \ \text{then} \ \mathrm{F = J \times B} \qquad ...(1\text{-}26)$$

1.7 Magneto Hydrodynamics (MHD): [91]

It is the branch of continum mechanics which deals with the motion of electrically conducting fluid in the presence of magnetic field. The motion of conducting material across the magnetic lines of force creates potential differences which, in general, cause electric currents to flow. The magnetic field associated with these currents modify the magnetic field which creates them. In other words, the fluid flow alters the electromagnetic state of the system. On the other hand, the flow of electric current across a magnetic field is associated with a body force that is called Lorentz force, which influence the fluid flow. It is this intimate interdependence of hydrodynamics and electrodynamics which really defines and characterizes MHD.

The MHD Navier- Stokes equations represented by:

In the x-direction:

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=\rho F_x-\frac{\partial p}{\partial x}+\mu\nabla^2 u+(J\times B)_x \qquad ...(1\text{-}27)$$

In the y- direction:

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=\rho F_y-\frac{\partial p}{\partial y}+\mu\nabla^2 v+(J\times B)_y \qquad ...(1\text{-}28)$$

In the z-direction:

$$\rho(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=\rho F_z-\frac{\partial p}{\partial z}+\mu\nabla^2 w+(J\times B)_z \qquad ...(1\text{-}29)$$

where $(J\times B)_x$, $(J\times B)_y$, $(J\times B)_z$ are the components of Lorentz force (electromagnetic force) in the x-direction, y-direction and z-direction respectively, B is the magnetic field, J is the current density or conduction current and μ is dynamic viscosity.

1.8 Heat Transfer: [115]

When a temperature difference exists in a medium or between media, heat transfer must occur. Heat transfer is energy in transit due to a temperature difference. Three different types of heat transfer process are known when a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term conduction for the heat transfer that occurs across the medium. The term convection refers to heat transfer that occurs between a surface and a moving fluid when they are at different temperatures. The third kind of heat transfer is termed thermal radiation. Heat transfer is termed thermal radiation. All surfaces of finite temperature emit energy in the form of electromagnetic waves. Owing to radiation between two surfaces at different temperatures when there is an absence of an intervening medium and heat transfer occurs.

1.8.1 Specific Heat of Fluid at Constant Pressure

It is denoted by C_ρ which is defined as the ratio of heat flow to mass and temperature difference it is expressed by the equation:

$$C_\rho = \frac{dQ}{dT} \qquad ...(1\text{-}30)$$

1.8.2 Thermal Conductivity

It is denoted by k_1, which is defined as the flow in a unit time across unit area through unit thickness when a temperature difference of unity is mentioned between opposite forces.

1.8.3 Thermal Diffusivity

It is defined as:

$$k' = \frac{k_1}{\rho C_\rho} \qquad ...(1\text{-}31)$$

where, k_1 is thermal conductivity, ρ is density, and C_ρ is specific heat.

The thermal diffusivity is, therefore, the ratio of heat conducted through the material to the heat stored per unit volume. The larger of the thermal diffusivity makes the propagation of heat into the material faster. If the thermal diffusivity is small, it means that a big part of the heat is absorbed by conducted through.

1.8.4 Heat Flux

The heat flux Q is defined by the Fourier's law as, the rate of heat flow is proportional to the area of flow A and to the temperature difference dT, across the layer, and inversely proportional to the thickness dx, and varies only in one direction, say x it is expressed by the equation;

$$Q = -k_1 A \frac{dT}{dx} \qquad ...(1\text{-}32)$$

Where, k_1 is thermal conductivity and the negative sign indicates that the temperature change in the direction of heat flow (-dT).

1.8.5 Heat Capacity

It is denoted by ρC_ρ, which is defined by the product of density and specific heat.

1.8.6 Joulean Heating

The Joulean heating is defined as the ratio of the square of conducting current to the electrical conductivity and is given by:

$$\text{joulean heating} = \frac{(J)^2}{\sigma}$$

1.9 The Energy Equation: [36]

The magneto fluid dynamic (MFD) energy equation is represented by a nonlinear partial differential equation and is given by :

$$\rho C_\rho \frac{DT}{Dt} = \phi + \frac{(J)^2}{\sigma} + k_1 \nabla^2 T + PR \qquad ...(1\text{-}33)$$

Where

$$\frac{DT}{Dt} = \left(\frac{\partial}{\partial t} + u.\nabla\right)T \,, \qquad ...(1\text{-}34)$$

PR is rate at which heat is added by chemical reaction, radiation and electromagnetic action and

$$\phi = \lambda(\nabla - u)^2 + 2\mu D.D \qquad ...(1\text{-}35)$$

where $D = \frac{1}{2}(\nabla u + (\nabla u)^T)$ is the deformation tensor. Here, λ is the second coefficient of viscosity, and if stokes hypothesis is assumed to hold then $\lambda = -\frac{2}{3}\mu$

Also, here ϕ is called the dissipation function. It can be shown that ϕ, which represents the rate at which work is converted into heat, is always greater or equal to zero.

For incompressible flows, the divergence of the velocity field ∇u ,
is identically zero, so any questions about the validity of stokes hypotheses are irrelevant. Also, when the density of the material particle is constant, the term expressing work done by compressing the fluid is absent. The internal generation due to viscous dissipation is frequently small allowing us to ignore ϕ. With these assumptions the energy equation reduces to:

$$\rho C_\rho \frac{DT}{Dt} = k_1 \nabla^2 T \qquad ...(1\text{-}36)$$

It is clearly that while T depends on the velocity field, the velocity field does not depend on temperature.

1.10 Flexible Wall: [71]

The governing equation of motion of the flexible wall may be expressed as:

$$L(H) = P - P_0 \qquad ...(1\text{-}37)$$

Where P_0 is the pressure on the outside of the wall due to tension in the muscles, and L is an operator that is used to represent the motion of the stretched membrane with damping forces such that:

$$L = -k\frac{\partial^2}{\partial x^2} + m_1\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t} \qquad ...(1\text{-}38)$$

Here, k is the elastic tension in the membrane, m_1 is mass per unit area and C is the coefficient of viscous damping forces.

1.11 Dimensions: [90]

A dimension is the measure by which a physical variable is expressed quantitatively in fluid mechanic, there are only three primary dimensions from which all the dimensions can be derived, namely; mass (m), Length (l) and time (t). All other variables in fluid mechanics can be expressed in terms of m, l and t. for example, acceleration has the dimension lt^{-2}. Force is directly related to mass, length and time by Newton's second law, force =mass ×acceleration (f=m×a), and from this we see that, the force has the dimension mlt^{-2}.

1.12 Dimensional Analysis: [114]

The method of dimensional analysis aims to deriving similarity parameters, which can be used to apply data measured with a model configuration to the geometrical similar full- scale configuration there by the number of necessary experiments can be reduced, which depends on the number of the physical quantities influencing the problem. The dimensional analysis also offers the advantage, that the physical quantities can be combined in such a way, that the results are independent of the measuring units. The physical quantities are combined in a product such that dimension less combinations result.

1.13 Dimensional Parameters: [114]

If the number of variable affecting a flow problem are known, these can be arranged into a suitable dimensionless parameters by the method of dimensional analysis. From experience and judgment, less important parameters may be dropped out. Thus we are left with the most important parameters which have a far greater influence upon the phenomenon than the parameters dropped out.

There are some important parameters of dimensionless number in fluid mechanics, which are:

1.13.1 Reynolds Number

It is denoted by (Re) it is the most common dimensionless number in fluid mechanics. Low Re flows involve small sizes, low speeds and high kinematic viscosity such as bacteria swimming through mucous. High Re flows involve large sizes, high speeds and low kinematics viscosity such as an ocean linear steaming at full speed. And represented the ratio of inertia force to the viscous force, given by :

$$\mathrm{Re} = \frac{\text{inertia force}}{\textit{viscous}\ \text{force}} = \frac{\rho c a}{\mu} \qquad ...(1\text{-}39)$$

where c, a represents, uniform velocity, dimension of the channel, respectively.

1.13.2 Prandtl's Number

It is a ratio of two molecular transport properties. Therefore a fluid property and independent of flow geometry, denoted by Pr, is a dimensionless and represents the ratio of kinematic viscosity to the thermal diffusivity and given by:

$$\mathrm{Pr} = \frac{\nu}{k'} = \frac{\mu/\rho}{k_1/\rho C_\rho} = \frac{\mu C_\rho}{k_1} \qquad ...(1\text{-}40)$$

1.13.3 Schmidt's Number

It is denoted by Sc, which relates viscous diffusion to mass diffusion, and given by:

$$Sc = \frac{\mu}{\rho D} = \frac{\nu}{D} \qquad ...(1\text{-}41)$$

where D is the coefficient of mass diffusivity has dimension L^2T^{-1}.

1.13.4 Grashof's Number

It is denoted by Gr, which is a measure of buoyancy or free convection effects in a flow, and given by:

$$Gr = \frac{g\,\rho a^2 \alpha (T_1 - T_0)}{c\,\mu} \qquad ...(1\text{-}42)$$

Where T_0 a temperature at lower wall of channel is, T_1 is a temperature at upper wall of channel, g is an acceleration due to gravity, α is a coefficient of linear thermal expansion of fluid and c is a wave velocity.

1.13.5 Darcy Number

In fluid dynamics through porous medium. The Darcy number, denoted by Da, represented the relative effect of the permeability of the medium versus it is cross- sectional commonly the diameter squared. The number is named after Henry Darcy and is found from non- dimensionalzing the differential form of Darcy's law. This number should not be confused with the Darcy friction factor. Which applies to pressure drop in pipe. It is defined as:

$$Da = \frac{k_0}{a^2} \qquad ...(1-43)$$

Where k_0 is the permeability of the medium and a is the diameter of the particle.

1.13.6 Soret's Number

The soret effect is mass flux due to temperature gradient and appears in the species continuity equation when you have a multi-component mixture where each species has its own diffusion velocity. The soret number, denoted by Sr, relates thermal diffusion coefficient to ordinary diffusion coefficient and given by:

$$Sr = \frac{D_m k_T (T_1 - T_0)}{T_m (c_1 - c_0)} \qquad ...(1-44)$$

where D_m the coefficient of mass diffusivity is, T_m is mean fluid temperature, k_T is the thermal diffusion ratio.

1.13.7 Froude Number

The Froude number, denoted by Fr, is the dimensionless nuber and represents the ratio of inertia force to the gravity force and given by:

$$Fr = \frac{\text{inertia force}}{\text{gravity force}} = \frac{c^2}{gd} \qquad ...(1-45)$$

where g, d represents, gravity of acceleration and dimension of the channel.

1.13.8 Eckert Number: [28]

The Eckert number Ec is a dimensionless number used in fluid dynamics. It expressed the relationship between a flow's kinetic energy and enthalpy, and is used to characterize dissipation. It is defined as:

$$Ec = \frac{c^2}{C_\rho \Delta T} \qquad ...(1-46)$$

where ΔT is characteristic temperature difference of the flow.

1.13.9 **<u>Hartman number:</u>** **[55]**

Denoted by (M), it is the dimensionless number and represents the ratio of magnetic force to the inertia force, or the square root of Stuart number (interaction parameter) product the Reynolds number is given by

$$M = \sqrt{\frac{\sigma}{\mu}} B_0 l \qquad ...(1\text{-}47)$$

Where l is a characteristic length.

1.14 **<u>Regular Perturbation Expansions:</u>** **[13]**

Perturbation methods, also known as asymptotes allow the simplification of complex mathematical problems. Use of perturbation theory will allow approximate solutions to be determined for problems which cannot solved by traditional analytical methods. Second order ordinary linear differential equations are solved by engineers and scientists routinely. However in many cases, real life situations can require much more difficult mathematical models, such as non-linear differential equations.

We are all familiar with the principle of the Taylor expansion. For an analytic function f(x), we can expand close to a point (x=a) as:

$$f(a+x) = f(a) + \varepsilon f'(a) + \frac{1}{2} f''(a) + ... \qquad ...(1\text{-}48)$$

for general function f(x) there are many ways this expansion can fail, including lack of convergence of the series or simply an in ability of the series to capture the behavior of the function, but the paradigm of the expansion in which a small change to x makes a small change to f(x) is powerful one, and the basis of regular perturbation expansions.

The basic principle and practice of the regular perturbation expansion is:

1 Set $\varepsilon = 0$ and solve the resulting system (solution f_0 for definiteness)

2 Perturb the system by allowing ε to be nonzero (but small in some sense)

3 Formulate the solution to the new perturb bed system as series:

$f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + ...$

4 Expand the governing equation as a series in ε, collecting terms with equal powers of ε, solve them in turn as far as the solution is required.

Chapter Two

Effect of Magnetic Field on Peristaltic Flow of Walters –B Fluid Through a Porous Medium in a Tapered Asymmetric Channel.

Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of contraction and expansion along the length of distensible tube mixing and transporting the fluid in the direction of the wave propagation. This kind of phenomenon is termed as peristaltic. It plays an indispensable role in transporting many physiological fluids in the body under various situations as urine transport from kidney to bladder, the movement of chyme in the gastrointestinal tracts, transport of spermatozoa in the ductus efferent's of the male reproductive tract, movement of ovum in the fallopian tubes, swallowing of food through esophagus and the vasomation of small blood vessels many modern mechanical devices have been designed on the principle of peristaltic pumping to transport the fluids without internal moving parts, for example the blood pump in the heart-lung machine and peristaltic transport of naxious fluid in nuclear industry. The mechanism of peristaltic transport has attracted the attention of many investigators since its investigation by Latham [61], Burns and Pareks[23], Shapero et all.[96], Fung and Yih [34], Takabatake and Ayukawa [109], Akram and Nedeem [12], Mekheimer and Elkot [67], Mekheimor and al –Arabi[66], Mekheimer[64], Nadeam and Akbar [73], Kothandapani et al.[58], of peristaltic flow for different fluids have been reported under various conditions with reference to physiological and mechanical situations. Most of these investigations are confined to the peristaltic flow only in a symmetric channel or tube.

Among the many suggested models, Walters [113] has developed a physically accurate mathematical model for the rhedogical equation of state of a viscoelastic fluid with short memory. This model has been shown to capture the characteristic of actual viscoelastic polymer solutions, hydrocarbons, paints and other chemical engineering fluids. The Walter's–B fluid model generates highly non-linear flow equations which have order higher than that of the Navier-stokes equations. It also incorporates elastic properties of the fluid which are important in extensional behavior of polymers. Peristalsis of Walters-B fluid with wall properties has never been addressed previously. Thus Margiam Javed et al. [53] is undertaking to fill this void by incorporating velocity slip and temperature jump conditions.

In the present study, and the purpose of this chapter is to investigated the peristaltic transport of Walters-B fluid under the effect of magnetic field through a porous medium in a tapered a symmetric channel. Regular perturbation technique are used under long-wave length (wave number is small) and law-Reynolds assumption. Series solutions for stream function, axial velocity and pressure rise are given, numerical computations have been performed for pressure rise per wave length. The influence of the physical parameters of the problem are discussed and illustrated graphically.

2-1 The Mathematical Model of the Problem

Let us consider the MHD flow of an incompressible and electrically conducting walters –B fluid through a porous medium of two–dimensional tapered a symmetric channel. We assume that infinite wave train traveling with velocity c along the non–uniform walls. We choose a rectangular coordinate system for the channel with $\overline{X}$ along the direction of wave propagation and parallel to the center line and $\overline{Y}$ transverse to it. The wall of the tapered a symmetric channel are given in fig. (2-1) by the equations: [58]

$$\overline{H}_1(\overline{x},\overline{t}) = -d - m'\overline{x} - a_1 \sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t}) + \phi] \ldots\ldots lower \text{ wall}$$

$$\overline{H}_2(\overline{x},\overline{t}) = d + m'\overline{x} + a_2 \sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t})] \ldots\ldots upper \text{ wall} \qquad \ldots\ldots\ldots(2\text{-}1)$$

where a_1, a_2 are the amplitudes of the waves, 2d is the width of the channel at the inlet, $m'(m' << 1)$ is the non-uniform parameters, the phase difference ϕ varies in the range $0 \le \phi \le \pi$, $\phi = 0$ represents to symmetric channel in which the waves are out of phase and when $\phi = \pi$ the waves are in phase, and further a_1, a_2, d and ϕ satisfies the condition :

$$a_1^2 + a_2^2 + 2a_1a_2\cos\phi \le (2d)^2 \qquad \ldots\ldots\ldots(2-2)$$

2-2 The Governing Equations of the Problem

The constitutive equations for Walters-B fluid are: [53]

$$\overline{S} = -PI + \overline{\zeta}, \qquad \ldots\ldots\ldots\ldots(2-3)$$

$$\overline{\zeta} = 2\eta_0 e_1 - 2k_0 \frac{\overline{\delta e_1}}{\overline{\delta t}}, \qquad \ldots\ldots\ldots(2-4)$$

$$e = \nabla\overline{W} + (\nabla\overline{W})^T, \qquad ....(2-5)$$

$$\frac{\overline{\delta e_1}}{\overline{\delta t}} = \frac{\partial e_1}{\partial t} + (\overline{V}.\nabla)e_1 - e_1(\nabla\overline{W}) - (\nabla\overline{W})^T e_1, \qquad ....(2-6)$$

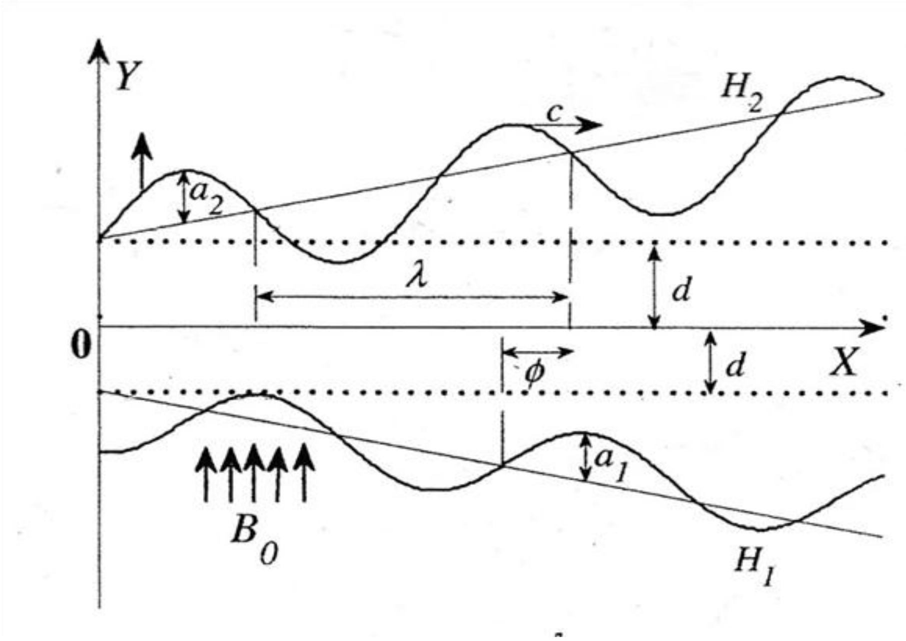


Fig. (2-1): physical Sketch of the problem

In which $\overline{S}$ is the Cauchy stress tensor, -PI is the spherical part of the stress due to constrain of incompressibility, ζ is the extra stress tensor, η_0 is the coefficient of viscosity, e_1 is the rate of strain tensor and $\frac{\overline{\delta}}{\overline{\delta t}}$ denotes the convicted differentiation of a tensor quantity in relation to the material motion, k_0 is the short memory coefficient. $(\nabla\overline{W})$ is the fluid velocity gradient in the Cartesian coordinated system $(\overline{X},\overline{Y})$ and $(\nabla\overline{W})^T$ is the transpose of the fluid velocity gradient in the Cartesian coordinates $(\overline{X},\overline{Y})$, they are defined as:

$$\nabla\overline{W} = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{U}}{\partial\overline{Y}} \\ \frac{\partial\overline{V}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix}, \quad (\nabla\overline{W})^T = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{X}} \\ \frac{\partial\overline{U}}{\partial\overline{Y}} & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ....(2-7)$$

Then

$$e_1 = \begin{pmatrix} 2\dfrac{\partial \overline{U}}{\partial \overline{X}} & (\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{X}}) \\ (\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{X}}) & 2\dfrac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix} \quad ....(2-8)$$

Also we have:

$$\frac{\partial e_1}{\partial \overline{t}} = \begin{pmatrix} 2\dfrac{\partial^2 \overline{U}}{\partial \overline{X}\,\partial \overline{t}} & (\dfrac{\partial^2 \overline{U}}{\partial \overline{Y}\,\partial \overline{t}} + \dfrac{\partial^2 \overline{V}}{\partial \overline{X}\,\partial \overline{t}}) \\ (\dfrac{\partial^2 \overline{V}}{\partial \overline{X}\,\partial \overline{t}} + \dfrac{\partial^2 \overline{U}}{\partial \overline{Y}\,\partial \overline{t}}) & 2\dfrac{\partial^2 \overline{V}}{\partial \overline{Y}\,\partial \overline{t}} \end{pmatrix} \quad ....(2-9)$$

$$(\overline{V}.\nabla)e_1 = (\overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}})\begin{pmatrix} 2\dfrac{\partial \overline{U}}{\partial \overline{X}} & (\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{X}}) \\ (\dfrac{\partial \overline{V}}{\partial \overline{X}} + \dfrac{\partial \overline{U}}{\partial \overline{Y}}) & 2\dfrac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix}$$

$$\begin{pmatrix} (\overline{U}\dfrac{\partial}{\partial \overline{X}} + \overline{V}\dfrac{\partial}{\partial \overline{Y}})2\dfrac{\partial \overline{U}}{\partial \overline{X}} & (\overline{U}\dfrac{\partial}{\partial \overline{X}} + \overline{V}\dfrac{\partial}{\partial \overline{Y}})(\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{X}}) \\ (\overline{U}\dfrac{\partial}{\partial \overline{X}} + \overline{V}\dfrac{\partial}{\partial \overline{Y}})(\dfrac{\partial \overline{V}}{\partial \overline{X}} + \dfrac{\partial \overline{U}}{\partial \overline{Y}}) & (\overline{U}\dfrac{\partial}{\partial \overline{X}} + \overline{V}\dfrac{\partial}{\partial \overline{Y}})2\dfrac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix}$$

$$\begin{pmatrix} 2\overline{U}\dfrac{\partial^2 \overline{U}}{\partial \overline{X}^2} + 2\overline{V}\dfrac{\partial^2 \overline{U}}{\partial \overline{X}\,\partial \overline{Y}}) & (\overline{U}\dfrac{\partial^2 \overline{V}}{\partial \overline{X}^2} + \overline{U}\dfrac{\partial^2 \overline{U}}{\partial \overline{X}\,\partial \overline{Y}} + \overline{V}\dfrac{\partial^2 \overline{V}}{\partial \overline{Y}\,\partial \overline{X}} + \overline{V}\dfrac{\partial^2 \overline{U}}{\partial \overline{Y}^2}) \\ (\overline{U}\dfrac{\partial^2 \overline{V}}{\partial \overline{X}^2} + \overline{U}\dfrac{\partial^2 \overline{U}}{\partial \overline{X}\,\partial \overline{Y}} + \overline{V}\dfrac{\partial^2 \overline{V}}{\partial \overline{Y}\,\partial \overline{X}} + \overline{V}\dfrac{\partial^2 \overline{U}}{\partial \overline{Y}^2}) & 2\overline{U}\dfrac{\partial^2 \overline{V}}{\partial \overline{X}\,\partial \overline{Y}} + 2\overline{V}\dfrac{\partial^2 \overline{V}}{\partial \overline{Y}^2}) \end{pmatrix}$$

.........(2-10)

$$e_1(\nabla\overline{V}) = \begin{pmatrix} 2\dfrac{\partial \overline{U}}{\partial \overline{X}} & (\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{X}}) \\ (\dfrac{\partial \overline{V}}{\partial \overline{X}} + \dfrac{\partial \overline{U}}{\partial \overline{Y}}) & 2\dfrac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix}\begin{pmatrix} \dfrac{\partial \overline{U}}{\partial \overline{X}} & \dfrac{\partial \overline{U}}{\partial \overline{Y}} \\ \dfrac{\partial \overline{V}}{\partial \overline{X}} & \dfrac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix}$$

$$= \begin{pmatrix} 2(\dfrac{\partial \overline{U}}{\partial \overline{X}})^2 + (\dfrac{\partial \overline{V}}{\partial \overline{X}})^2 + \dfrac{\partial \overline{U}}{\partial \overline{Y}}\dfrac{\partial \overline{V}}{\partial \overline{X}} & 2\dfrac{\partial \overline{U}}{\partial \overline{X}}\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{Y}}\dfrac{\partial \overline{U}}{\partial \overline{Y}} + \dfrac{\partial \overline{V}}{\partial \overline{Y}}\dfrac{\partial \overline{V}}{\partial \overline{X}} \\ \dfrac{\partial \overline{U}}{\partial \overline{X}}\dfrac{\partial \overline{V}}{\partial \overline{X}} + \dfrac{\partial \overline{U}}{\partial \overline{X}}\dfrac{\partial \overline{U}}{\partial \overline{Y}} + 2\dfrac{\partial \overline{V}}{\partial \overline{Y}}\dfrac{\partial \overline{V}}{\partial \overline{X}} & (\dfrac{\partial \overline{U}}{\partial \overline{Y}})^2 + \dfrac{\partial \overline{V}}{\partial \overline{X}}\dfrac{\partial \overline{U}}{\partial \overline{Y}} + 2(\dfrac{\partial \overline{V}}{\partial \overline{Y}})^2 \end{pmatrix} \quad ....(2-11)$$

$$(\nabla\overline{V})^T e_1 = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{X}} \\ \frac{\partial\overline{U}}{\partial\overline{Y}} & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \begin{pmatrix} 2\frac{\partial\overline{U}}{\partial\overline{X}} & (\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) \\ (\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{U}}{\partial\overline{Y}}) & 2\frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix}$$

$$= \begin{pmatrix} 2(\frac{\partial\overline{U}}{\partial\overline{X}})^2 + (\frac{\partial\overline{V}}{\partial\overline{X}})^2 + \frac{\partial\overline{V}}{\partial\overline{X}}\frac{\partial\overline{U}}{\partial\overline{Y}} & \frac{\partial\overline{U}}{\partial\overline{X}}\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{U}}{\partial\overline{X}}\frac{\partial\overline{V}}{\partial\overline{X}} + 2\frac{\partial\overline{V}}{\partial\overline{X}}\frac{\partial\overline{V}}{\partial\overline{Y}} \\ 2\frac{\partial\overline{U}}{\partial\overline{X}}\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{Y}}\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{V}}{\partial\overline{Y}}\frac{\partial\overline{U}}{\partial\overline{Y}} & (\frac{\partial\overline{U}}{\partial\overline{Y}})^2 + \frac{\partial\overline{U}}{\partial\overline{Y}}\frac{\partial\overline{V}}{\partial\overline{X}} + 2(\frac{\partial\overline{V}}{\partial\overline{Y}})^2 \end{pmatrix} \qquad ....(2\text{-}12)$$

$$2\eta_0 e_1 = \begin{pmatrix} 4\eta_0\frac{\partial\overline{U}}{\partial\overline{X}} & 2\eta_0\frac{\partial\overline{U}}{\partial\overline{Y}} + 2\eta_0\frac{\partial\overline{V}}{\partial\overline{X}} \\ 2\eta_0\frac{\partial\overline{V}}{\partial\overline{X}} + 2\eta_0\frac{\partial\overline{U}}{\partial\overline{Y}}) & 4\eta_0\frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ....(2\text{-}13)$$

Now, write $\overline{\zeta} = \begin{pmatrix} \overline{\zeta}_{\overline{X}\overline{X}} & \overline{\zeta}_{\overline{X}\overline{Y}} \\ \overline{\zeta}_{\overline{Y}\overline{X}} & \overline{\zeta}_{\overline{Y}\overline{Y}} \end{pmatrix}$ ....(2-14)

And substitute (2-9), (2-10), (2-11), (2-12), (2-13) into eq. (2-4), thus we have the components of shear tensor $\overline{\zeta}$ as follows:

$$\overline{\zeta}_{\overline{X}\overline{X}} = 4\eta_0\frac{\partial\overline{U}}{\partial\overline{X}} - 2k_0(2\frac{\partial^2\overline{U}}{\partial\overline{X}\,\partial\overline{t}} + 2(\overline{U}\frac{\partial^2\overline{U}}{\partial\overline{X}^2} + \overline{V}\frac{\partial^2\overline{U}}{\partial\overline{X}\,\partial\overline{Y}}) - 4(\frac{\partial\overline{U}}{\partial\overline{X}})^2 - 2\frac{\partial\overline{V}}{\partial\overline{X}}$$
$$(\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{U}}{\partial\overline{Y}}) \qquad ....(2-15)$$

$$\overline{\zeta}_{\overline{X}\overline{Y}} = \overline{\zeta}_{\overline{Y}\overline{X}} = 2\eta_0(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - 2k_0(\frac{\partial^2\overline{U}}{\partial\overline{Y}\,\partial\overline{t}} + \frac{\partial^2\overline{V}}{\partial\overline{X}\,\partial\overline{t}} - 2\frac{\partial\overline{U}}{\partial\overline{X}}\frac{\partial\overline{U}}{\partial\overline{Y}} + (\overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}})$$
$$(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - \frac{\partial\overline{V}}{\partial\overline{Y}}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - \frac{\partial\overline{U}}{\partial\overline{X}}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - 2\frac{\partial\overline{V}}{\partial\overline{X}}\frac{\partial\overline{V}}{\partial\overline{Y}}) \qquad ....(2-16)$$

$$\overline{\zeta}_{\overline{Y}\overline{Y}} = 4\eta_0\frac{\partial\overline{V}}{\partial\overline{Y}} - 2k_0(2\frac{\partial^2\overline{V}}{\partial\overline{Y}\,\partial\overline{t}} + 2(\overline{U}\frac{\partial^2\overline{V}}{\partial\overline{X}\,\partial\overline{Y}} + \overline{V}\frac{\partial^2\overline{V}}{\partial\overline{Y}^2}) - 4(\frac{\partial\overline{V}}{\partial\overline{Y}})^2 - 2\frac{\partial\overline{U}}{\partial\overline{Y}}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}})$$
$$.........(2-17)$$

2-3 Calculation of Lorentz Force: [60]

To calculate the Lorentz force $(\bar{J} \times \bar{B})$, we will apply a magnetic field just in $\bar{Y}$ -direction. The effect of this force on the fluid flow, will be analyzed. Now, apply magnetic field in $\bar{Y}$ -direction $(0, B_0, 0)$ and to calculate Lorentz force we start with:

$$\bar{V} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \bar{U} & \bar{V} & 0 \\ 0 & B_0 & 0 \end{vmatrix} = B_0 \bar{U}\,\bar{k} \qquad ....(1)$$

By definition (1.6.4) we have:

$$\bar{J} = \sigma(\bar{V} \times \bar{B}) = \sigma B_0 \bar{U}\,\bar{k} \qquad ....(2)$$

and so;

$$\bar{J} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & \sigma\bar{U}\,B_0 \\ 0 & B_0 & 0 \end{vmatrix} = -\sigma B_0{}^2 \bar{U}\,\bar{i} \qquad ....(3)$$

where $\bar{J}$ is the induced current and $\bar{B}$ is the magnetic field. It is observed that the effect of the magnetic field is appear on the flow in the $\bar{X}$ -direction only.

2-4 Basic Equations of the Problem

The equations governing of the non Newtonian incompressible viscous Walter's-B fluid are:

The continuity equation is given by:

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \qquad ....(2-18)$$

The momentum equations are given by:

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} + \frac{\partial}{\partial \bar{X}}(\bar{\zeta}_{\overline{XX}}) + \frac{\partial}{\partial \bar{Y}}(\bar{\zeta}_{\overline{XY}}) - \sigma B_0^2 \bar{U} - \frac{\eta_0}{K}\bar{U}. \qquad ....(2-19)$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} + \frac{\partial}{\partial \bar{X}}(\bar{\zeta}_{\overline{XY}}) + \frac{\partial}{\partial \bar{Y}}(\bar{\zeta}_{\overline{YY}}) - \frac{\eta_0}{K}\bar{V} \qquad ....(2-20)$$

2-5 Method of Solution of the Problem

In order to simplify the governing equations of continuity and motion, we may introduce the following dimensionless transformations as follows:

$$x=\frac{\overline{X}}{\lambda},y=\frac{\overline{Y}}{d},t=\frac{c\overline{t}}{\lambda},u=\frac{\overline{U}}{c},v=\frac{\overline{V}}{\delta c},h_1=\frac{\overline{H_1}}{d},h_2=\frac{\overline{H_2}}{d}$$

$$\rho=\frac{d^2\overline{P}}{c\lambda\eta_0},a=\frac{a_1}{d},b=\frac{a_2}{d},m=\frac{m'\lambda}{d},\zeta=\frac{d}{\eta_0 c}\overline{\zeta},\delta=\frac{d}{\lambda}$$

$$M=\sqrt{\frac{\sigma}{\eta_0}}B_0 d,\mathrm{Re}=\frac{\rho e d}{\eta_0},\overline{k}=\frac{k_0 c}{\eta_0 d},k^2=\frac{d^2}{K},u=\frac{\partial\psi}{\partial y},v=\frac{-\partial\psi}{\partial x} \qquad ....(2-21)$$

Where a, b are the amplitudes of the waves at the lower and upper walls of channel, ψ is stream function.

Substituting (2-21) into Equations.(2-18),(2-19) and (2-20)we get:

From eq. (2-18) we have:

$$\frac{\partial\overline{U}}{\partial\overline{X}}+\frac{\partial\overline{V}}{\partial\overline{Y}}=0$$

$$\frac{c}{\lambda}\frac{\partial u}{\partial x}+\frac{c\delta}{d}\frac{\partial v}{\partial y}=0 \qquad ....(2-22)$$

Multiplying both sides of (2-22) by (λ/c) yields to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad ....(2-23)$$

From eq. (2-19):

$$\rho\left(\frac{\partial\overline{U}}{\partial\overline{t}}+\overline{U}\frac{\partial\overline{U}}{\partial\overline{X}}+\overline{V}\frac{\partial\overline{U}}{\partial\overline{Y}}\right)=-\frac{\partial\overline{P}}{\partial\overline{X}}+\frac{\partial}{\partial\overline{X}}(\overline{\zeta}_{\overline{XX}})+\frac{\partial}{\partial\overline{Y}}(\overline{\zeta}_{\overline{XY}})-\sigma B_0^2\overline{U}-\frac{\eta_0}{K}\overline{U}.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+Cu\frac{C}{\lambda}\frac{\partial u}{\partial x}+C\delta v\frac{C}{d}\frac{\partial u}{\partial y}\right)=-\frac{C\eta_0}{d^2}\frac{\partial P}{\partial x}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xx}+\frac{C\eta_0}{d^2}\frac{\partial}{\partial y}\zeta_{xy}$$

$$-\sigma B_0^2 Cu-\frac{\eta_0 C}{K}u.$$

$$\rho(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}u\frac{\partial u}{\partial x}+\frac{C^2}{\lambda}v\frac{C}{d}\frac{\partial u}{\partial y})=-\frac{C\eta_0}{d^2}\frac{\partial P}{\partial x}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xx}+\frac{C\eta_0}{d^2}\frac{\partial}{\partial y}\zeta_{xy}$$
$$-\sigma B_0^2 Cu-\frac{\eta_0 C}{K}u.$$

$$\rho\frac{C^2}{\lambda}(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{C\eta_0}{d^2}\frac{\partial P}{\partial x}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xx}+\frac{C\eta_0}{d^2}\frac{\partial}{\partial y}\zeta_{xy}-\sigma B_0^2 Cu-\frac{\eta_0 C}{K}u. \quad ....(2-24)$$

Now multiplying both sides of equation (2-24) by $(\frac{d^2}{C\eta_0})$ we have to get:

$$\rho\frac{C^2}{\lambda}\frac{d^2}{C\eta_0}(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+\frac{\eta_0 C}{\lambda d}\frac{d^2}{C\eta_0}\frac{\partial}{\partial x}\zeta_{xx}+\frac{C\eta_0}{d^2}\frac{d^2}{C\eta_0}\frac{\partial}{\partial y}\zeta_{xy}$$
$$-\sigma B_0^2 uC\frac{d^2}{C\eta_0}-\frac{\eta_0 C}{K}\frac{d^2}{\eta_0 C}u.$$
$$\frac{\rho Cd}{\eta_0}\frac{d}{\lambda}(\frac{du}{dt}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+\delta\frac{\partial}{\partial x}\zeta_{xx}+\frac{\partial}{\partial y}\zeta_{xy}-\frac{\sigma B_0 d^2}{\eta_0}u-\frac{d^2}{K}u.$$

Which may be written as:

$$\operatorname{Re}\delta(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+\delta\frac{\partial}{\partial x}\zeta_{xx}+\frac{\partial}{\partial y}\zeta_{xy}-M^2u-K^2u.$$

That is:

$$\operatorname{Re}\delta(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+\delta\frac{\partial}{\partial x}\zeta_{xx}+\frac{\partial}{\partial y}\zeta_{xy}-(M^2+K^2)u \quad ....(2-25)$$

From eq. (2-20):

$$\rho(\frac{\partial\overline{V}}{\partial\overline{t}}+\overline{U}\frac{\partial\overline{V}}{\partial\overline{X}}+\overline{V}\frac{\partial\overline{V}}{\partial\overline{Y}})=-\frac{\partial\overline{P}}{\partial\overline{Y}}+\frac{\partial}{\partial\overline{X}}(\overline{\zeta}_{\overline{XY}})+\frac{\partial}{\partial\overline{Y}}(\overline{\zeta}_{\overline{YY}})-\frac{\eta_0}{K}\overline{V}$$
$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+Cu\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}+C\delta v\frac{C\delta}{d}\frac{\partial v}{\partial y})=-\frac{\lambda\eta_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xy}+\frac{\eta_0 C}{d^2}\frac{\partial}{\partial y}\zeta_{yy}-\frac{\eta_0}{K}C\delta v$$
$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+\frac{C^2\delta}{\lambda}u\frac{\partial v}{\partial x}+\frac{C^2\delta}{\lambda}v\frac{\partial v}{\partial y})=-\frac{\lambda\eta_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xy}+\frac{\eta_0 C}{d^2}\frac{\partial}{\partial y}\zeta_{yy}-\frac{\eta_0 C\delta}{K}v$$

$$\frac{\rho C^2\delta}{\lambda}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\lambda\eta_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{\eta_0 C}{\lambda d}\frac{\partial}{\partial x}\zeta_{xy}+\frac{\eta_0 C}{d^2}\frac{\partial}{\partial y}\zeta_{yy}-\frac{\eta_0 C\delta}{K}v \quad ....(2-26)$$

Now multiplying both sides of (2-26) by $(\frac{d^3}{C\lambda\eta_0})$ will produce:

$$\frac{\rho C^2\delta}{\lambda}\frac{d^3}{\lambda\eta_0 C}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}+\frac{C\eta_0}{\lambda d}\frac{d^3}{\lambda C\eta_0}\frac{\partial}{\partial x}\zeta_{xy}+\frac{C\eta_0}{d^2}$$

$$\frac{d^3}{\lambda C\eta_0}\frac{\partial}{\partial y}\zeta_{yy}-\frac{C\eta_0\delta}{K}\frac{d^3}{\lambda C\eta_0}v\,.$$

$$\frac{\rho C d}{\eta_0}\delta\frac{d^2}{\lambda^2}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}+\frac{d^2}{\lambda^2}\frac{\partial}{\partial x}\zeta_{xy}+\frac{d}{\lambda}\frac{\partial}{\partial y}\zeta_{yy}-\delta^2\frac{d^2}{K}v\,.$$

Which can be written as:

$$\mathrm{Re}\,\delta^3(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}+\delta^2\frac{\partial}{\partial x}\zeta_{xy}+\delta\frac{\partial}{\partial y}\zeta_{yy}-\delta^2 k^2\mathrm{v} \quad ....(2-27)$$

From eq. (2-15) we have:

$$\frac{\eta_0 C}{d}\zeta_{xx}=\frac{4\eta_0 C}{\lambda}\frac{\partial u}{\partial x}-2k_0(2\frac{C^2}{\lambda^2}\frac{\partial^2 u}{\partial x\,\partial t}+2(\frac{C^2}{\lambda^2}u\frac{\partial^2 u}{\partial x^2}+\frac{C^2\delta}{\lambda d}v\frac{\partial^2 u}{\partial x\,\partial y})-4\frac{C^2}{\lambda^2}(\frac{\partial u}{\partial x})^2$$

$$-2\frac{\partial v}{\partial x}(\frac{C\delta}{\lambda}\frac{C}{d}\frac{\partial u}{\partial y}+\frac{C\delta}{\lambda}\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}).$$

$$\frac{\eta_0 C}{d}\zeta_{xx}=\frac{4\eta_0 C}{\lambda}\frac{\partial u}{\partial x}-2k_0(2\frac{C^2}{\lambda^2}\frac{\partial^2 u}{\partial x\,\partial t}+2(\frac{C^2}{\lambda^2}u\frac{\partial^2 u}{\partial x^2}+\frac{C^2}{\lambda d}\frac{d}{\lambda}v\frac{\partial^2 u}{\partial x\,\partial y})-4\frac{C^2}{\lambda^2}(\frac{\partial u}{\partial x})^2$$

$$-2\frac{\partial v}{\partial x}(\frac{C^2}{\lambda d}\frac{d}{\lambda}\frac{\partial u}{\partial y}+\frac{C^2}{\lambda^2}\delta^2\frac{\partial v}{\partial x})).$$

$$\frac{\eta_0 C}{d}\zeta_{xx}=\frac{4\eta_0 C}{\lambda}\frac{\partial u}{\partial x}-2k_0\frac{C^2}{\lambda^2}(2\frac{\partial^2 u}{\partial x\,\partial t}+2(u\frac{\partial^2 u}{\partial x^2}+v\frac{\partial^2 u}{\partial x\,\partial y})-4(\frac{\partial u}{\partial x})^2-2\frac{\partial v}{\partial x}(\frac{\partial u}{\partial y}+$$

$$\delta^2\frac{\partial v}{\partial x})) \quad ....(2-28)$$

Multiplying both sides of (2-28) by $(\frac{d}{\eta_0 C})$ we get:

$$\zeta_{xx} = \frac{4C\eta_0}{\lambda}\frac{d}{C\eta_0}\frac{\partial u}{\partial x} - 2k_0\frac{C^2}{\lambda^2}\frac{d}{\eta_0 C}\frac{d}{d}(2\frac{\partial^2 u}{\partial x\,\partial t} + 2(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial x\,\partial y}) - 4(\frac{\partial u}{\partial x})^2 - 2$$
$$\frac{\partial v}{\partial x}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})).$$

$$\zeta_{xx} = 4\delta\frac{\partial u}{\partial x} - 2\frac{k_0 C}{\eta_0 d}\frac{d^2}{\lambda^2}(2\frac{\partial^2 u}{\partial x\,\partial t} + 2(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial x\,\partial y}) - 4(\frac{\partial u}{\partial x})^2 - 2\frac{\partial v}{\partial x}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}))$$

Thus we have:

$$\zeta_{xx} = 4\delta\frac{\partial u}{\partial x} - 2\overline{K}\delta^2(2\frac{\partial^2 u}{\partial x\,\partial t} + 2(u\frac{\partial^2 u}{\partial x^2} + v\frac{\partial^2 u}{\partial x\,\partial y}) - 4(\frac{\partial u}{\partial x})^2 - 2\frac{\partial v}{\partial x}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}))$$
$$.........(2-29)$$

From eq. (2-16):

$$\overline{\zeta}_{\overline{XY}} = 2\eta_0(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - 2k_0(\frac{\partial^2\overline{U}}{\partial\overline{Y}\,\partial\overline{t}} + \frac{\partial^2\overline{V}}{\partial\overline{X}\,\partial\overline{t}} - 2\frac{\partial\overline{U}}{\partial\overline{X}}\frac{\partial\overline{U}}{\partial\overline{Y}} + (\overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}})$$
$$(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - \frac{\partial\overline{V}}{\partial\overline{Y}}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - \frac{\partial\overline{U}}{\partial\overline{X}}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) - 2\frac{\partial\overline{V}}{\partial\overline{X}}\frac{\partial\overline{V}}{\partial\overline{Y}}).$$

$$\frac{\eta_0 C}{d}\zeta_{xy} = 2\eta_0(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C\delta}{\lambda}\frac{\partial v}{\partial x}) - 2k_0(\frac{C^2}{\lambda d}\frac{\partial^2 u}{\partial y\,\partial t} + \frac{C^2\delta}{\lambda^2}\frac{\partial^2 v}{\partial x\,\partial t} - 2\frac{C}{\lambda}\frac{\partial u}{\partial x}\frac{C}{d}\frac{\partial u}{\partial y} +$$
$$(\frac{C}{\lambda}u\frac{\partial}{\partial x} + \frac{C\delta}{d}v\frac{\partial}{\partial y})(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C\delta}{\lambda}\frac{\partial v}{\partial x}) - \frac{C\delta}{d}\frac{\partial v}{\partial y}(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C\delta}{\lambda}\frac{\partial v}{\partial x}) - \frac{C}{\lambda}\frac{\partial u}{\partial x}(\frac{C}{d}\frac{\partial u}{\partial y}$$
$$+\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}) - 2\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}\frac{C\delta}{d}\frac{\partial v}{\partial y})$$

$$\frac{\eta_0 C}{d}\zeta_{xy} = 2\eta_0(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C}{\lambda}\frac{d}{\lambda}\frac{\partial v}{\partial x}) - 2k_0(\frac{C^2}{\lambda d}\frac{\partial^2 u}{\partial y\,\partial t} + \frac{C^2}{\lambda^2}\frac{d}{\lambda}\frac{\partial^2 v}{\partial x\,\partial t} - \frac{2C^2}{\lambda d}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + (\frac{C}{\lambda}u$$
$$\frac{\partial}{\partial x} + \frac{C}{d}\frac{d}{\lambda}v\frac{\partial}{\partial y})(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C}{\lambda}\frac{d}{\lambda}\frac{\partial v}{\partial x}) - \frac{C}{d}\frac{d}{\lambda}\frac{\partial v}{\partial y}(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C}{\lambda}\frac{d}{\lambda}\frac{\partial v}{\partial x}) - \frac{C}{\lambda}\frac{\partial u}{\partial x}(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C}{\lambda}$$
$$\frac{d}{\lambda}\frac{\partial v}{\partial x}) - \frac{2C^2}{\lambda d}\frac{d^2}{\lambda^2}\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}).$$

$$\frac{\eta_0 C}{d}\zeta_{xy} = 2\eta_0(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{C}{\lambda}\frac{d}{\lambda}\frac{d}{d}\frac{\partial v}{\partial x})-2k_0(\frac{C^2}{\lambda d}\frac{\partial^2 u}{\partial y\,\partial t}+\frac{C^2}{\lambda^2}\frac{d}{\lambda}\frac{d}{d}\frac{\partial^2 v}{\partial x\,\partial t}-\frac{2C^2}{\lambda d}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+$$
$$\frac{C}{\lambda}(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{C}{\lambda}\frac{d}{\lambda}\frac{d}{d}\frac{\partial v}{\partial x})-\frac{C}{\lambda}\frac{\partial v}{\partial y}(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{C}{\lambda}\frac{d}{\lambda}\frac{d}{d}\frac{\partial v}{\partial x})-\frac{C}{\lambda}\frac{\partial u}{\partial x}(\frac{C}{d}\frac{\partial u}{\partial y}+$$
$$\frac{C}{\lambda}\frac{d}{\lambda}\frac{d}{d}\frac{\partial v}{\partial x})-\frac{2C^2}{\lambda d}\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}).$$

$$\frac{\eta_0 C}{d}\zeta_{xy} = 2\eta_0\frac{C}{d}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2k_0(\frac{C^2}{\lambda d}\frac{\partial^2 u}{\partial y\,\partial t}+\frac{C^2}{\lambda d}\delta^2\frac{\partial^2 v}{\partial x\,\partial t}-\frac{2C^2}{\lambda d}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+$$
$$\frac{C}{\lambda}\frac{C}{d}(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{C}{\lambda}\frac{C}{d}\frac{\partial v}{\partial y}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{C}{\lambda}\frac{C}{d}\frac{\partial u}{\partial x}(\frac{\partial u}{\partial y}+$$
$$\delta^2\frac{\partial v}{\partial x})-\frac{2C^2}{\lambda d}\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}).$$

$$\frac{\eta_0 C}{d}\zeta_{xy} = \frac{2\eta_0 C}{d}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2k_0\frac{C^2}{\lambda d}(\frac{\partial^2 u}{\partial y\,\partial t}+\delta^2\frac{\partial^2 v}{\partial x\,\partial t}-2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})$$
$$(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial v}{\partial y}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial u}{\partial x}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}). \qquad ....(2-30)$$

Multiplying both sides of eq. (2-30) by $(\frac{d}{\eta_0 C})$ we get:

$$\zeta_{xy} = \frac{2C\eta_0}{d}\frac{d}{C\eta_0}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2k_0\frac{C^2}{\lambda d}\frac{d}{C\eta_0}(\frac{\partial^2 u}{\partial y\,\partial t}+\delta^2\frac{\partial^2 v}{\partial x\,\partial t}-2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+(u\frac{\partial}{\partial x}$$
$$+v\frac{\partial}{\partial y})(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial v}{\partial y}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial u}{\partial x}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}).$$

$$\zeta_{xy} = 2(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{2k_0 C}{\eta_0 d}\delta(\frac{\partial^2 u}{\partial y\,\partial t}+\delta^2\frac{\partial^2 v}{\partial x\,\partial t}-2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})(\frac{\partial u}{\partial y}+$$
$$\delta^2\frac{\partial v}{\partial x})-\frac{\partial v}{\partial y}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial u}{\partial x}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}).$$

$$\zeta_{xy} = 2(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\bar{k}\,\delta(\frac{\partial^2 u}{\partial y\,\partial t}+\delta^2\frac{\partial^2 v}{\partial x\,\partial t}-2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})(\frac{\partial u}{\partial y}+$$
$$\delta^2\frac{\partial v}{\partial x})-\frac{\partial v}{\partial y}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-\frac{\partial u}{\partial x}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})-2\delta^2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}). \qquad ....(2-31)$$

From eq. (2-17):

$$\bar{\zeta}_{\bar{Y}\bar{Y}} = 4\eta_0 \frac{\partial \bar{V}}{\partial \bar{Y}} - 2k_0 (2\frac{\partial^2 \bar{V}}{\partial \bar{Y}\,\partial t} + 2(\bar{U}\,\frac{\partial^2 \bar{V}}{\partial \bar{X}\,\partial \bar{Y}} + \bar{V}\,\frac{\partial^2 \bar{V}}{\partial \bar{Y}^2}) - 4(\frac{\partial \bar{V}}{\partial \bar{Y}})^2 - 2\frac{\partial \bar{U}}{\partial \bar{Y}}(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}})$$

$$\frac{\eta_0 C}{d}\zeta_{yy} = 4\eta_0 \frac{C\,\delta}{d}\frac{\partial v}{\partial y} - 2k_0 (\frac{2C^2\delta}{\lambda d}\frac{\partial^2 v}{\partial y\,\partial t} + 2(Cu\,\frac{C\,\delta}{\lambda d}\frac{\partial^2 v}{\partial x\,\partial y} + C\,\delta v\,\frac{C\,\delta}{d^2}\frac{\partial^2 v}{\partial y^2}) - 4\frac{\delta^2 C^2}{d^2}$$
$$(\frac{\partial v}{\partial y})^2 - 2\frac{C}{d}\frac{\partial u}{\partial y}(\frac{C}{d}\frac{\partial u}{\partial y} + \frac{C\,\delta}{\lambda}\frac{\partial v}{\partial x})).$$

$$\frac{\eta_0 C}{d}\zeta_{yy} = \frac{4\eta_0 C}{\lambda}\frac{\partial v}{\partial y} - 2k_0 (2\frac{C^2}{\lambda^2}\frac{\partial^2 v}{\partial y\,\partial t} + 2(\frac{C^2}{\lambda^2}u\,\frac{\partial^2 v}{\partial x\,\partial y} + \frac{C^2}{\lambda^2}v\,\frac{\partial^2 v}{\partial y^2}) - 4\frac{C^2}{\lambda^2}(\frac{\partial v}{\partial y})^2 -$$
$$2\frac{C}{d}\frac{C}{d}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}))$$

$$\frac{\eta_0 C}{d}\zeta_{yy} = \frac{4\eta_0 C}{\lambda}\frac{\partial v}{\partial y} - 2k_0 (2\frac{C^2}{\lambda^2}\frac{\partial^2 v}{\partial y\,\partial t} + 2(\frac{C^2}{\lambda^2}u\,\frac{\partial^2 v}{\partial x\,\partial y} + \frac{C^2}{\lambda^2}v\,\frac{\partial^2 v}{\partial y^2}) - 4\frac{C^2}{\lambda^2}(\frac{\partial v}{\partial y})^2 -$$
$$2\frac{C^2}{d^2}\frac{\lambda^2}{\lambda^2}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})).$$

$$\frac{\eta_0 C}{d}\zeta_{yy} = \frac{4\eta_0 C}{\lambda}\frac{\partial v}{\partial y} - 2k_0\frac{C^2}{\lambda^2}(2\frac{\partial^2 v}{\partial y\,\partial t} + 2(u\,\frac{\partial^2 v}{\partial x\,\partial y} + v\,\frac{\partial^2 v}{\partial y^2}) - 4(\frac{\partial v}{\partial y})^2 - \frac{1}{\delta^2}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y}$$
$$+\delta^2\frac{\partial v}{\partial x})). \qquad ....(2\text{-}32)$$

Multiplying both sides of (2-32) by $(\frac{d}{\eta_0 C})$ implies to:

$$\zeta_{yy} = \frac{4C\eta_0}{\lambda}\frac{d}{C\eta_0}\frac{\partial v}{\partial y} - 2k_0\frac{C^2}{\lambda^2}\frac{d}{\eta_0 C}(2\frac{\partial^2 v}{\partial y\,\partial t} + 2(u\,\frac{\partial^2 v}{\partial x\,\partial y} + v\,\frac{\partial^2 v}{\partial y^2}) - 4(\frac{\partial v}{\partial y})^2 - \frac{2}{\delta^2}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y}$$
$$+\delta^2\frac{\partial v}{\partial x})).$$

$$\zeta_{yy} = 4\delta\frac{\partial v}{\partial y} - 2\frac{k_0 C}{\eta_0}\frac{d}{\lambda^2}\frac{d}{d}(2\frac{\partial^2 v}{\partial y\,\partial t} + 2(u\,\frac{\partial^2 v}{\partial x\,\partial y} + v\,\frac{\partial^2 v}{\partial y^2}) - 4(\frac{\partial v}{\partial y})^2 - \frac{2}{\delta^2}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})).$$

$$\zeta_{yy} = 4\delta\frac{\partial v}{\partial y} - 2\frac{k_0 C}{\eta_0 d}\delta^2(2\frac{\partial^2 v}{\partial y\,\partial t} + 2(u\,\frac{\partial^2 v}{\partial x\,\partial y} + v\,\frac{\partial^2 v}{\partial y^2}) - 4(\frac{\partial v}{\partial y})^2 - \frac{2}{\delta^2}\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})).$$

Which can be written as

$$\zeta_{yy} = 4\delta \frac{\partial v}{\partial y} - 2\overline{K}(\delta^2(2\frac{\partial^2 v}{\partial y\,\partial t} + 2(u\frac{\partial^2 v}{\partial x\,\partial y} + v\frac{\partial^2 v}{\partial y^2}) - 4(\frac{\partial v}{\partial y})^2) - 2\frac{\partial u}{\partial y}(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x})).$$

$$\zeta_{yy} = 4\delta \frac{\partial v}{\partial y} - 2\overline{K}[\delta^2(2\frac{\partial^2 v}{\partial y\,\partial t} + 2u\frac{\partial^2 v}{\partial x\,\partial y} + 2v\frac{\partial^2 v}{\partial y^2} - 4(\frac{\partial v}{\partial y})^2 - 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}) - 2(\frac{\partial u}{\partial y})^2] \qquad ....(2-33)$$

Now, under the assumption of long wave length $(\delta << 1)$ and low Reynolds number, the Eqs. (2-25), (2-27) can be written as:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\zeta_{xy} - (M^2 + k^2)u \qquad ....(2-34)$$

$$\frac{\partial p}{\partial y} = 0 \qquad ....(2-35)$$

Introduce the stream function $u = \frac{\partial \psi}{\partial y}, v = \frac{-\partial \psi}{\partial x}$ in eqs. (2-31), (2-34) we have:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\zeta_{xy} - (M^2 + k^2)\frac{\partial \psi}{\partial y} \qquad ....(2-36)$$

$$\zeta_{xy} = 2(\frac{\partial^2 \psi}{\partial y^2} - \delta^2\frac{\partial^2 \psi}{\partial x^2}) - 2\overline{K}(\delta\frac{\partial^3 \psi}{\partial y^2\partial t} - \delta^3\frac{\partial^3 \psi}{\partial x^2\partial t} - 2\delta\frac{\partial^2 \psi}{\partial x\,\partial y}\frac{\partial^2 \psi}{\partial y^2} + \delta\frac{\partial \psi}{\partial y}\frac{\partial^3 \psi}{\partial x\,\partial y^2} -$$
$$\delta^3\frac{\partial \psi}{\partial y}\frac{\partial^3 \psi}{\partial x^3} - \delta\frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial y^3} + \delta^3\frac{\partial \psi}{\partial x}\frac{\partial^3 \psi}{\partial x^2\partial y} - 2\delta^3\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial x\,\partial y}) \qquad ....(2-37)$$

2-6 Rate of Volume Flow and Boundary Conditions

In laboratory frame, the dimensional volume flow rate is:[58]

$$Q(\overline{X},\bar{t}) = \int_{\overline{H_1}(\overline{X},\bar{t})}^{\overline{H_2}(\overline{X},\bar{t})} \overline{U}(\overline{X},\overline{Y},\bar{t})d\overline{Y}, \qquad ....(2-38)$$

In which $\overline{H}_1$ and $\overline{H}_2$ are functions of $\overline{X}$ and $\bar{t}$. In wave frame, the dimensional volume flow rate is

$$q = \int_{\overline{H_1}(\overline{X})}^{\overline{H_2}(\overline{X})} \overline{U}(\overline{X},\overline{Y})d\overline{Y}, \qquad ....(2-39)$$

If we introduce the wave frame having coordinates $(\overline{X}, \overline{Y})$ which travel in the $\overline{X}$-direction with the same wave velocity C. Then the unsteady flow in the laboratory frame $(\overline{X}, \overline{Y})$ can be treated as steady. The coordinates and velocities in the two frames are related by:

$$\overline{x} = \overline{X} - C\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u}(\overline{x}, \overline{y}) = \overline{U}(\overline{X}, \overline{Y}, \overline{t}), \quad \overline{v}(\overline{x}, \overline{y}) = \overline{V}(\overline{X}, \overline{Y}, \overline{t}) \qquad ....(2\text{-}40)$$

Substituting Eq. (2-40) in Eq. (2-38) we obtain:

$$Q = q + C\overline{H}_2 - C\overline{H}_1, \qquad ....(2\text{-}41)$$

The time averaged flow over a period $(T_2 = \lambda / C)$ at a fixed position $\overline{X}$ is defined as:

$$\overline{Q} = \frac{1}{T_2} \int_0^{T_2} Q\,d\overline{t}, \qquad ....(2\text{-}42)$$

If we substitute Eq. (2-41) into (2-42) and by integration, we get:

$$\overline{Q} = q - a_2 c \sin[\frac{2\pi}{\lambda}(\overline{x} - \overline{ct})] - a_1 c \sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t}) + \phi] \qquad ....(2\text{-}43)$$

If we find the dimensionless mean flow F, in the laboratory frame and θ, in the wave frame, according to: $F = \frac{\overline{Q}}{-cd}, \; \theta = \frac{q}{-cd}$ one can find eq.(2-43) to be:

$$F(x,t) = \theta + a\sin[2\pi(x - t) + \phi] + b\sin[2\pi(x - t)] \qquad ....(2-44)$$

In which,

$$F = \int_{h_1(x)}^{h_2(x)} \frac{\partial\psi}{\partial y} dy = \psi(h_2) - \psi(h_1) \qquad ....(2-45)$$

Here it is pointed out that the conditions on ψ satisfy Eq.(2-45) and the conditions on $\partial\psi / \partial y$ are no-slip.

Selecting $\psi(h_2)=\frac{F}{2}$, then we have $\psi(h_1)=\frac{-F}{2}$. The boundary conditions in dimensionless stream function will take the following form:

$$\psi=\frac{F}{2},\ \frac{\partial\psi}{\partial y}=0,\ at\ \left(y=h_2\right)$$
$$\psi=\frac{-F}{2},\ \frac{\partial\psi}{\partial y}=0,\ at\ \left(y=h_1\right) \qquad ....(2-46)$$

In which

$$h_2=1+mx+b\sin(2\pi(x-t))and$$
$$h_1=-1-mx-a\sin(2\pi(x-t)+\phi)$$

the non-dimensional expression for the average rise in pressure Δp is given as follows:

$$\Delta p=(\int_0^1\int_0^1\frac{\partial p}{\partial x}dxdt\ )_{y=0} \qquad ....(2-47)$$

<u>2-7 Perturbation Analysis of the problem</u>

It is clear that from the resulting equation of motion Eq. (2-35), p is independent of y and the Eq. (2-36) is nonlinear. It seems to be impossible to obtain the general solution in closed form for orbitary values of all parameters arising in this nonlinear equation. We seek the solution of the problem as a power series expansion in terms of small parameter δ . (regular perturbation technique), thus we expand $\psi, F, \zeta_{xy}\ and$ p as follows:

$$\psi=\psi_0+\delta\psi_1+............$$
$$F=F_0+\delta F_1+.............$$
$$\zeta_{xy}=(\zeta_{xy})_0+\delta(\zeta_{xy})_1+..........$$
$$p=p_0+\delta p_1+........ \qquad ....(2-48)$$

Now substituting Eq. (2-48) back into Eqs. (2-36) and (2-37), (2-44) and (2-46). Thus we get:

$$(\frac{\partial p_0}{\partial x}+\delta\frac{\partial p_1}{\partial x})=\frac{\partial}{\partial y}[(\zeta_{xy})_0+\delta(\zeta_{xy})_1]-(M^2+K^2)(\frac{\partial \psi_0}{\partial y}+\delta\frac{\partial \psi_1}{\partial y}).$$

That is

$$\begin{aligned}(\zeta_{xy})_0+\delta(\zeta_{xy})_1 &= 2[(\frac{\partial^2\psi_0}{\partial y^2}+\delta\frac{\partial^2\psi_1}{\partial y^2})-\delta^2(\frac{\partial^2\psi_0}{\partial x^2}+\delta\frac{\partial^2\psi_1}{\partial x^2})-2\overline{K}[\delta(\frac{\partial^3\psi_0}{\partial y^2\partial t}+\delta\frac{\partial^3\psi_1}{\partial y^2\partial t})\\ &-\delta^3(\frac{\partial^3\psi_0}{\partial x^2\partial t}+\delta\frac{\partial^3\psi_1}{\partial x^2\partial t})-2\delta(\frac{\partial^2\psi_0}{\partial x\partial y}+\delta\frac{\partial^2\psi_1}{\partial x\partial y}).(\frac{\partial^2\psi_0}{\partial y^2}+\delta\frac{\partial^2\psi_1}{\partial y^2})+\delta(\frac{\partial\psi_0}{\partial y}+\delta\frac{\partial\psi_1}{\partial y})\\ &(\frac{\partial^3\psi_0}{\partial x\partial y^2}+\delta\frac{\partial^3\psi_1}{\partial x\partial y^2})-\delta^3(\frac{\partial\psi_0}{\partial y}+\delta\frac{\partial\psi_1}{\partial y}).(\frac{\partial^3\psi_0}{\partial x^3}+\delta\frac{\partial^3\psi_1}{\partial x^3})-\delta(\frac{\partial\psi_0}{\partial x}+\delta\frac{\partial\psi_1}{\partial x})(\frac{\partial^3\psi_0}{\partial y^3}\\ &+\delta\frac{\partial^3\psi_1}{\partial y^3})+\delta^3(\frac{\partial\psi_0}{\partial x}+\delta\frac{\partial\psi_1}{\partial x}).(\frac{\partial^3\psi_0}{\partial x^2\partial y}+\delta\frac{\partial^3\psi_1}{\partial x^2\partial y})-2\delta^3(\frac{\partial^2\psi_0}{\partial x^2}+\delta\frac{\partial^2\psi_1}{\partial x^2})(\frac{\partial^2\psi_0}{\partial x\partial y}\\ &+\delta\frac{\partial^2\psi_1}{\partial x\partial y})] \qquad \text{.........}(2-49)\end{aligned}$$

Now, collecting the coefficient of like power of δ, thus one can get the zeroth and first order equations as:

<u>2-7-1 Zero's- order system</u> $(\delta^{(0)})$

$$\frac{\partial p_0}{\partial x}=\frac{\partial}{\partial y}(\zeta_{xy})_0-N_1^2\frac{\partial\psi_0}{\partial y} \qquad \text{.........}(2-50)$$

$where \quad N_1^2=(M^2+k^2);$

$$(\zeta_{xy})_0=2\frac{\partial^2\psi_0}{\partial y^2} \qquad \text{.........}(2-51)$$

Differentiating eq. (2-50) with respect to y implies to:

$$0=\frac{\partial^2}{\partial y^2}(\zeta_{xy})_0-N_1^2\frac{\partial^2\psi_0}{\partial y^2}$$

Which can be written as:

$$0=2(\frac{\partial^4\psi_0}{\partial y^4})-N_1^2\frac{\partial^2\psi_0}{\partial y^2} \qquad \text{.........}(2-52)$$

With the corresponding boundary conditions:

$$\psi_0 = \frac{F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, \; at\left(y = h_2\right)$$

$$\psi_0 = \frac{-F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, \; at\left(y = h_1\right) \qquad ....(2-53)$$

2-7-2 First order system $(\delta^{(1)})$

$$\frac{\partial p_1}{\partial x} = \frac{\partial}{\partial y}(\zeta_{xy})_1 - N_1^{\,2}\frac{\partial \psi_1}{\partial y} \qquad ....(2-54)$$

$$(\zeta_{xy})_1 = 2\frac{\partial^2 \psi_1}{\partial y^2} - 2\overline{K}\left[\frac{\partial \psi_0}{\partial y}\frac{\partial^3 \psi_0}{\partial x \partial y^2} - \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_0}{\partial y^3} - 2\frac{\partial^2 \psi_0}{\partial x \partial y}\frac{\partial^2 \psi_0}{\partial y^2}\right] \qquad ....(2-55)$$

Differentiating eq. (2-54) with respect to y, we have:

$$0 = 2\frac{\partial^4 \psi_1}{\partial y^4} - N_1^{\,2}\frac{\partial^2 \psi_1}{\partial y^2} - 2\overline{K}\frac{\partial^2}{\partial y^2}\left[\frac{\partial \psi_0}{\partial y}\frac{\partial^3 \psi_0}{\partial x \partial y^2} - \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_0}{\partial y^3} - 2\frac{\partial^2 \psi_0}{\partial x \partial y}\frac{\partial^2 \psi_0}{\partial y^2}\right] \qquad ....(2-56)$$

The corresponding boundary conditions are:

$$\psi_1 = \frac{F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, \; at\left(y = h_2\right)$$

$$\psi_1 = \frac{-F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, at\left(y = h_1\right) \qquad ....(2-57)$$

2-8 Solution of the Problem

In this section we have given the solution of the zero and first order systems:

2-8-1 Solution for the zeroth order system $(\delta^{(0)})$

It is found that the solution of equation (2-52) under the associated boundary condition (2-53) is given by:

$$\psi_0 = n_2 e^{n_1 y} a_1 + n_2 e^{-n_1 y} a_2 + a_3 + a_4 y \qquad ....(2-58)$$

$$where\,(n_1 = \frac{N_1}{\sqrt{2}}; n_2 = \frac{2}{N_1^{\,2}});$$

$a_i, (i = 1, 2, 3, 4)$ are constants can be obtained by using the boundary conditions in Eq.(2-53).

2-8-2 Solution of the first order system $(\delta^{(1)})$

If we solve the equation (2-56) under the associated boundary conditions (2-57) we can find the solution of the first order system as follows:

$$\psi_1 = \frac{e^{n_1 y}}{n_1^2}c_1 + \frac{e^{-n_1 y}}{n_1^2}c_2 + c_3 + c_4 y + \frac{\begin{aligned}&(e^{n_1 y} F_0^2 n_1^2 (2e^{(h_1+h_2)n_1}(-(h_1-h_2)h_6 n_1(-5+\\ &2n_1 y) + h_4(23-5h_5 n_1 - 6n_1 y + 2h_5 n_1^2 y - 2n_1^2 y + h_2 n_1(5-2n_1 y) + h_1 n_1(-5+2n_1 y)) +\\ &h_3(-23+5h_5 n_1 + 6n_1 y - 2h_5 n_1^2 y + 2n_1^2 y + h_2 n_1(5-2n_1 y) + h_1 n_1(-5+2n_1 y))) + e^{2h_1 n_1}\\ &(-h_6(-2+h_1 n_1 - h_2 n_1)(-5+2n_1 y) + h_4(-13+2n_1 y + 2n_1^2 y^2 + h_5 n_1(5-2n_1 y)) + h_3\\ &(33-5h_5 n_1 - 10 n_1 y + 2h_5 n_1^2 y + 2n_1^2 y^2 + 2h_2 n_1(5-2n_1 y) + 2h_1 n_1(-5+2n_1 y))) + e^{2h_2 n_1}\\ &(-h_6(2+h_1 n_1 - h_2 n_1)(-5+2n_1 y) + h_4(-33+2n_1^2 y^2 + 5h_5 n_1 + 10 n_1 y - 2h_5 n_1^2 y +\\ &2h_2 n_1(5-2n_1 y) + 2h_1 n_1(-5+2n_1 y) + h_3(13-2n_1 y - 2n_1^2 y^2 + h_5 n_1(5-2n_1 y)))\overline{K}\end{aligned}}{(8(e^{h_1 n_1}(-2+h_1 n_1 - h_2 n_1) + e^{2h_2 n_1}(2-h_1 n_1 - h_2 n_1))^3)} \qquad \text{.........(2-59)}$$

where

$h_3 = -m - 2a\pi\cos(2\pi(x-t)+\phi);$

$h_4 = m + 2b\pi\cos(2\pi(x-t));$

$h_5 = b\sin(2\pi(x-t)) - a\sin(2\pi(x-t)+\phi);$

$h_6 = 2b\pi\cos(2\pi(x-t)) - 2a\pi\cos(2\pi(x-t)+\phi);$

And c_1, c_2, c_3, c_4 are constants can be determinates by using the boundary conditions in Eq. (2-57) and software of "MATHEMATICA" program.

2-9 Results and Discussion

In this section, the numerical and computational results are discussed for the problem of peristaltic transport of an incompressible non Newtonian Walter's-B fluid under the effect of normal magnetic field through porous medium in a tapered asymmetric channel with the help of using non-slip conditions. Analytical results are shown by using regular perturbation technique for small value of wave number δ under the assumption of long wave length and low Reynolds number approximations, and using series for stream functions, axial velocity, pressure gradient and mean flow rate F. The effects of some important various parameters are displayed graphically.

2-9-1 Pumping characteristics

We plot the expression for Δp in Eq. (2-47) against θ for various values of parameters of interest in Figs. (2-2)- (2-7). Numerical calculations for several values of Hartmann number (M), the phase difference (ϕ), the non- uniform parameter of the channel (m), porosity parameter (K), the amplitudes of upper and lower walls ($a \& b$) have been carried out. The effect of these parameters on Δp have been evaluated numerically using "MATHEMATICA" programm and the results are presented graphically. In fig. (2-2), the effect of Hartmann number M on Δp are seen, observed that in the pumping $\Delta p > 0$ and the co-pumping ($\Delta p < 0$)for the Walters-B fluid, an increase in M causes decreasing in the pumping $\Delta p > 0$ and increasing in pumping $\Delta p < 0$. In Fig. (2-3), the effect of phase difference ϕ on Δp is showed, observed that an increase in ϕ causes increasing in the co-pumping ($\Delta p < 0$) and decreasing in the pumping $\Delta p > 0$. The effects of non-uniform parameter m as well as the amplitude of lower wall of channel a are plotted respectively in Figs.(4) and (5), it examined that an increase in m and a causes an increase in the pumping $\Delta p > 0$ and decrease in the free pumping $\Delta p = 0$ and co-pumping $\Delta p < 0$. The influence of amplitude of upper wall of channel b and porosity parameter K on Δp are illustrated respectively in Figs.(6) and (7), it noticed that there is rise up in the pumping $\Delta p > 0$ and free pumping $\Delta p = 0$ and the pumping will be reduce in the region of $\Delta p < 0$ with an increase of privous parameters.

2-9-2 Velocity distribution

Influences of geometric parameters on the velocity distribution have been illustrated in Figs.(2-8)-(2-17), these figures are scratched at the fixed values of x=0.3, the change in values of m on the axial velocity u is shown in fig.(2-8), it can be found that the axial velocity u decrease with an increase in m at the center of channel but after y=0.6, y=-0.7 of the upper and lower parts of the channel respectively, the velocity u will be increased. Fig.(2-9) shows the influence of ϕ on the axial velocity u, it observed that an increase in ϕ causes an increase in magnitude of u at the core and walls of the channel opposite behavior is seen for the effect of a on the axial velocity u, it which is plotted in .Fig.(2-10), (2-11) illustrated the influence of b on the axial velocity u, it is examined that an increase in b results an increase in u at the center and walls of channel , but after y=0.5, The velocity will be reduce at the upper wall of channel. The influence of M on the axial velocity u is shown in Fig. (2-12), it noticed that an increase in M yield an decrease in u at the center of channel, but the flow of fluid will be inflected after the values of y=o.4, y=-o.5 at the upper and lower sides of the channel and so the velocity will be increased. Similar behavior is shown for the effect of k on the axial velocity, and its effects is plotted in figure (2-13), we can say that the reason behind this behavior is due to the obstruction that is obtained by the porosity parameter, also because of resistive nature of the Lorentz force when the magnetic field of strength B_0 is applied in the normal direction of the flow fluid. Figure (2-14) give the impact of volume flow rate θ on the velocity, which in turn increase the amount of velocity at all regions of flow. Conversely conduct is observed for the effect of t on the velocity of fluid and it is noted it's graph in figure (2-15). In figure (2-16), the impact of perturbation parameter (δ) is noticed, it is examined that the fluids flow will be increase at center but it is decreased after y=0.6 and y=-0.6 of both sides of channel, which can say that the velocity of fluid in the non-Newtonian case is much more that in new case of the fluid. Like manar is showed for the effect of $\overline{K}$ on the axial velocity, its graph can be seen in figure (2-17).

2-9-3 Trapping phenomenon

The phenomenon of trapping is another interesting topic in peristaltic transport. The formation of an internally circulating bolus of fluid through closed stream lines is called trapping and this trapped bolus is pushed a head along with the peristaltic waves. The trapping for different values of m,ϕ,a, b, M, K and θ are shown in Figs.(2-18)-(2-27). The stream lines for different values of m are shown in Fig.(2-18), it has been noticed that the bolus decreasing in size in the lower and upper wall of the tapered channel with increasing m. the streams for different values of ϕ are shown in fig.(2-19), it is examined that the size of bolus increase with an increase of ϕ. Effect of a are shown in fig.(2-20), it is noticed that the size of bolus reduced in the lower and upper part of channel with an increase of a, but the wobbling impact is shown on internal bolus. Figs. (2-21) and (2-22) shows the effects of b and θ

respectively and it is observed that the size and number of trapping bolus increase with an increase of these parameters. The influences of M and K are plotted in figures (2-23) and (2-24) respectively, an increase in the magnitude of these last parameters results small size and number of bolus in both parts of channel. Opposite behavior is noted for the effects of $\overline{K}$ and $\overline{\delta}$ and their graph are noticed in figs. (2-25) and (2-26) respectively. Figure (2-27) give the behavior of parameter t on bolus, which is showed that the bolus is unchanged in shape with an increase of t, we can explain this case because of steady treatment.

2-10 Concluding Remark

In this chapter, we investigated the peristaltic transport of Walters –B fluid through a porous medium in a tapered a symmetric channel under the influence of magnetic field. The channels a symmetry is produced by choosing the peristaltic waves train on the non- uniform walls to have different amplitudes and phases, along-wave length and low Reynolds number approximations are adopted. A regular perturbation method is employed to obtain the expression for stream function, axial velocity and pressure gradient. Numerical study has been conducted for average rise in pressure over a wave length. The effects of Hartmann number (M), porosity parameter (k), wave amplitudes (a &b), non-uniform parameter (m) and phase angle ϕ on the pressure rise, axial velocity and stream lines are also investigated in detail. It found that:

1. The pressure rise over a wave length Δp increase with an increase of m, a in the pumping $\Delta p > 0$ while the situation is reversed in the free pumping $\Delta p = 0$ and Co-pumping $\Delta p < 0$

2. The pressure rise over a wave length Δp increase with an increase in b, k in the pumping $\Delta p > 0$ and free pumping $\Delta p = o$ while the situation is reversed in the Co-pumping $\Delta p < 0$

3. The pressure rise over a wave length Δp increase with an increase of M in the pumping $\Delta p = 0$ while the situation is conversely in the pumping $\Delta p > 0$ and Co-pumping $\Delta p < 0$

4. The pressure rise over a wave length Δp increase with an increase in ϕ in the Co-pumping $\Delta p < 0$ and it's conversely in the pumping $\Delta p > 0$.

5. The lines of pressure rise against mean volume rate θ is intersected lines.

6. The relation between pressure rise and volume flow rate θ is some what to be linear by the effect of M and to be non-linear by the effect of a, m, k, b, ϕ.

7. The axial velocity u increased at all regions of flow with an increase of ϕ and θ but the case is conversed with an increase of a, t.

8. The axial velocity u increase at the center of channel with an increase of $\delta, \overline{K}, b$ but the flow is reflected at the walls of channel. Opposite behavior is noted with an increase of m, k, and M.

9. The size of trapping bolus increased with an increase of $\delta, \overline{K}$ and ϕ but they have small volume with an increase of m and M.

10. The number and size of bolus is rise up with an increase of b and θ but the conversely statement is seen with an increase of k.

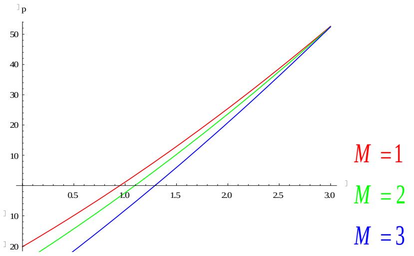


Fig.(2-2).Effect of Hartmann number M on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3$

$b = 0.2, \delta = 0.001, \bar{k} = 2, K = 1$

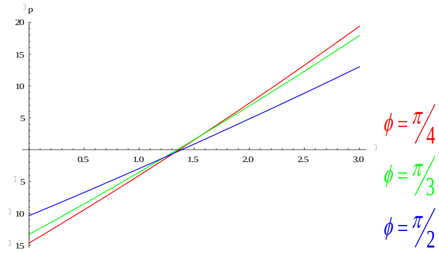


Fig.(2-3). Effect of phase difference ϕ on Δp

$m = 0.4, t = 0.5,, a = 0.2, b = 0.1,$

$M = 3, \delta = 0.001, \bar{k} = 2, K = 1$

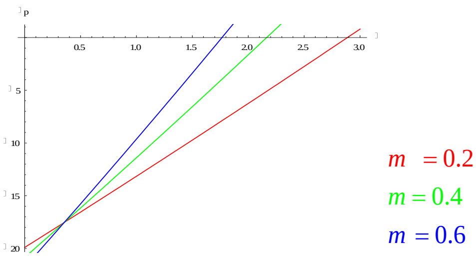


Fig.(2-4) Effect of non-uniform parameter m on Δp

$t = 0.5, \phi = \frac{\pi}{2}, a = 0.2, b = 0.1,$

$M = 5, \delta = 0.001, \bar{k} = 2, K = 1$

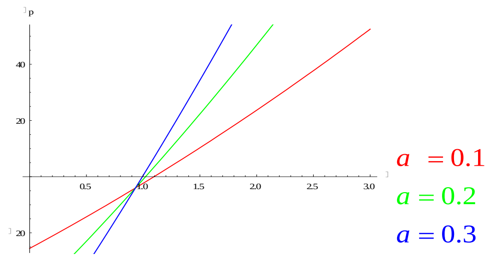


Fig.(2-5) Effect of the amplitude of lower wall of channel a on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.2$

$M = 2, \delta = 0.001, \bar{k} = 2, K = 1$

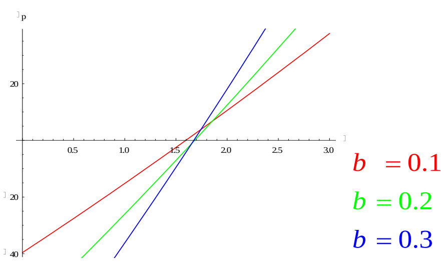


Fig.(2-6) Effect of amplitude of upper wall of channel b on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$M = 5, \delta = 0.001, \bar{k} = 2, K = 1$

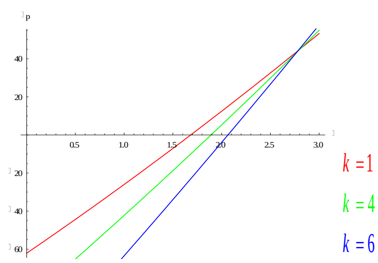


Fig.(2-7) Effect of the inverse of porosity parameter K on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, M = 5, \delta = 0.001, \bar{k} = 2,$

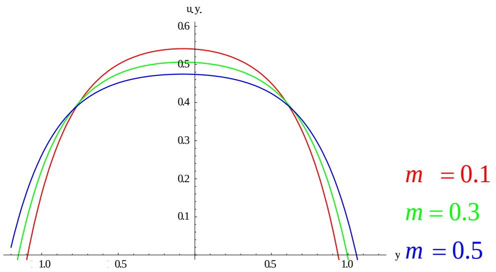


Fig.(2-8) Effect of non –uniform parameter m on axial velocity u(y)

$$t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 5,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$

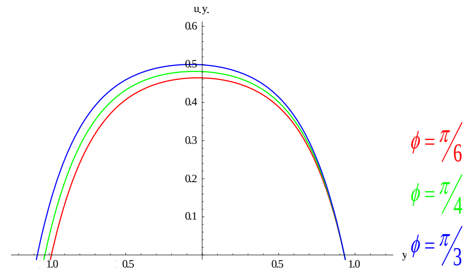
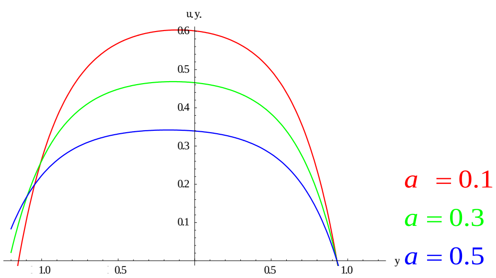


Fig.(2-9) Effect of phase difference on the axial velocity on u(y)

$$m = 0.4, t = 0.5, a = 0.2, b = 0.2, M = 5,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$



Fig(2-10) Effect of amplitude of lower wall of channel (a) on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, b = 0.2, M = 5,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$

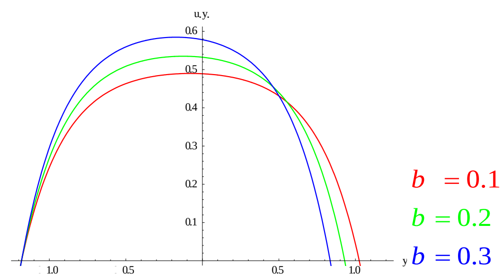


Fig.(2-11) Effect of amplitudes of upper wall channel (b) on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, M = 5,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$

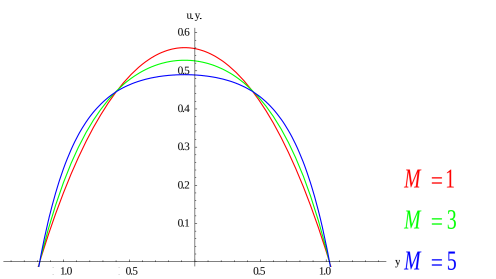
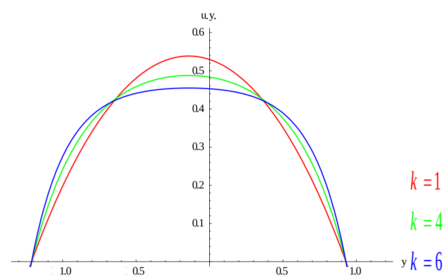


Fig.(2-12) Effect of Hartmann number M on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$



Fig(2-13) Effect of the inverse of the Dercy number K on axial velocity u(y).

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2,$$
$$M = 0.1, \delta = 0.0001, \bar{k} = 2,, \theta = 1, x = 0.3$$

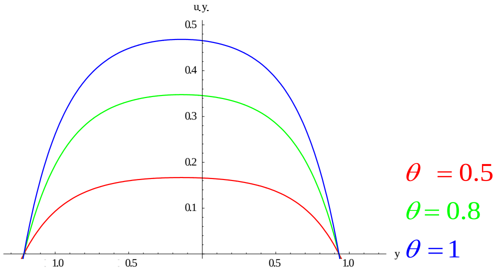


Fig.(2-14) Effect of the time average θ on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2,$$
$$M = 5, \delta = 0.0001, \bar{k} = 2, K = 1, x = 0.3$$

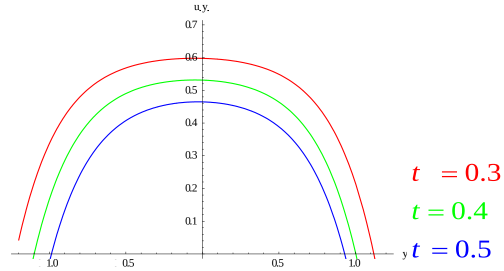


Fig.(2-15) Effect of the time average (t) on axial velocity u(y)

$$m = 0.4, \phi = \pi/6, a = 0.2, b = 0.2, M = 5,$$
$$\delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$

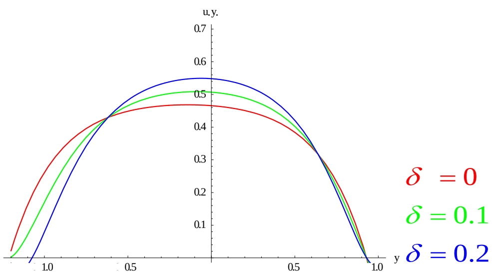


Fig.(2-16) Effect of the time average (δ) on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2,$$
$$M = 5, , \bar{k} = 2, K = 1, \theta = 1, x = 0.3$$

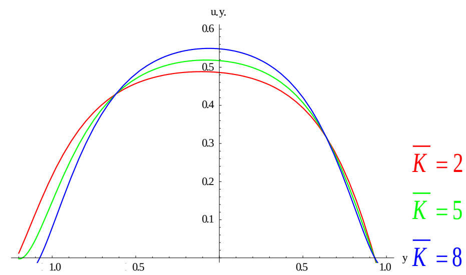


Fig.(2-17) Effect of the time average ($\bar{K}$) on axial velocity u(y)

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2,$$
$$\delta = 0.05, M = 5, , K = 1, \theta = 1, x = 0.3$$

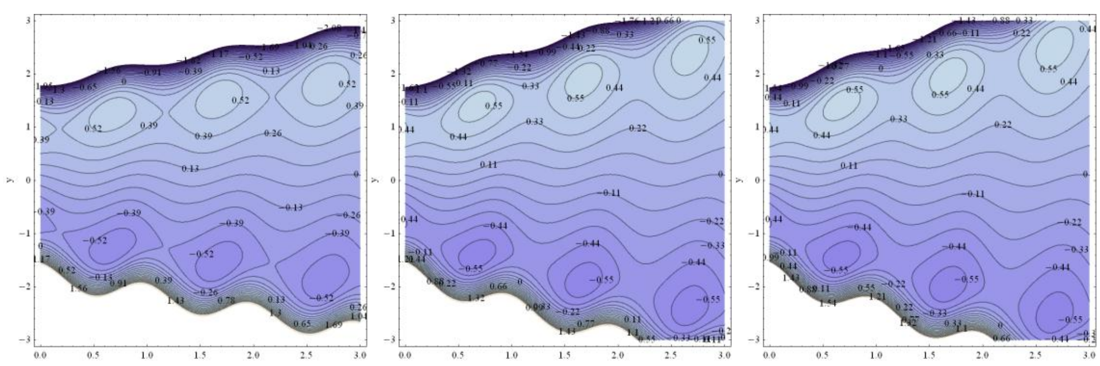


Fig.(2-18) :Stream lines for

$$t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 5, \delta = 0.0001, \bar{k} = 2, K = 1, \theta = 1$$
$$(a) m = 0.3, (b) m = 0.5, (c) m = 0.55$$

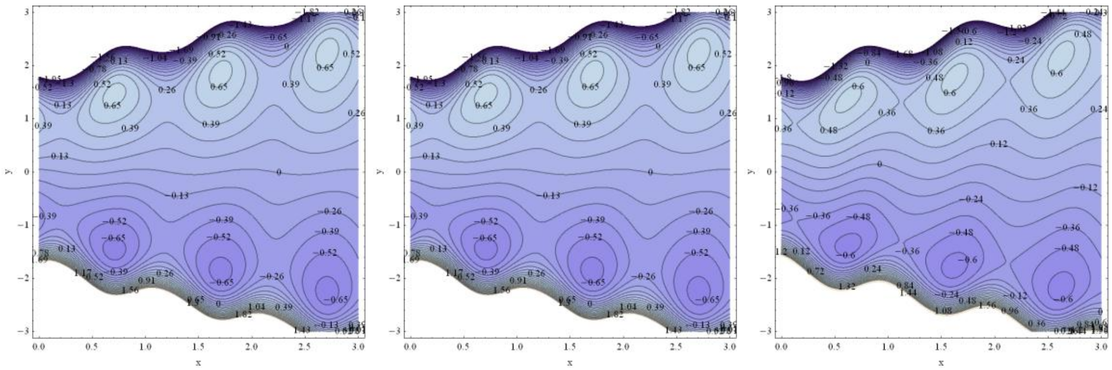


Fig.(2-19) :Stream lines for

$$m = 0.4, t = 0.5, a = 0.2, b = 0.2, M = 5, \delta = 0.0001, \overline{k} = 2, K = 1, \theta = 1$$

$$(a)\phi = \pi/6, (b)\phi = \pi/4, (c)\phi = \pi/2$$

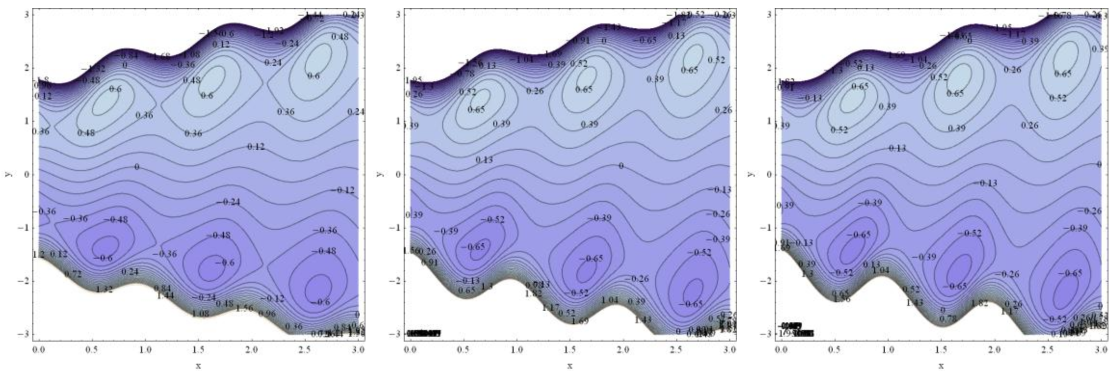


Fig.(2-20) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, b = 0.2, M = 5, \delta = 0.0001, \overline{k} = 2, K = 1, \theta = 1$$

$$(a)a = 0.2, (b)a = 0.3, (c)a = 0.35$$

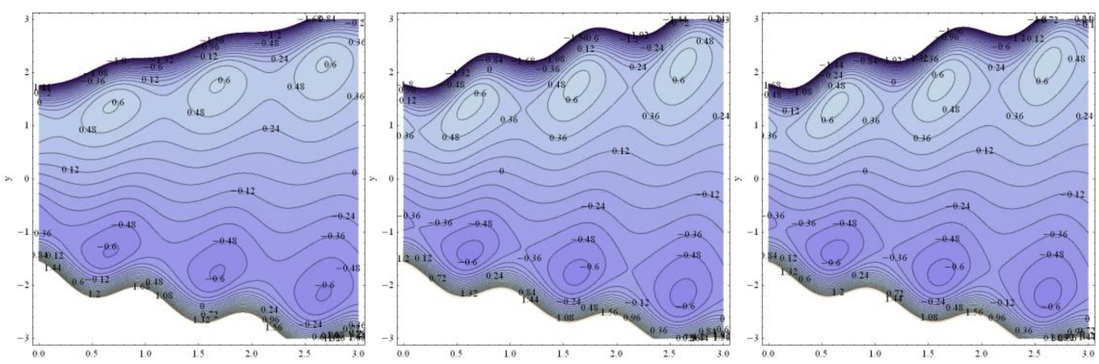


Fig.(2-21) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, M = 5, \delta = 0.0001, \overline{k} = 2, K = 1, \theta = 1$$

$$(a)b = 0.1, (b)b = 0.2, (c)b = 0.21$$

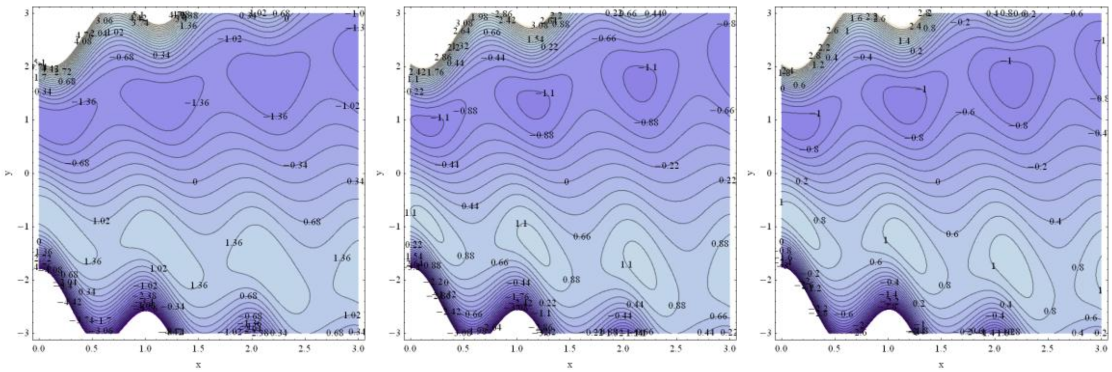


Fig.(2-22) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2, M = 1, \delta = 0.0001, \overline{k} = 2, K = 1$$

$$(a)\theta = -3, (b)\theta = -2, (c)\theta = -1$$

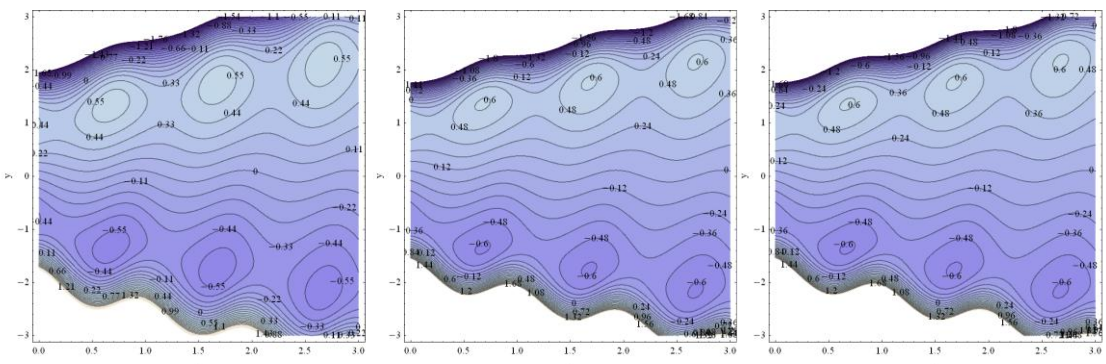


Fig.(2-23) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, \delta = 0.0001, \overline{k} = 2, K = 1, \theta = 1$$

$$(a)M = 3, (b)M = 5, (c)M = 5.1$$

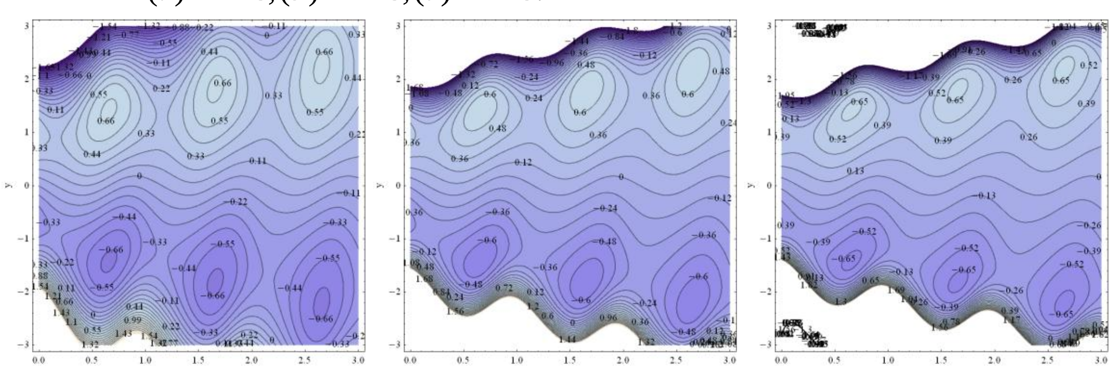


Fig.(2-24) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2, M = 0.1, \delta = 0.0001, \overline{k} = 2, \theta = 1$$

$$(a)K = 1, (b)K = 4, (c)K = 6$$

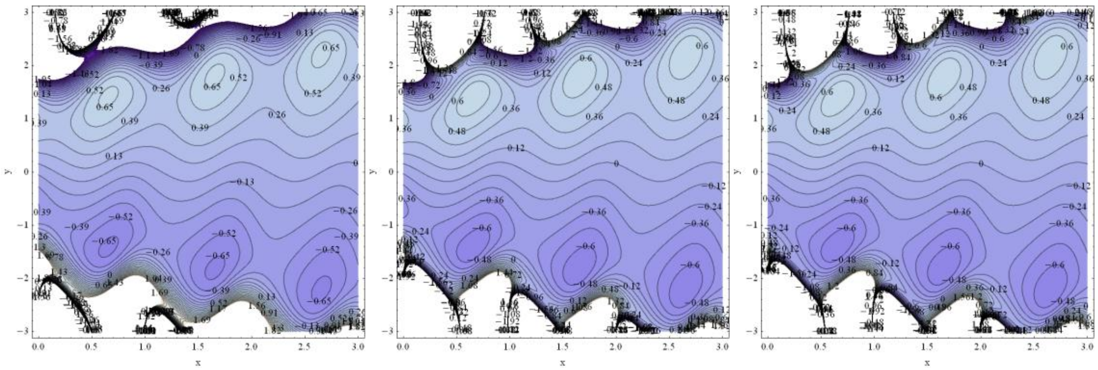


Fig.(2-25) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2, M = 5, \delta = 0.01, k = 1, \theta = 1$$
$$(a)\overline{K} = 0.5, (b)\overline{K} = 2, (c)\overline{K} = 2.5$$

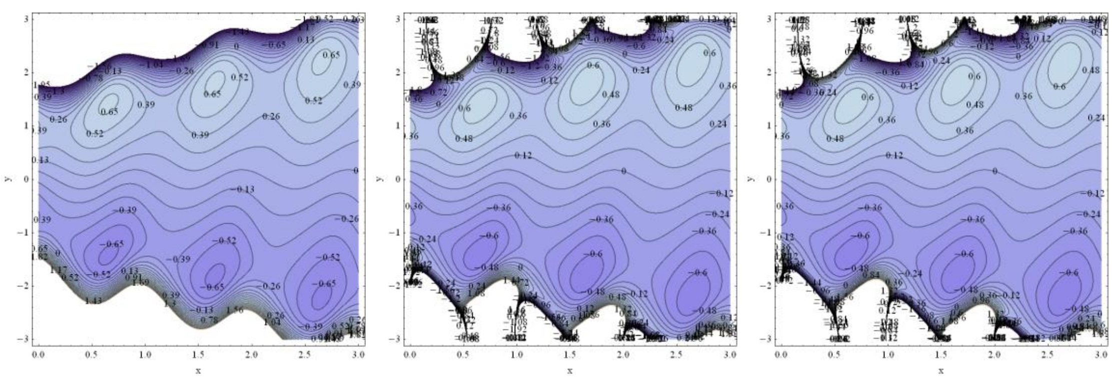


Fig.(2-26) :Stream lines for

$$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.3, b = 0.2, M = 5, \overline{k} = 2, K = 1, \theta = 1$$
$$(a)\delta = 0, (b)\delta = 0.01, (c)\delta = 0.012$$

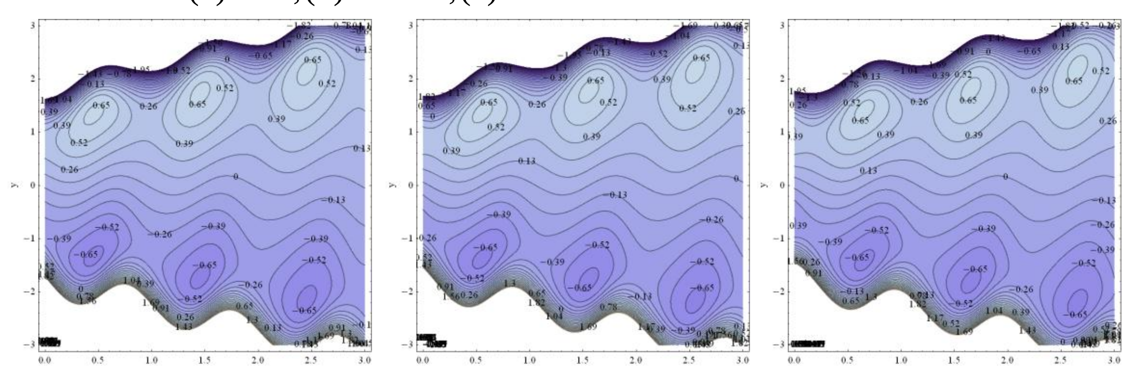


Fig.(2-27) :Stream lines for

$$m = 0.4, \phi = \pi/2, a = 0.3, b = 0.2, M = 5, \delta = 0.0001, \overline{k} = 2, K = 1, \theta = 1$$
$$(a)t = 0.3, (b)t = 0.4, (c)t = 0.5$$

Chapter Three

Peristaltic Transport of MHD Flow of Blood and Heat Transfer in a Tapered Asymmetric Channel Through Porous Medium: Effect of Variable Viscosity, Velocity-Non Slip and Temperature-Non Slip

Introduction

Peristalsis is now well known to physiologists to be one of the major mechanism for fluid transport in many biological systems. In the living systems peristalsis is the mechanism to propel food stuffs through esophagus and the vasomotion of small blood vessels. Engineers developed pumps having industrial and physiological applications adapting the principle of peristalsis. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pumps internal surfaces. The problem of the mechanism of peristaltic transport has attracted. The attention of many investigators and the first investigation of Latham [61]. The fundamental studies on peristaltic transport were performed by fung and Yih[34] using laboratory frame of reference. And then by Shapiro et al.[96] using wave frame of references. A number of analytical, numerical and experimental studies of peristaltic flows of different fluids have been reported by [21, 109, 110, 104, 106, 107, 108], the peristaltic fluid flow through channels with flexible walls has been studied by Ravi Kumar et al [86, 87].

Peristalsis is an important physiological mechanism for mixing and transporting fluids, which is generated by a progressive wave of area contraction or expansion moving on the wall of the tube containing fluid. The peristaltic fluid flows involving Newtonian and non-Newtonian fluids have been studied by [111, 72, 14, 38, 7, 39] and others. The magneto hydrodynamic (MHD) flow of the fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids e.g. The blood and blood pump machines, and with the need for theoretical research on the operation of peristaltic MHD compressor. Blood is regarded as a suspension of small cells in plasma. Moreover, it is known that in blood flows in two layers, arteries, a core layer and the plasma layer near the wall consisting of suspension of cells in the plasma. The red blood cells, which contain iron, are magnetic in nature, the core may be treated as magnetic field. Abd El Hakeem et al [8] has been studied by effects of a magnetic field on trapping through peristaltic motion for generalize Newtonian fluid in a channel. Non-linear peristaltic flow of a non-Newtonian fluid has been studied by [51, 47]. Recently, the study of (MHD) flow of electrically conducting fluids on peristaltic motion has become a subject of growing interest for researchers and clinicians. This is due to the fact that such studies are useful particularly for pumping of blood and magnetic resonance imaging. Theoretical work of Agarwal and Anwaruddin [11] explored the effect of magnetic field on the flow of blood in atherosclerotic vessels of blood pump during cardiac operations ALi et al. [15] observed that an impulsive magnetic field can be used for a therapeutic treatment of patients who have stone fragments in their urinary tract. Many authors [23, 77] suggested the presence of red blood cell slip at the vessel wall. Misra and Kar [70] solved the problem of blood flow through a stenos vessel by taking into considering the slip velocity at the wall by using the momentum integral technique. While flowing through the arterial tree, blood carries a large quantity of heat to different parts of the body on the skin surface, the transfer of heat can take place by any of the four processes: radiation, evaporation, conduction and convection. It may further be mentioned that blood flow enhances when a man performs hard physical work and also when the body is exposed to excessive heat environment. In case like these, blood circulation cannot remain normal. In order to take care of the increase in blood flow, the dimensions of the artery have to increase suitably. It is known that when the temperature of the surrounding-exceeds $20\,^{0}c$, heat transfer takes place from the surface of the skin by the process of evaporation through sweating and when the temperature is below $20\,^{0}c$, the human body loses heat by conduction and radiation. Blood flow with radiative heat transfer was discussed by Ogulu and Bestman [80] on the basis of a theoretical study. The study of heat transfer analysis is an important area in connection with peristaltic motion, which has industrial applications such as sanitary fluid transport, blood pumps in heat lungs machine and transport of corrosive fluids where the contact of fluid with the machinery parts is prohibited. In the above mentioned studies fluids viscosity is assumed to be constant. There are few attempts [24] in which the effects of variable viscosity in the peristaltic mechanisms have been considered. These studies considered the viscosity to be a function of space variable in the form of an exponential function. In a typical situation must of the fluids have temperature dependent viscosity and this properly varies significantly when large temperature difference exists. Recently, Sinha et al. [98] have examined the peristaltic motion of (MHD) flow of blood with variable viscosity depend on space with effect of slip conditions on the velocity and temperature, therefore there is no attempt is available in the literature which deals with the problem of peristaltic transport in an asymmetric capillary blood vessel with variable viscosity and the effect of non-slip conditions on the velocity and temperature.

So, in the present work, the aim of this chapter is to examine the peristaltic motion of (MHD) flow of blood and heat transfer in a tapered asymmetric channel through porous medium with variable viscosity and non-slip conditions on the velocity and temperature. The energy equation is formulated by including a heat source term which simulates either absorption or generation the governing equations of motion and energy are simplified using long wave length and low Reynolds number approximation. The nonlinear differential equation are solved analytically by using of perturbation method for small values of Reynolds model viscosity parameter. Series solutions for stream function, axial velocity and pressure gradient are given by using the regular perturbation technique. Numerical computations have been performed for the pressure rise per wave length. The effects of the physical parameters on these distributions are discussed and illustrated graphically through a set of figures.

3-1 The Mathematical Model of the Problem

Let us consider the MHD flow of blood and heat transfer through a porous medium of two –dimensional tapered a symmetric channel. We assume that infinite wave train traveling with speed c along the non –uniform walls. We choose a rectangular coordinate system for the channel with $\overline{X}$ along the direction of wave propagation and parallel to the center line and $\overline{Y}$ transverse to it. The wall of the tapered a symmetric channel are given in eq. (2-1) by fig. (2-1).

3-2 The Governing Equations of the Problem

It is well known that the second grade fluid has extra stress tensor τ of the following form: [98]

$$\tau = 2\mu'(\overline{Y})e \qquad ......(3\text{-}1)$$

where $\mu'(\overline{Y})$ is the viscosity function and e is the strain.
The Rivilin-Ericksen tensors are given by:

$$e = \tfrac{1}{2}[\nabla\overline{V} + (\nabla\overline{V})^T] \qquad ......(3\text{-}2)$$

where ($\nabla\overline{V}$) is the fluid velocity gradient in the Cartesian coordinates $(\overline{X},\overline{Y})$ and $(\nabla\overline{V})^T$ is the transpose of the fluid velocity gradient in the Cartesian coordinates (x,y), they defined as:

$$\nabla\overline{V} = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{U}}{\partial\overline{Y}} \\ \frac{\partial\overline{V}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix}, \quad (\nabla\overline{V})^T = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{X}} \\ \frac{\partial\overline{U}}{\partial\overline{Y}} & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ......(3\text{-}3)$$

Then

$$e = \begin{pmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \tfrac{1}{2}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) \\ \tfrac{1}{2}(\frac{\partial\overline{U}}{\partial\overline{Y}} + \frac{\partial\overline{V}}{\partial\overline{X}}) & \frac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ......(3\text{-}4)$$

Now, substituting equation (3-4) into (3-1), we get:

$$\tau = 2\mu'(\overline{Y})\begin{pmatrix} \frac{\partial \overline{U}}{\partial \overline{X}} & \frac{1}{2}(\frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}}) \\ \frac{1}{2}(\frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}}) & \frac{\partial \overline{V}}{\partial \overline{Y}} \end{pmatrix} \qquad .....(3\text{-}5)$$

Thus the components of stress will be:

$$\tau_{\overline{XX}} = 2\mu'(\overline{Y})\frac{\partial \overline{U}}{\partial \overline{X}}$$

$$\tau_{\overline{XY}} = \tau_{\overline{YX}} = \mu'(\overline{Y})(\frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}})$$

$$\tau_{\overline{YY}} = 2\mu'(\overline{Y})\frac{\partial \overline{V}}{\partial \overline{Y}} \qquad .....(3\text{-}6)$$

<u>3-3 Basic Equations of the Problem</u>

In the laboratory frame, the equations governing of the two-dimensional motion of an incompressible MHD flow of blood and heat transfer through a porous medium with the effect of variable viscosity depend on space.

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0 \qquad ......(3-7)$$

$$\rho(\frac{\partial \overline{U}}{\partial \bar{t}} + \overline{U}\frac{\partial \overline{U}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{U}}{\partial \overline{Y}}) = -\frac{\partial \overline{P}}{\partial \overline{X}} + 2\frac{\partial}{\partial \overline{X}}(\mu'(\overline{Y})\frac{\partial \overline{U}}{\partial \overline{X}}) +$$
$$\frac{\partial}{\partial \overline{Y}}[\mu'(\overline{Y})(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}})] - \sigma B_0^2 \overline{U} + \rho g \alpha_1 (T - T_0) - \frac{\mu'(\overline{Y})}{k_0}\overline{U}. \qquad .....(3-8)$$

$$\rho(\frac{\partial \overline{V}}{\partial \bar{t}} + \overline{U}\frac{\partial \overline{V}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{V}}{\partial \overline{Y}}) = -\frac{\partial \overline{P}}{\partial \overline{Y}} + 2\frac{\partial}{\partial \overline{Y}}(\mu'(\overline{Y})\frac{\partial \overline{V}}{\partial \overline{Y}}) +$$
$$\frac{\partial}{\partial \overline{X}}[\mu'(\overline{Y})(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}})] - \frac{\mu'(\overline{Y})}{k_0}\overline{V}. \qquad ......(3-9)$$

$$\rho C_\rho(\frac{\partial T}{\partial \bar{t}} + \overline{U}\frac{\partial T}{\partial \overline{X}} + \overline{V}\frac{\partial T}{\partial \overline{Y}}) = k_1[\frac{\partial^2 T}{\partial (\overline{X})^2} + \frac{\partial^2 T}{\partial (\overline{Y})^2}] + Q_0 \qquad ....(3\text{-}10)$$

in which $(\overline{U}, \overline{V})$ are velocity component in the direction of the laboratory frame, $(\overline{X}, \overline{Y})$, α_1 is the Coefficient of thermal expansion, μ' is variable viscosity.

In order to simplify the governing equations of motion, and temperature, we may introduce the following dimensionless transformations as follows:

$$x=\frac{\overline{X}}{\lambda},\ y=\frac{\overline{Y}}{d},\ t=\frac{c\bar{t}}{\lambda},\ u=\frac{\overline{U}}{c},\ v=\frac{\overline{V}}{\delta c},\ h_1=\frac{\overline{H_1}}{d},\ h_2=\frac{\overline{H_2}}{d},\ p=\frac{d^2\overline{P}}{c\,\lambda\,\mu_0},\ a=\frac{a_1}{d},\ b=\frac{a_2}{d},$$

$$m=\frac{m'\lambda}{d},\theta=\frac{T-T_0}{T_0},\ \mu(y)=\frac{\mu'(\overline{Y})}{\mu_0},\ \delta=\frac{d}{\lambda},\ \mathrm{Re}=\frac{\rho c\,d}{\mu_0},\ M=\sqrt{\frac{\sigma}{\mu_0}}B_0\,d,\ k^2=\frac{d^2}{k_0},$$

$$Gr=\frac{\rho\,g\,\alpha_1(T_0)d^2}{c\mu_0},Pr=\frac{\mu_0 C_\rho}{k_1},\beta=\frac{Q_0 d^2}{k_1(T_0)} \qquad .....(3\text{-}11)$$

In which T_0 is the temperature of the lower wall and upper wall and β is source\sink parameter.

Substituting Eq. (3-11) into Eqs. (3-7)- (3-10) we get:

Eq. (3-7) is transformed automatically.

From eq. (3-8):

$$\rho\left(\frac{\partial\overline{U}}{\partial\bar{t}}+\overline{U}\frac{\partial\overline{U}}{\partial\overline{X}}+\overline{V}\frac{\partial\overline{U}}{\partial\overline{Y}}\right)=-\frac{\partial\overline{P}}{\partial\overline{X}}+2\frac{\partial}{\partial\overline{X}}\left(\mu'(\overline{Y})\frac{\partial\overline{U}}{\partial\overline{X}}\right)+\frac{\partial}{\partial\overline{Y}}\left[\mu'(\overline{Y})\left(\frac{\partial\overline{V}}{\partial\overline{X}}+\frac{\partial\overline{U}}{\partial\overline{Y}}\right)\right]-\sigma B_0^2\overline{U}$$
$$+\rho g\,\alpha_1(T-T_0)-\frac{\mu'(\overline{Y})}{K_0}\overline{U}.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+Cu\frac{C}{\lambda}\frac{\partial u}{\partial x}+C\delta v\frac{C}{d}\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}+2\frac{1}{\lambda}\frac{\partial}{\partial x}\left(\mu_0.\mu(y)\frac{C}{\lambda}\frac{\partial u}{\partial x}\right)+\frac{1}{d}\frac{\partial}{\partial y}[\mu_0.$$
$$\mu(y)\left(\frac{Cd}{\lambda^2}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{\partial u}{\partial y}\right)]-\sigma B_0^2Cu+\rho g\,\alpha_1(T-T_0)-\frac{\mu_0.\mu(y)}{K_0}Cu.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}u\frac{\partial u}{\partial x}+C\frac{d}{\lambda}v\frac{C}{d}\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}+\frac{2C}{\lambda^2}\frac{\partial}{\partial x}\left(\mu_0.\mu(y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}[\mu_0.\mu(y)$$
$$\left(\frac{Cd}{\lambda^2}\frac{1}{d}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{1}{d}\frac{\partial u}{\partial y}\right)]-\sigma B_0^2Cu+\rho g\,\alpha_1\theta(T_0)-\frac{\mu_0.\mu(y)}{K_0}Cu.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}u\frac{\partial u}{\partial x}+\frac{C^2}{\lambda}v\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}+\frac{2C}{\lambda^2}\frac{\partial}{\partial x}\left(\mu_0.\mu(y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}[\mu_0.\mu(y)$$
$$\left(\frac{C}{\lambda^2}\frac{\partial v}{\partial x}+\frac{\lambda^2}{\lambda^2}\frac{C}{d^2}\frac{\partial u}{\partial y}\right)]-\sigma B_0^2Cu+\rho g\,\alpha_1\theta(T_0)-\frac{\mu_0.\mu(y)}{K_0}Cu.$$

Thus we have after some simplification:

$$\rho\frac{C^2}{\lambda}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}+\frac{2C}{\lambda^2}\frac{\partial}{\partial x}\left(\mu_0.\mu(y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}[\mu_0.\mu(y)$$

$$\frac{C}{\lambda^2}(\frac{\partial v}{\partial x}+\frac{1}{\delta^2}\frac{\partial u}{\partial y})]-\sigma B_0^2 Cu+\rho g\,\alpha_1\theta(T_0)-\frac{\mu_0.\mu(y)}{K_0}Cu. \qquad ....(3\text{-}12)$$

Now multiplying both sides of equation (3-12) by $(\frac{d^2}{C\,\mu_0})$ we have:

$$\rho\frac{C^2}{\lambda}\frac{d^2}{C\,\mu_0}(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+\frac{2C}{\lambda^2}\mu_0\frac{d^2}{C\,\mu_0}\frac{\partial}{\partial x}(\mu(y)\frac{\partial u}{\partial x})+\frac{C\,\mu_0}{\lambda^2}\frac{d^2}{C\,\mu_0}$$

$$\frac{\partial}{\partial y}[\mu(y)(\frac{\partial v}{\partial x}+\frac{1}{\delta^2}\frac{\partial u}{\partial y})]-\sigma B_0^2 Cu.\frac{d^2}{C\,\mu_0}+\rho g\,\alpha_1\theta(T_0)\frac{d^2}{C\,\mu_0}-\frac{\mu_0.\mu(y)}{K_0}Cu.\frac{d^2}{C\,\mu_0}$$

Thus we obtain that:

$$\frac{\rho Cd}{\mu_0}\frac{d}{\lambda}(\frac{du}{dt}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+2\delta^2\frac{\partial}{\partial x}(\mu(y)\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}[\mu(y)(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]-$$

$$\mu^2 u+Gr\theta-\frac{d^2}{K_0}\mu(y)u.$$

Which can be written as:

$$\mathrm{Re}\,\delta(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}+2\delta^2\frac{\partial}{\partial x}(\mu(y)\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}[\mu(y)(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]-\mu^2 u+Gr\theta$$

$$-K^2\mu(y)u. \qquad .....(3\text{-}13)$$

From eq. (3-9):

$$\rho(\frac{\partial\overline{V}}{\partial\overline{t}}+\overline{U}\,\frac{\partial\overline{V}}{\partial\overline{X}}+\overline{V}\,\frac{\partial\overline{V}}{\partial\overline{Y}})=-\frac{\partial\overline{P}}{\partial\overline{Y}}+2\frac{\partial}{\partial\overline{Y}}(\mu'(\overline{Y})\frac{\partial\overline{V}}{\partial\overline{Y}})+\frac{\partial}{\partial\overline{X}}[\mu'(\overline{Y})(\frac{\partial\overline{V}}{\partial\overline{X}}+\frac{\partial\overline{u}}{\partial\overline{Y}})]-\frac{\mu'(\overline{Y})}{k_0}\overline{V}.$$

$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+Cu\,\frac{C\,\delta}{\lambda}\frac{\partial v}{\partial x}+C\,\delta v\,\frac{C\,\delta}{d}\frac{\partial v}{\partial y})=-\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{2}{d}\frac{\partial}{\partial y}(\mu_0.\mu(y)\frac{C\,\delta}{d}\frac{\partial v}{\partial y})+\frac{1}{\lambda}\frac{\partial}{\partial x}$$

$$[\mu_0.\mu(y).(\frac{C\,\delta}{\lambda}\frac{\partial v}{\partial x}+\frac{C}{d}\,\frac{\partial u}{\partial y})]-\frac{\mu_0.\mu(y)}{k_0}C\,\delta v.$$

$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+\frac{C^2\delta}{\lambda}u\,\frac{\partial v}{\partial x}+C^2\delta\frac{d}{\lambda}\frac{1}{d}v\,\frac{\partial v}{\partial y})=-\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{2C\,\delta\mu_0}{d^2}\frac{\partial}{\partial y}(\mu(y)\frac{\partial v}{\partial y})+\frac{\partial}{\partial x}$$

$$[\mu_0.\mu(y).(\frac{C\,\delta}{\lambda^2}\frac{\partial v}{\partial x}+\frac{C}{\lambda d}\,\frac{\partial u}{\partial y})]-\frac{\mu_0.\mu(y)}{k_0}C\,\delta v.$$

$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+\frac{C^2\delta}{\lambda}u\,\frac{\partial v}{\partial x}+\frac{C^2\delta}{\lambda}v\,\frac{\partial v}{\partial y})=-\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y}+2C\,\frac{d}{\lambda}\frac{\mu_0}{d^2}\frac{\partial}{\partial y}(\mu(y)\frac{\partial v}{\partial y})+\frac{\partial}{\partial x}$$

$$[\mu_0.\mu(y).(\frac{C}{\lambda^2}\frac{d}{\lambda}\frac{\partial v}{\partial x}+\frac{C}{\lambda d}\,\frac{\partial u}{\partial y})]-\frac{\mu_0.\mu(y)}{k_0}C\,\delta v.$$

Thus we have after some simplification:

$$\rho \frac{C^2\delta}{\lambda}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{2C\mu_0}{\lambda d}\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}[\mu_0.\mu(y).$$
$$\left(\frac{C}{\lambda^2}\frac{d}{\lambda}\frac{d}{d}\frac{\partial v}{\partial x}+\frac{C}{\lambda d}\frac{\partial u}{\partial y}\right)]-\frac{\mu_0.\mu(y)}{k_0}C\,\delta v.$$

$$\frac{\rho C^2\delta}{\lambda}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y}+\frac{2C\mu_0}{\lambda d}\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}[\mu_0.\mu(y).$$
$$\frac{C}{\lambda d}\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)]-\frac{\mu_0.\mu(y)}{k_0}C\,\delta v. \qquad ......(3\text{-}14)$$

Now multiplying both sides of (3-14) by $\left(\frac{d^3}{C\lambda\mu_0}\right)$ we get:

$$\frac{\rho C^2\delta}{\lambda}\frac{d^3}{\lambda\mu_0 C}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\frac{2C\mu_0}{\lambda d}\frac{d^3}{\lambda C\mu_0}\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\frac{C\mu_0}{\lambda d}$$
$$\frac{d^3}{\lambda C\mu_0}\frac{\partial}{\partial x}[\mu(y).(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]-\frac{\mu_0\mu(y)}{k_0}C\,\delta v\,\frac{d^3}{\lambda C\mu_0}.$$

Thus we have:

$$\frac{\rho C d}{\mu_0}\frac{d^2}{\lambda^2}\delta\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+2\delta^2\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\delta^2\frac{\partial}{\partial x}[\mu(y).(\delta^2\frac{\partial v}{\partial x}$$
$$+\frac{\partial u}{\partial y})]-\frac{d^2}{k_0}\delta^2 v\,\mu(y).$$

Which can be written as:

$$\operatorname{Re}\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+2\delta^2\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\delta^2\frac{\partial}{\partial x}\mu(y).(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})-$$
$$k^2\delta^2\mu(y)v. \qquad ......(3\text{-}15)$$

From eq. (3-10):

$$\rho C_\rho\left(\frac{\partial T}{\partial \bar{t}}+\bar{U}\frac{\partial T}{\partial \bar{X}}+\bar{V}\frac{\partial T}{\partial \bar{Y}}\right)=k_1\left[\frac{\partial^2 T}{\partial \bar{X}^2}+\frac{\partial^2 T}{\partial \bar{Y}^2}\right]+Q_0$$
:
$$\rho C_\rho\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\frac{\partial T}{\partial x}+C\,\delta v\,\frac{1}{d}\frac{\partial T}{\partial y}\right)=k_1\left[\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}\right]+Q_0$$

$$\rho C_\rho(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\,\frac{\partial T}{\partial x}+C\,\frac{d}{\lambda}v\,\frac{1}{d}\frac{\partial T}{\partial y})=k_1[\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}]+Q_0$$

$$\rho C_\rho\frac{C}{\lambda}(\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y})=k_1[\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}]+Q_0$$

$$\rho C_\rho\frac{C}{\lambda}(\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y})=k_1[\frac{1}{\lambda^2}\frac{d^2}{d^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}]+Q_0$$

$$\rho C_\rho\frac{C}{\lambda}(\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y})=\frac{k_1}{d^2}[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}]+Q_0 \qquad .....(3\text{-}16)$$

Multiplying both sides of (3-16) by $(\frac{d^2}{k_1})$ we get

$$\rho C_\rho.\frac{C}{\lambda}\frac{d^2}{k_1}(\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y})=[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}]+Q_0\frac{d^2}{k_1}$$

Now since $\theta=\frac{T-T_0}{T_0}\Rightarrow T-T_0=\theta T_0\Rightarrow T=\theta T_0+T_0\Rightarrow\partial T=T_0\partial\theta$

Thus we have:

$$\rho C_\rho.\frac{C}{\lambda}\frac{d^2}{k_1}(T_0\frac{\partial\theta}{\partial t}+uT_0\frac{\partial\theta}{\partial x}+vT_0\frac{\partial\theta}{\partial y})=[\delta^2 T_0\frac{\partial^2\theta}{\partial x^2}+T_0\frac{\partial^2\theta}{\partial y^2}]+Q_0\frac{d^2}{k_1} \qquad ......(3\text{-}17)$$

Multiplying both sides of (3-21) by $(\frac{1}{T_0})$ we get:

$$\rho C_\rho\frac{C}{\lambda}\frac{d^2}{k_1}T_0\frac{1}{T_0}(\frac{d\theta}{dt}+u\,\frac{\partial\theta}{\partial x}+v\,\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+Q_0\frac{d^2}{k_1}.\frac{1}{T_0}$$

$$\rho C_\rho\frac{C}{\lambda}\frac{d^2}{k_1}\frac{\mu_0}{\mu_0}(\frac{d\theta}{dt}+u\,\frac{\partial\theta}{\partial x}+v\,\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+\frac{Q_0 d^2}{T_0 k_1}$$

Thus we have after some simplification:

$$\frac{\rho Cd}{\mu_0}\frac{d}{\lambda}\frac{\mu_0 C_P}{k_1}(\frac{\partial\theta}{\partial t}+u\,\frac{\partial\theta}{\partial x}+v\,\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+\beta$$

Which can be written as:

$$\mathrm{Re}\,\delta\,\mathrm{Pr}(\frac{\partial\theta}{\partial t}+u\,\frac{\partial\theta}{\partial x}+v\,\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+\beta \qquad ......(3\text{-}18)$$

Now, under the assumption of length $(\delta<<1)$ and low Reynolds number, the Eqs. (3-13), (3-15) and (3-18) can be written as:

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}[\mu(y).\frac{\partial u}{\partial y}]-M^{2}u+Gr\theta-k^{2}\mu(y)u \qquad .....(3\text{-}19)$$

$$0=\frac{\partial p}{\partial y} \qquad .....(3\text{-}20)$$

$$0=\frac{\partial^2\theta}{\partial y^2}+\beta \qquad .....(3\text{-}21)$$

Introduce the stream function $u=\frac{\partial\psi}{\partial y},v=\frac{-\partial\psi}{\partial x}$ into eq. (3-19) then we have:

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}[\mu(y).\frac{\partial^2\psi}{\partial y^2}]-k^{2}\mu(y)\frac{\partial\psi}{\partial y}-\mu^2\frac{\partial\psi}{\partial y}+Gr\theta \qquad .....(3\text{-}22)$$

3-4 Rate of Volume Flow and Boundary Conditions

In order to discuss the results quantitatively, we assume the instantaneous volume rate of the flow F(x, t) is periodic in (x-t), [58]

$$F(x,t)=\theta'+a\sin(2\pi(x-t)+\phi)+b\sin(2\pi(x-t)) \qquad .....(3-23)$$

In which θ' is the mean flow rate in the wave frame, F is the mean flow rate in the laboratory frame and

$$F=\int_{h_1}^{h_2}udy$$

$$=\int_{h_1}^{h_2}\frac{\partial\psi}{\partial y}dy=\psi(h_2)-\psi(h_1)$$

Selecting $\psi(h_2)=\frac{F}{2}$, we have $\psi(h_1)=\frac{-F}{2}$

The boundary conditions in dimensionless stream function with now take the following form:

$$\left.\begin{array}{l}\psi=\frac{F}{2},\frac{\partial\psi}{\partial y}=0\ \text{ and } \theta=0\ at\left(y=h_2\right)\\ \psi=\frac{-F}{2},\frac{\partial\psi}{\partial y}=0\ \text{ and } \theta=0\ at\left(y=h_1\right)\end{array}\right] \qquad ......(3-24)$$

In which

$$h_2 = 1 + mx + b\sin(2\pi(x - t))$$
$$h_1 = -1 - mx - a\sin(2\pi(x - t) + \phi)$$

The non-dimensional expression for the average rise pressure Δp is given in eq. (2-47):

3-5 Reynolds Model of Viscosity

The Reynolds model of viscosity is used to describe the function of space (y) which is defined as: [98]

$$\mu(y) = e^{-\alpha y} \qquad .....(3\text{-}25)$$

Using the maclaurin series expansion the above expression can be written as:

$$\mu(y) = 1 - \alpha y, \quad \text{for}\ \alpha << 1 \qquad ......(3\text{-}26)$$

Here $\alpha = 0$ Corresponds to the constant viscosity case where α is Reynolds model viscosity parameter.

Compensating equation (3-26) into equation (3-22) we have:

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}[(1 - \alpha y).\frac{\partial^2 \psi}{\partial y^2}] - k^2(1 - \alpha y)\frac{\partial \psi}{\partial y} - M^2\frac{\partial \psi}{\partial y} + Gr\theta \qquad ......(3\text{-}27)$$

3-6 Perturbation Analysis of the Problem

It is clear that the resulting equation of motion Eq.(3-27) is not linear because it contains unknown ψ of some powers which must be solved to yield the desired stream function of fluid which is we have in our problem "blood" and to yield the desired velocity profiles. Due to that non linearity it is difficult to solve it. However, the Reynolds model viscosity parameter α is considered to be very small, so in order to solve Eq. (3-27) with the help of boundary conditions (3-24), we consider the perturbation technique as a power series expansion in small parameter α and writing ψ , F, and P as:

$$\psi = \psi_0 + \alpha\psi_1 + ...$$
$$F = F_0 + \alpha F_1 + ...$$
$$p = p_0 + \alpha p_1 + ... \qquad ......(3\text{-}28)$$

Now substituting Eq. (3-28) into Eq. (3-27) and (3-24) we see that:

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}[(1-\alpha y).\frac{\partial^2 \psi}{\partial y^2}] - k^2(1-\alpha y)\frac{\partial \psi}{\partial y} - M^2\frac{\partial \psi}{\partial y} + Gr\theta$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}[\frac{\partial^2 \psi}{\partial y^2} - \alpha y.\frac{\partial^2 \psi}{\partial y^2}] - k^2[\frac{\partial \psi}{\partial y} - \alpha y\frac{\partial \psi}{\partial y}] - M^2\frac{\partial \psi}{\partial y} + Gr\theta$$

$$\frac{\partial p}{\partial x} = \frac{\partial^3 \psi}{\partial y^3} - \alpha(y.\frac{\partial^3 \psi}{\partial y^3} + \frac{\partial^2 \psi}{\partial y^2}) - k^2[\frac{\partial \psi}{\partial y} - \alpha y\frac{\partial \psi}{\partial y}] - M^2\frac{\partial \psi}{\partial y} + Gr\theta$$

$$\frac{\partial(p_0+\alpha p_1)}{\partial x} = \frac{\partial^3(\psi_0+\alpha\psi_1)}{\partial y^3} - \alpha(y.\frac{\partial^3}{\partial y^3}(\psi_0+\alpha\psi_1) + \frac{\partial^2}{\partial y^2}(\psi_0+\alpha\psi_1)) -$$

$$k^2[\frac{\partial}{\partial y}(\psi_0+\alpha\psi_1) - \alpha y\frac{\partial}{\partial y}(\psi_0+\alpha\psi_1)] - M^2(\frac{\partial}{\partial y}(\psi_0+\alpha\psi_1)) + Gr\theta.$$

$$\frac{\partial p_0}{\partial x} + \alpha\frac{\partial p_1}{\partial x} = \frac{\partial^3\psi_0}{\partial y^3} + \alpha\frac{\partial^3\psi_1}{\partial y^3} - \alpha(y.\frac{\partial^3\psi_0}{\partial y^3} + \alpha y\frac{\partial^3\psi_1}{\partial y^3} + \frac{\partial^2\psi_0}{\partial y^2} + \alpha\frac{\partial^2\psi_1}{\partial y^2}) - k^2[\frac{\partial\psi_0}{\partial y}$$

$$+\alpha\frac{\partial\psi_1}{\partial y} - \alpha y\frac{\partial\psi_0}{\partial y} - \alpha^2 y\frac{\partial\psi_1}{\partial y}] - M^2(\frac{\partial\psi_0}{\partial y} + \alpha\frac{\partial\psi_1}{\partial y}) + Gr\theta.$$

$$(\frac{\partial p_0}{\partial x} + \alpha\frac{\partial p_1}{\partial x}) = \frac{\partial^3\psi_0}{\partial y^3} + \alpha\frac{\partial^3\psi_1}{\partial y^3} - \alpha(y.\frac{\partial^3\psi_0}{\partial y^3} + \alpha y\frac{\partial^3\psi_1}{\partial y^3} + \frac{\partial^2\psi_0}{\partial y^2} + \alpha\frac{\partial^2\psi_1}{\partial y^2}) - k^2[\frac{\partial\psi_0}{\partial y}$$

$$+\alpha\frac{\partial\psi_1}{\partial y} - \alpha y\frac{\partial\psi_0}{\partial y} - \alpha^2 y\frac{\partial\psi_1}{\partial y}] - M^2(\frac{\partial\psi_0}{\partial y} + \alpha\frac{\partial\psi_1}{\partial y}) + Gr\theta. \qquad ......(3-29)$$

Thus if we collect the Coefficient of like power of α, one can gets the zeroth-order and first-order equation as:

3-6-1 Zero's- order system $(\alpha^{(0)})$

$$\frac{\partial p_0}{\partial x} = (\frac{\partial^3\psi_0}{\partial y^3} - N_1\frac{\partial\psi_0}{\partial y}) + \mathrm{Gr}\theta \qquad ....(3-30)$$

where $N_1 = (k^2 + M^2)$

Differentiating eq. (3-30) with respect to y will give:

$$0 = \frac{\partial^4\psi_0}{\partial y^4} - N_1\frac{\partial^2\psi_0}{\partial y^2} + \mathrm{Gr}\frac{\partial\theta}{\partial y} \qquad ....(3-31)$$

Along with the corresponding boundary conditions:

$$\psi_0 = \frac{F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, at \left(y = h_2 \right)$$

$$\psi_0 = \frac{-F_0}{2}, \frac{\partial \psi_0}{\partial y} = 0, at \left(y = h_1 \right) \qquad ....(3-32)$$

3-6-2 First order system $(\alpha^{(1)})$

$$\frac{\partial p_1}{\partial x} = \frac{\partial^3 \psi_1}{\partial y^3} - y \frac{\partial^3 \psi_0}{\partial y^3} - \frac{\partial^2 \psi_0}{\partial y^2} - N_1 \frac{\partial \psi_1}{\partial y^2} + k^2 y \frac{\partial \psi_0}{\partial y} \qquad ......(3-33)$$

Differentiable eq. (3-33) with respect to y we have:

$$0 = \frac{\partial^4 \psi_1}{\partial y^4} - y \frac{\partial^4 \psi_0}{\partial y^4} - 2\frac{\partial^3 \psi_0}{\partial y^3} - N_1 \frac{\partial^2 \psi_1}{\partial y^2} + k^2 y \frac{\partial \psi^2{}_0}{\partial y^2} + \frac{\partial \psi_0}{\partial y} \qquad ......(3-34)$$

The corresponding boundary conditions are:

$$\psi_1 = \frac{F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, at \left(y = h_2 \right)$$

$$\psi_1 = \frac{-F_1}{2}, \frac{\partial \psi_1}{\partial y} = 0, at \left(y = h_1 \right) \qquad ......(3-35)$$

3-7 Solution of the Problem

In this section, let us give the solution of the temperature and motion equations:

3-7-1 Solution of temperature equation

The solution of temperature in Eq. (3-21) that satisfy the boundary conditions (3-24) is found in the form of:

$$\theta = \frac{-\beta y^2}{2} + c_1 y + c_2 \quad ; \qquad ......(3-36)$$

c_1, c_2 are constants can be determinates by using the boundary conditions in Eq.(3-24) such that:

$$c_1 = -\frac{1}{2} h_1 h_2 \beta;$$

$$c_2 = \frac{1}{2}(h_1 \beta + h_2 \beta) \qquad ......(3-37)$$

3-7-2 Solution of motion equations

1 Solution for the zeroth order system $(\alpha^{(0)})$

We substitute Eq. (3-36) into Eq. (3-31) and solve the resulting equation one can find the solution of the zeroth order system which is:

$$\psi_0 = a_3 + a_2 e^{-n_1 y} n_2 + a_1 e^{n_1 y} n_2 + a_4 y + \frac{1}{4} Gr\, h_1 n_2 y^2 \beta + \frac{1}{4} Gr\, h_2 n_2 y^2 \beta - \frac{1}{6} Gr\, n_2 y^3 \beta; \qquad ......(3\text{-}38)$$

$$where \quad (n_1 = \sqrt{N_1}; n_2 = \frac{1}{N_1};\ a_i, (i = 1,2,3,4)$$

a_i, (i = 1, 2, 3, 4) are constants can be determinates by using the boundary conditions in Eq. (3-32) such that:

$$a_1 = \frac{12 f_0 + Grh_1^3 n_2 \beta - 3Grh_1^2 h_2 n_2 \beta + 3Grh_1 h_2^2 n_2 \beta - Grh_2^3 n_2 \beta}{12(-2e^{h_1 n_1} + 2e^{h_2 n_1} + e^{h_1 n_1} h_1 n_1 + e^{h_2 n_1} h_1 n_1 - e^{h_1 n_1} h_2 n_1 - e^{h_2 n_1} h_2 n_1) n_2 Grh_2^3 n_2 \beta}$$

$$a_2 = \frac{e^{h_1 n_1 + h_2 n_1}(12 f_0 + Grh_1^3 n_2 \beta - 3Grh_1^2 h_2 n_2 \beta + 3Grh_1 h_2^2 n_2 \beta - Grh_2^3 n_2 \beta}{12(-2e^{h_1 n_1} + 2e^{h_2 n_1} + e^{h_1 n_1} h_1 n_1 + e^{h_2 n_1} h_1 n_1 - e^{h_1 n_1} h_2 n_1 - e^{h_2 n_1} h_2 n_1) n_2 Grh_2^3 n_2 \beta}$$

$$a_3 = \frac{1}{12}(6 f_0 + 3Grh_1 h_2^2 n_2 \beta - Grh_2^3 n_2 \beta) +$$
$$\frac{e^{h_1 n_1} - e^{h_2 n_1} + e^{h_1 n_1} h_2 n_1 + e^{h_2 n_1} h_2 n_1)(12 f_0 + Grh_1^3 n_2 \beta - 3Grh_1^2 h_2 n_2 \beta + 3Grh_1 h_2^2 n_2 \beta - Grh_2^3 n_2 \beta)}{12(-2e^{h_1 n_1} + 2e^{h_2 n_1} + e^{h_1 n_1} h_1 n_1 + e^{h_2 n_1} h_1 n_1 - e^{h_1 n_1} h_2 n_1 - e^{h_2 n_1} h_2 n_1)}$$

$$a_4 = -\frac{1}{12} Grh_1 h_2 n_2 \beta -$$
$$\frac{(e^{h_1 n_1} + e^{h_2 n_1}) n_1 (12 f_0 + Grh_1^3 n_2 \beta - 3Grh_1^2 h_2 n_2 \beta + 3Grh_1 h_2^2 n_2 \beta - Grh_2^3 n_2 \beta)}{12(-2e^{h_1 n_1} + 2e^{h_2 n_1} + e^{h_1 n_1} h_1 n_1 + e^{h_2 n_1} h_1 n_1 - e^{h_1 n_1} h_2 n_1 - e^{h_2 n_1} h_2 n_1)} \qquad ....(3\text{-}39)$$

2 Solution of the first order system $(\alpha^{(1)})$

We substitute the expression for ψ_0 into Eq. (3-34) and solve the resulting equation one can find the solution of the first order system in the form:

$$\psi_1 = e^{-n_1 y}(-e^{2n_1 y} n_1 (n_1^2 (3 + 2n_1 y - 2n_1^2 y^2) + k^2 (7 - 6n_1 y + 2n_1^2 y^2))(12F_0 + Gr(h_1 - h_2)^3 n_2 \beta)$$
$$-e^{(h_1 + h_2) n_1} n_1 (n_1^2 (3 - 2n_1 y - 2n_1^2 y^2) + k^2 (7 + 6n_1 y + 2n_1^2 y^2))(12F_0 + Gr(h_1 - h_2)^3 n_2 \beta) + 4e^{n_1 (h_2 + y)}$$
$$y^2 (-12F_0 k^2 n_1^3 + Grn_2 (24n_1^2 (2 + h_1 n_1 - h_2 n_1) + k^2 (-h_1^3 n_1^3 + h_2^3 n_1^3 - 4h_2^2 n_1^3 - 4h_2^2 n_1^3 y + h_1^2 n_1^3$$

$$(-3h_2+4y)+h_1n_1(-36-12h_2n_1+3h_2^2n_1^2+8n_1y-3n_1^2y^2)-6(12+n_1^2y^2)+h_2n_1$$
$$(36+8n_1y+3n_1^2y^2)))\beta)-4e^{n_1(h_1+y)}y^2(12F_0k^2n_1^3+Gm_2(24n_1^2(2-h_1n_1-h_2n_1)+k^2$$
$$(h_1^3n_1^3-h_2^3n_1^3+h_1^2n_1^3(3h_2-4y)+4h_2^2n_1^3y+h_2n_1(-36+8n_1y-3n_1^2y^2)-6(12+n_1^2y^2)+h_1n_1$$
$$(36-12h_2n_1-3h_2^2n_1^2+8n_1y+3n_1^2y^2)))\beta)+96e^{n_1(h_1+2y)}n_1^2(-2+h_1n_1-h_2n_1)b_1-96e^{n_1(h_2+2y)}$$
$$n_1^2(-2-h_1n_1+h_2n_1)b_1+96e^{h_1n_1}n_1^2(-2+h_1n_1-h_2n_1)b_2-96e^{h_2n_1}n_1^2(-2-h_1n_1+h_2n_1)b_2))$$
$$/e^{h_1n_1}(-2+h_1n_1-h_2n_1)+e^{h_2n_1}(2+h_1n_1-h_2n_1))+b_3+yb_4; \qquad ....(3\text{-}40)$$

b_1, b_2, b_3, b_4 are constants can be determinates by using the boundary conditions in Eq. (3-35) and using "MATHEMATICA" program software.

3-8 Results and Discussion

In this section, the numerical and computational results are discussed for the problem of peristaltic transport of incompressible non-Newtonian and electrically conducting which is consider by "blood" with variable viscosity of space in a tapered asymmetric channel through porous medium with the effect of heat transfer. The numerical evaluations of the analytical results which is showed by using the perturbation technique for small values of Reynolds model viscosity parameter under the assumption of long wave length and low Reynolds number approximation. Some important results are displayed graphically in figures (3-2)-(3-28).

3-8-1 Pumping Characteristic

We plot the expression for Δp in Eq. (2-47) against θ' for various values of parameters of interesting in Figs. (3-2)- (3-10). Numerical calculations for several values of the Hartmann number (M), the phase difference ϕ, the non-uniform parameter of the channel (m), the porosity parameter (K), the amplitudes of upper and lower walls of the channel ($a\,\&\,b$), the source/ sink parameter (β), Garshof number (Gr) and Reynolds model of viscosity have been carried out. Pumping regions can be divided into three regions which are (retrograde pumping that is described by $(\Delta p>0,\ \theta'<0)$,co-pumping or augmented pumping described by $(\Delta p<0,\ \theta'>0)$ and free pumping described by ($\Delta p=0$). In fig.(3-2), The effects of non-uniform parameter m on Δp against θ' is seen, observed that pressure rise increase in the retrograde pumping region and decrease in the augmented pumping. The effects of a and b on Δp are seen, observed that pressure rise behaved similar to effect of (m) and we noted there is slightly increase on the free pumping region and their behavior are displayed in figures (3-3) and (3-4) respectively. Figure (3-5) illustrated the influence of ϕ on pressure rise and it is noticed that pressure rise decrease in the retrograde pumping region and increase in the co-pumping region. The impacts of M and K are noticed in figures (3-6) and (3-7) respectively, it is examined that in back ward pumping region or retrograde pumping, the pumping rate enhances with an increase of M and k while in augmented pumping region, the pumping decrease via M and k. currents are induced in the tissue or medium by the moving ions. This interaction serves as a basis of magnetically induced blood flow. Figures (3-8) and (3-9) displayed the effects of parameters β and Gr respectively, it is noticed that the pressure rise increase in the all regions of pumping with an increase of these parameters. The impact of α on Δp is seen in figure (3-10), it is observed that the pumping reduced in the case of variable viscosity in the retrograde pumping and there is no change in pressure rise in all regions of pumping. However, we can see small distance between the curves for different values of α .

<u>3-8-2 Velocity Distribution</u>

Influences of geometric parameters on the velocity distribution have been illustrated in Fig.(3-11)-(3-20). These figures are scratched at the fixed values of x=0.3, t=0.5. The change in values of m on the axial velocity u is shown in fig.(3-11), it is interesting to note that an increase in m causes an increase in the magnitude of u at the boundaries, however, at the center of the channel the magnitude of u gets decrease. A similar behavior is seen for the case of the Hartmann parameter and it is projected in figure (3-12), this observation agrees with the theory because with the increase in Hartmann number, the Lorentz force increase, it is well known that Lorentz force opposes the flow, this implies that if we increase the strength of magnetic field, the flow of blood will be impeded. From figure (3-13), it appears that the velocity profile traces a parabolic path, it increases with an increase of Reynolds parameter (α) in the upper wall and converse in behavior is observed at the lower part of channel. Figure (3-14), shows that with an increase in mean volume flow rate θ', the axial velocity increases. The influence of the amplitudes of the upper wall b on the velocity is depicted in figure (3-15) for a fixed values of other parameters it could be observed that an increase in the value of upper amplitude b increases the magnitude of the velocity at the center and the lower wall of channel and decreases at the upper wall of channel. Figure (3-16) displays the effect of lower amplitude a of the channel, it observed that an increase in this parameter lead to drop in value of velocity at the center and walls of channel. The axial velocity for the phase angle ϕ is shown in figure (3-17), it has been noticed that an increase in ϕ values results as rising up in the magnitude of axial velocity at the center and lower wall of channel and hardily reduction in the upper part of channel. A similar manner of behavior of Hartmann parameter that is the effect of porous parameter (k), which is procure obstruction in the flow of blood, and is plotted in the figure (3-18). The effects of Grashof- number Gr and source/sink parameter β are illustrated in figures (3-19) and (3-20) respectively, it can be increase at the center of channel and decrease at the walls with an increase of these parameters. However, increase in velocity behind enhancing Grashof no. back to fact is the decreased viscosity results in increased velocity, on the other hand we can say that the velocity will be extended after rising up the value of β since the manner of velocity and temperature are interdependent because the last one is increased the temperature. With the effects of m, a, b, M, k, Gr and β we observed that for any values of these parameters, the axial velocity vanishes at points of inflexion.

3-8-3 Trapping Phenomenon

The formation of an internally circulating bolus under certain conditions due to splitting of some streamlines is named as trapping phenomenon. Physically this phenomenon appears in thrombus in blood and the movement of food bolus in the gastrointestinal tract. The trapping for different values of m, ϕ, a, b, M, β , K and θ' are shown in Figs.(3-21)-(3-30) at fixed values of (t=0.5). The stream lines for different values of m are shown in Fig.(3-21), it has been noticed that the size and number of bolus increase in the lower and upper of the tapered channel. The stream lines for different values of ϕ are shown in fig.(3-22), it is examined that the size and number of bolus increase in the both parts of channel with an increasing of ϕ. Fig.(3-23), showed the effect of parameter (a) on trapping, it is found that the bolus decrease in number but it is increase in size with an increase of a. The influences of parameters b, Gr, β and θ on the trapping are plotted in figures (3-24), (3-25), (3-26) and (3-27)respectively, which is noticed that an increase on the values of this parameters lead to rise up in the size and number of the trapping bolus in the walls of the channel. Figures (3-28) and (3-29) displayed the impact of parameters M and k on trapping, which is increasing in these parameters causes reduce in size and number of bolus in the upper and lower sides of channel. The effect of Reynolds model α on trapping is shown in figure (3-30), it is showed that an increase in this parameter yields decreasing in the number and size of circulation bolus in the walls of channel, in this case we can say that the bolus in the case of constant viscosity is bigger than variable viscosity.

3-8-4 Temperature Characteristics

The expressions for temperature are given by Eq.(3-36). To explicitly see the effects of various parameters on temperature for fixed values of (x=0.3, t=0.5), eq. (3-36) has been solved by exact solution and the results of these parameters presented graphically by using "MATHEMATICA" program and illustrated in fig. (3-31)- (3-35). Figure (3-31) emphasizes that as heat generates during blood flow in arterioles, there is a significant rise in thickness of the boundary layer enhanced by appreciable extend. It is also noticed from this figure that the maximum value of temperature attains in the central and walls of the of the channel. Further it can be noticed that the increase of m, ϕ lead to similar behavior of effect β on temperature and showed it in figs.(3-32) and (3-33) respectively. Fig.(3-34) display the influence of parameter (a) on temperature profile, it is examined that an increase in a causes conversely behavior as the effect of above parameters. The effect of b on temperature is plotted in Fig.(3-35), we note that an increasing in b causes similar behavior as the effect of a on temperature. In all figures of temperature profile the curves are parabolic.

3-9 Concluding Remarks

In this chapter, we investigated the peristaltic transport of electrically conducting fluid which is considered by 'blood" through porous medium in the tapered asymmetric channel under the effect of magnetic fields and heat transfer by using variable viscosity, velocity non slip conditions. The channel asymmetry is produced by choosing the peristaltic waves drain on the non-uniform walls to have different amplitudes and phases. Along wave length and low Reynolds number approximations are adopted. A regular perturbation method for small values of Reynolds model viscosity parameter is employed to obtain the expression for stream function, axial velocity and pressure rise . numerical study has been conduct for average rise in pressure over a wave length. The effects of Hartmann number (M), porosity parameter (K), wave amplitudes (a& b), channel non-uniform parameter (m), phase difference ϕ and source/sink parameter β are also investigated in details, it found that :

1. The pressure rise Δp increase in the pumping $(\Delta p > 0,\ \theta' < 0)$ with an increase of m, a, b, M, k, β and Gr and decrease with an increase of ϕ, α
2. The pressure rise Δp increase in the pumping $(\Delta p < 0, \theta' > 0)$ with an increase of Gr, ϕ, β and decrease with an increase of m, a, b, k, M.
3. The pressure rise Δp increase in the pumping $(\Delta p = 0)$ with an increase of m, a, β and Gr and decreasing with an increase of M, k.
4. The relation between mean flow rate θ' and Δp is linear in the case of increasing of Gr and in β and it is seen nonlinear in the case of rising values of M, k, m, a and b, $\phi,\ \alpha$.
5. It is observed that the curves pumping is intersected at different points by increasing of M, k, m, a, b, α and ϕ and they are parallel curves in the sense of enhancing value of β and Gr.
6. At the center and walls of channels (upper and lower parts), we found that axial velocity increase in magnitude with an increase of ϕ , b, Gr , β , θ and decrease with an increase of m, a, M, k
7. The effect of Reynolds model α has oscillating influence on the velocity at the upper and lower parts of channel.
8. Velocity profiles have inflexion points at the upper and lower parts of channel at different values of increasing of m, a, b, M, Gr, β and k, α .
9. Velocity profiles at most are parabolic and symmetric with an increasing of m, M, Gr, β and θ, α ,k.
10. The temperature profile increases with an increase of m, ϕ, β while the temperature decrease with an increase of a, b.
11. The curves of temperature profiles at all figures are parabolic.
12. The size and number of the trapped bolus increase with an increase of ϕ, β, θ , b, Gr decrease with an increase of m, M, k, α .
13. The influence of a on trapping bolus is wobbling that is these bolus decrease in number with an increase of a but its size be extended.

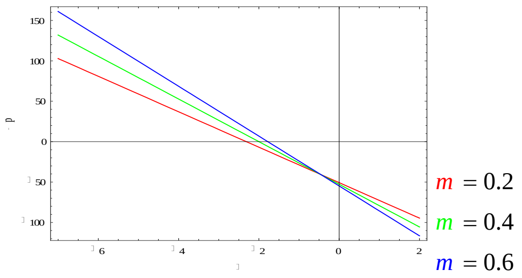


Fig.(3-2) Effect of non-uniform parameter on Δp

$t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, M = 5$

$\alpha = 0.1, K = 1, Gr = 5, \beta = 0.1, x = 0.5$

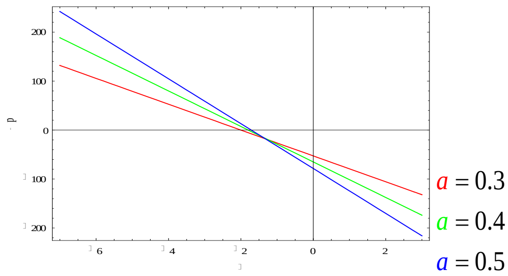


Fig.(3-3) Effect of the amplitude of lower wall of channel a on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, M = 5, b = 0.2,$

$\alpha = 0.1, K = 1, Gr = 5, \beta = 0.1, x = 0.5$

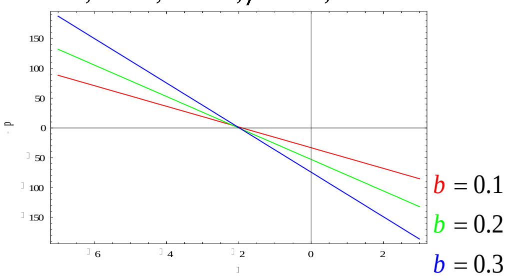


Fig.(3-4) Effect of amplitude of upper wall of channel b on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, M = 5$

$\alpha = 0.1, K = 1, Gr = 5, \beta = 0.1, x = 0.5$

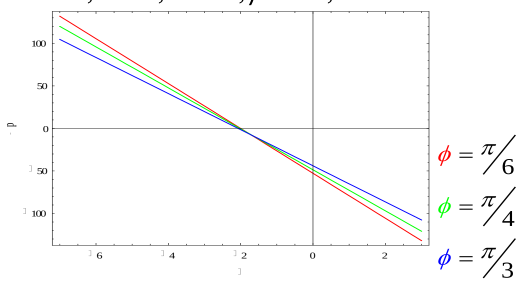


Fig.(3-5) Effect of phase difference on Δp

$m = 0.4, t = 0.5, a = 0.3, b = 0.2, M = 5$

$\alpha = 0.1, K = 1, Gr = 5, \beta = 0.1, x = 0.5$

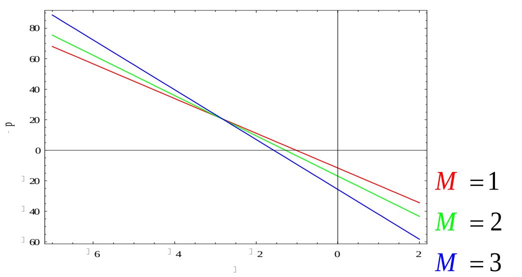


Fig.(3-6) Effect of Hartmann number M on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$\alpha = 0.1, K = 1, Gr = 5, \beta = 0.1, x = 0.5$

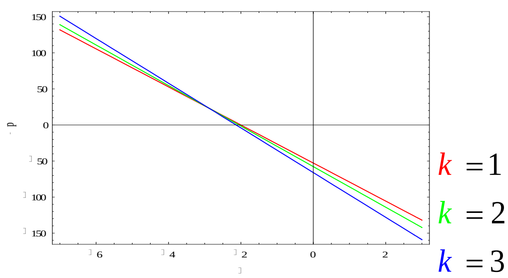


Fig.(3-7) Effect of k on Δp

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$\alpha = 0.1, M = 5, Gr = 5, \beta = 0.1, x = 0.5$

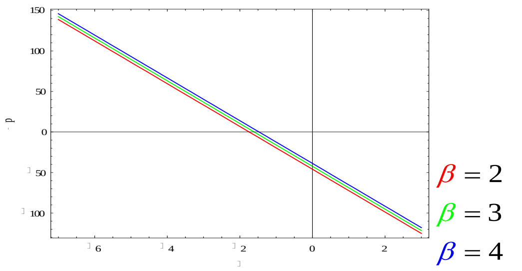


Fig.(3-8) Effect of source/ sink (β) 0n Δp .

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$\alpha = 0.1, M = 5, K = 1, Gr = 5, x = 0.5$

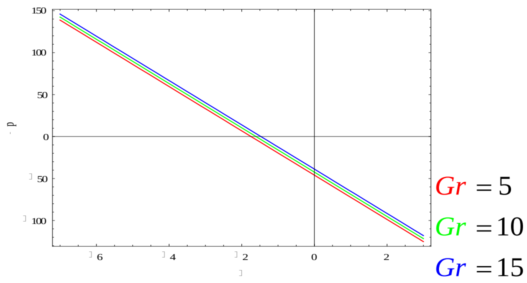


Fig.(3-9) Effect of (Gr) 0n Δp .

$m = 0.01, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$\alpha = 0.1, M = 5, K = 1, \beta = 3, x = 0.5$

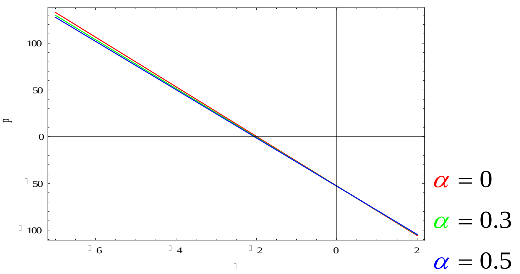


Fig.(3-10) Effect of (α) On Δp.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$Gr = 5, M = 5, K = 1, \beta = 0.1, x = 0.5$

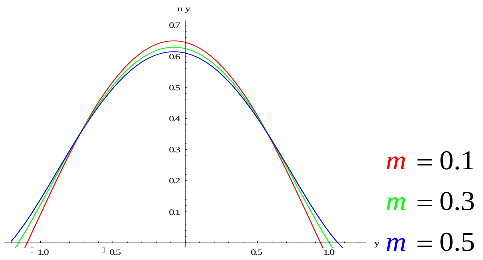


Fig.(3-11) Effect of (m) on the axial velocity

$t = 0.5, a = 0.2, b = 0.1, \phi = \pi/2, M = 2,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

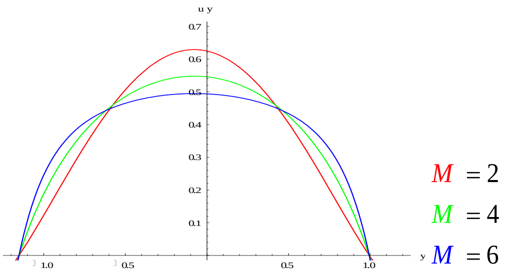


Fig.(3-12) Effect of (M) on the axial velocity

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

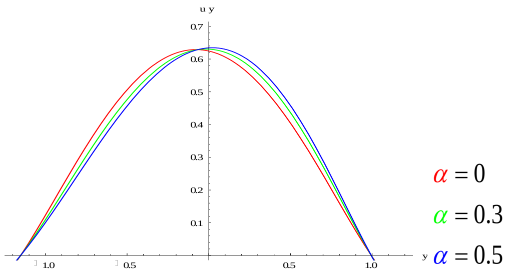


Fig.(3-13) Effect of (α) on the axial velocity u(y)

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1$

$M = 2, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

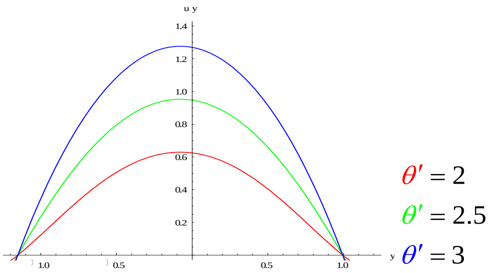


Fig.(3-14) Effect of (θ') on the axial velocity

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, M = 2, x = 0.3$

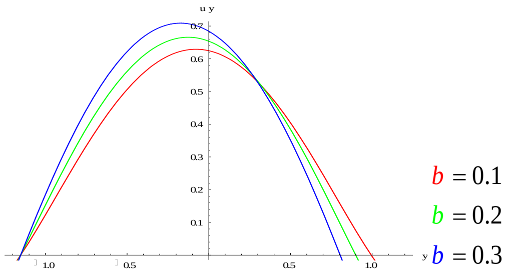


Fig.(3-15) Effect of (b) on the axial velocity u(y)

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, M = 2,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

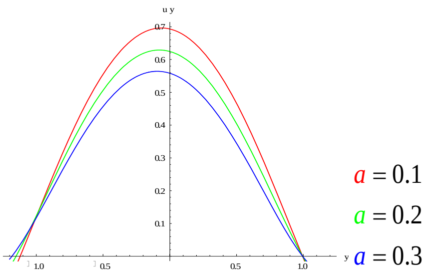


Fig.(3-16) Effect of (a) on the axial velocity u(y)

$m = 0.3, t = 0.5, \phi = \pi/2, b = 0.1, M = 2,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

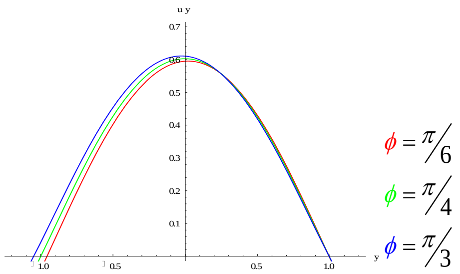


Fig.(3-17) Effect of (ϕ) on the axial velocity u(y)

$m = 0.3, t = 0.5, a = 0.2, b = 0.1, M = 2,$

$\alpha = 0.0001, Gr = 5, \beta = 2, K = 1, \theta' = 1, x = 0.3$

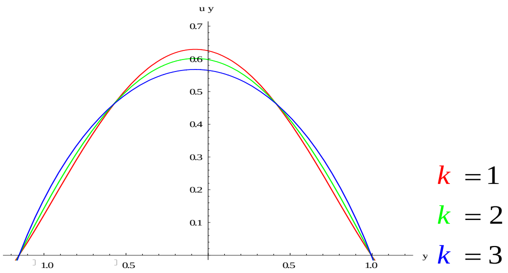


Fig.(3-18) Effect of (k) on the axial velocity

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1$

$\alpha = 0.0001, \beta = 2, M = 2, Gr = 5, \theta' = 1, x = 0.3$

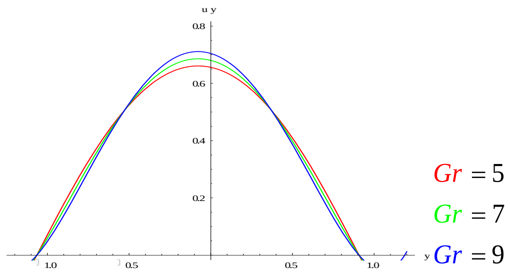


Fig.(3-19) Effect of (Gr) on the axial velocity

$m = 0.01, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 2$

$\alpha = 0.0001, \beta = 2, K = 1, \theta' = 1, x = 0.3$

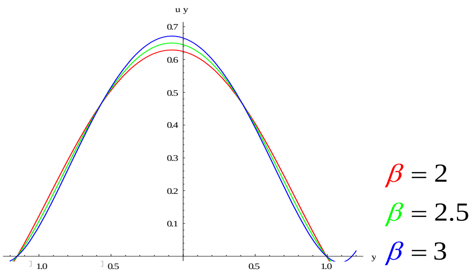


Fig.(3-20) Effect of (β) on the axial velocity u(y)

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1$

$\alpha = 0.0001, M = 2, Gr = 5, K = 1, \theta' = 1, x = 0.3$

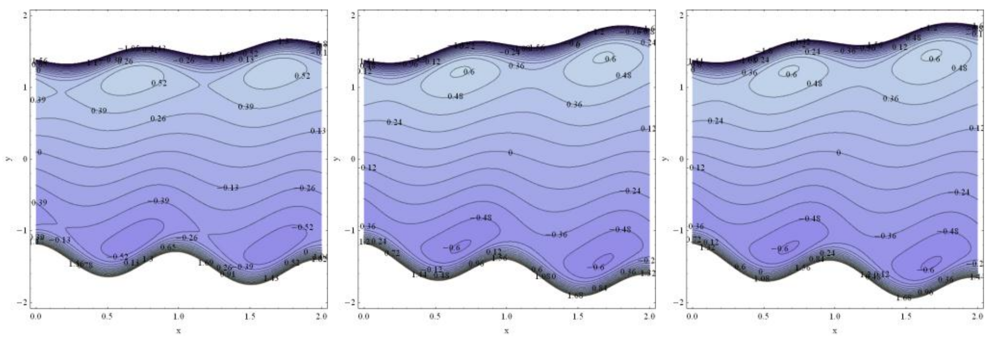


Fig.(3-21) Stream lines for

$t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 10, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1, \theta' = 1$

$(a) m = 0.1, (b) m = 0.2, (c) m = 0.22$

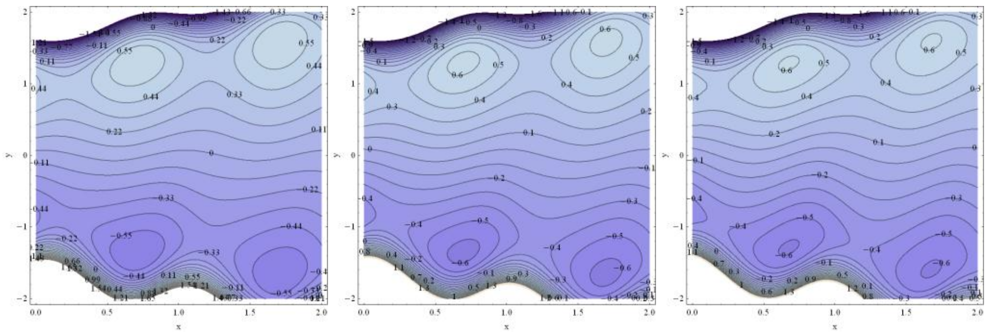


Fig.(3-22) Stream lines for

$m = 0.3, t = 0.5, a = 0.2, b = 0.1, M = 5, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1, \theta' = 1$

$(a) \phi = \pi/4, (b) \phi = \pi/3, (c) \phi = \pi/2$

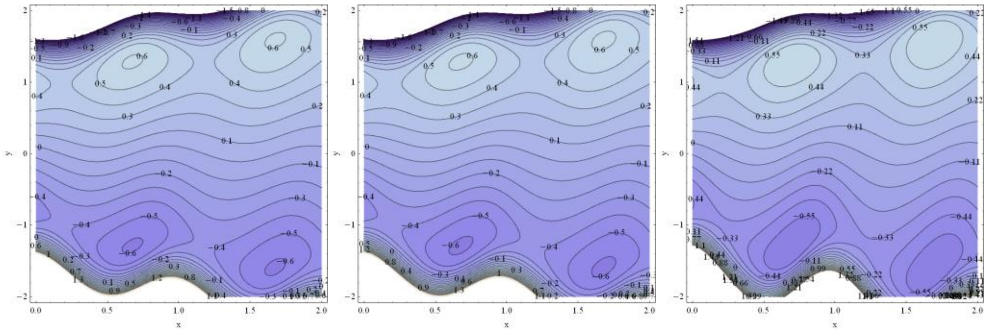


Fig.(3-23) Stream lines for

$m = 0.3, t = 0.5, \phi = \pi/2, b = 0.1, M = 5, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1, \theta' = 1$

$(a) a = 0.2, (b) a = 0.21, (c) a = 0.3$

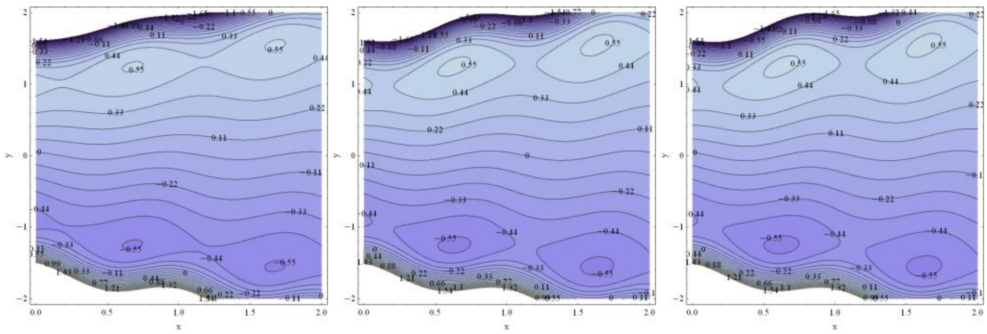


Fig.(3-24) Stream lines for

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.1, M = 5, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1, \theta' = 1$

$(a) b = 0.05, (b) b = 0.09, (c) b = 0.1$

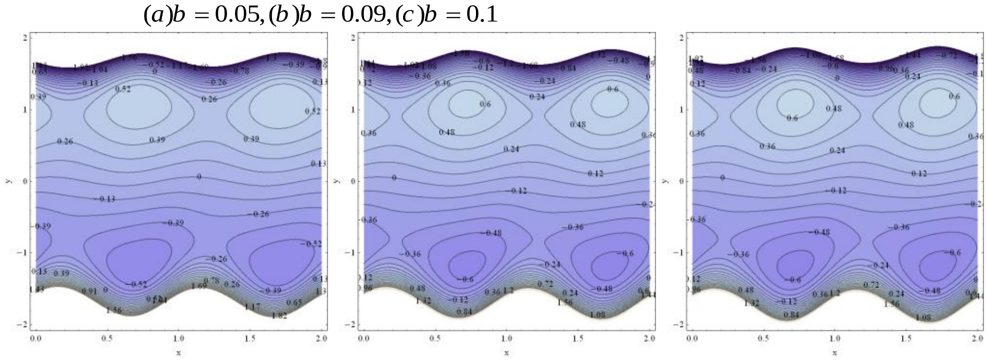


Fig.(3-25) Stream lines for

$m = 0.01, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 5, \alpha = 0.0001, \beta = 2, K = 1, \theta' = 1$

$(a) Gr = 5, (b) Gr = 8, (c) Gr = 10$

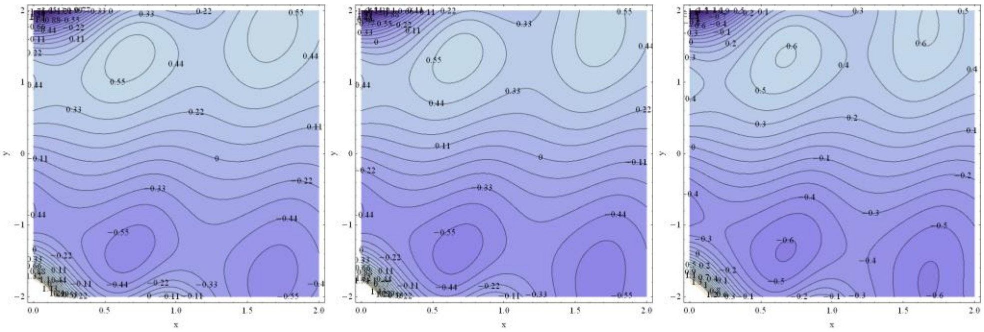


Fig.(3-26) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 2, \alpha = 0.0001, Gr = 5,, K = 1, \theta' = 1$

$(a)\beta = 0.3, (b)\beta = 0.5, (c)\beta = 0.55,$

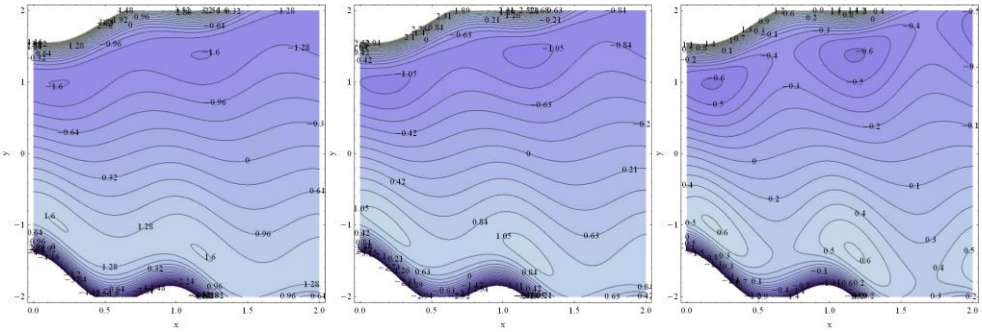


Fig.(3-27) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 5, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1,$

$(a)\theta' = -3, (b)\theta' = -2, (c)\theta' = -1,$

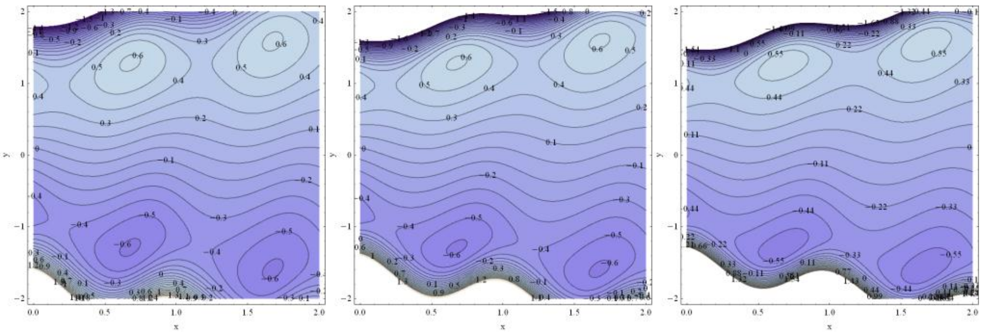


Fig.(3-28) Stream lines for

$m = 0.3, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, \alpha = 0.0001, Gr = 5, \beta = 0.1, K = 1, \theta' = 1$

$(a)M = 3, (b)M = 5, (c)M = 7$

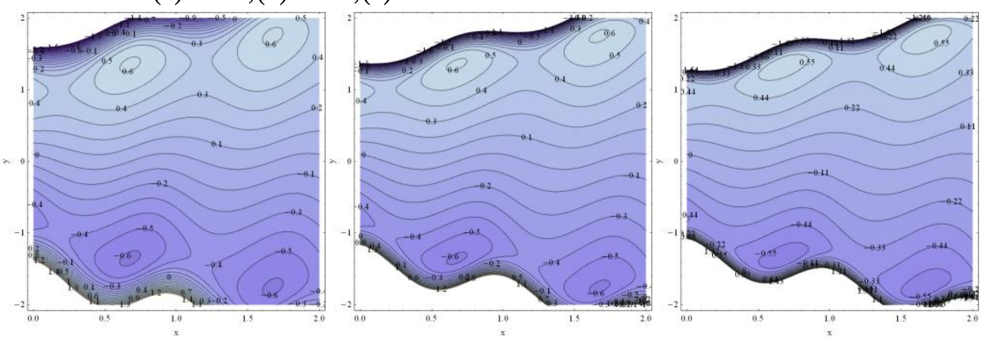


Fig.(3-29) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/2, a = 0.2, b = 0.1, M = 1, \alpha = 0.0001, Gr = 5, \beta = 0.1, \theta' = 1$

$(a)K = 5 (b)K = 10, (c)K = 15,$

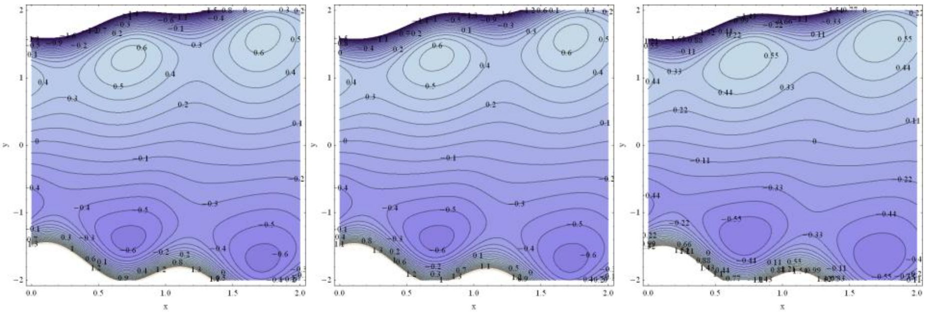


Fig.(3-30) Stream lines for

$m=0.3, t=0.5, \phi=\pi/6, a=0.2, b=0.1, M=5, Gr=5, \beta=0.1, \theta'=1, k=1$

$(a)\alpha=0 (b)\alpha=0.05, (c)\alpha=0.1,$

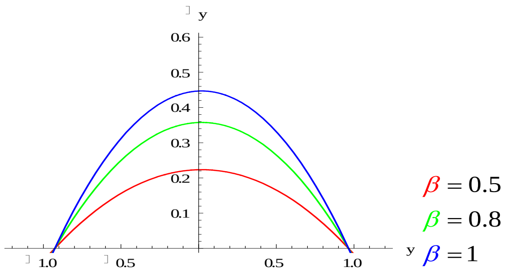


Fig.(3-31) : Effect of source/ sink parameter β on temperature

$m=0.2, t=0.5, \phi=\pi/6, a=0.2, b=0.1, x=0.3$

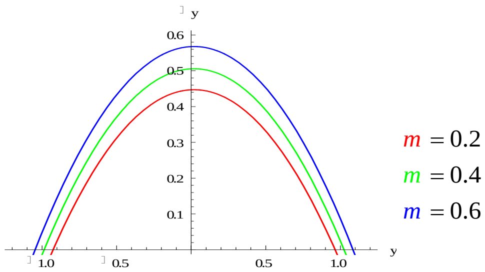


Fig.(3-32) Effect of non-uniform parameter m on temperature

$t=0.5, \phi=\pi/6, a=0.2, b=0.1, \beta=1, x=0.3$

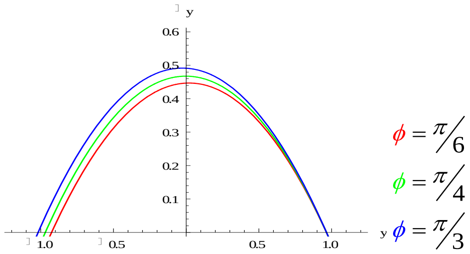


Fig.(3-33) Effect of phase difference parameter on temperature

$m=0.2, t=0.5, a=0.2, b=0.1, \beta=1, x=0.3$

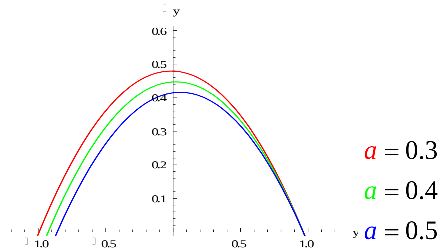


Fig.(3-34) Effect of(a)on temperature

$m=0.2, t=0.5, \phi=\pi/6, b=0.1, \beta=1, x=0.3$

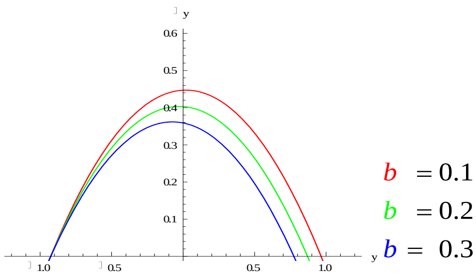


Fig.(3-35) Effect of (b) on temperature.

$m=0.2, t=0.5, \phi=\pi/6, a=0.2, \beta=1, x=0.3$

Chapter Four

Effect of Inclined Magnetic Field on Peristaltic Flow of Williamson Fluid Through Porous Medium in an Inclined Tapered Asymmetric Channel

Introduction

Peristaltic is well known mechanism for pumping biological and industrial fluids. Even though it is observed in living systems for many centuries, the mathematical modeling of peristaltic transport has begun with important works by fung and Yih[35] using laboratory frame of reference. Many of the contributors to the area of peristaltic pumping have either followed Shapiro, [95] or fung, [33]. Most of the studies on peristaltic flow deal with Newtonian fluids. The complex rheology of biological fluids has motivated investigations involving different non Newtonian fluids. Peristaltic flow of non Newtonian fluids. Peristaltic flow of Non Newtonian fluids in a tube was first studies by Raju and Devanathan [82]. Ravi Kumar et al. [87] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Y. V. K. Ravi Kumar et. al. [88] studied the peristaltic pumping of a magneto hydrodynamic casson fluid in an inclined channel. Ravi Kumar et.al. [89] Studied the peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel. Mekheimer[64].studied the peristaltic transport of MHD flow in an inclined planner channel. Hayat et.al. [46] extended the idea of Elshehawey et. al. [30] for partial slip condition. Srinivas et al. [101] studied the peristaltic transport in an asymmetric channel with heat transfer. Srinivas et al. [103] studied the non-linear peristaltic transport in an inclined asymmetric channel. Vajravelu et al. [111] analyzed peristaltic transport of a casson fluid in contact with a Newtonian fluid in circular tube with permeable wall. Nadeem and Akram [75] discussed peristaltic flow of a Williamson fluid in an asymmetric channel. It is observed that most of the physiological fluids for example, blood cannot be described by Newtonian model. Hence, several non Newtonian models are being proposed by various researchers to investigate the flow behavior in Physiological system of a living body. Among them Williamson model is expected to explain most of the features of a physiological fluid. Moreover, this model is nonlinear and Newtonian fluid model may be deduced as a special case of this model.

In this chapter, we will present the peristaltic motion of MHD flow and heat transfer of Williamson fluid in an inclined tapered asymmetric channel through porous medium with the effects of non-slip conditions. By using the perturbation technique for small values of weissenberg number, the nonlinear governing equations are solved under long wave length and low Reynolds number assumption. The stream function, temperature distribution, coefficient of heat transfer, frictional forces at the walls of channel, pressure gradient and pressure rise are calculated. Effect of involved parameters on the flow characteristics have been plotted and examined.

4-1 The Mathematical Model of the Problem

Let us consider the MHD flow and heat transfer of Williamson fluid through a porous medium of two –dimensional inclined tapered a symmetric channel. We assume that infinite wave train traveling with velocity c along the non – uniform walls. We choose a rectangular coordinate system for the channel with $\overline{X}$ along the direction of wave propagation and parallel to the center line and $\overline{Y}$ transverse to it. The wall of the tapered a symmetric channel are given in fig. (4-1) by the eq. (2-1).

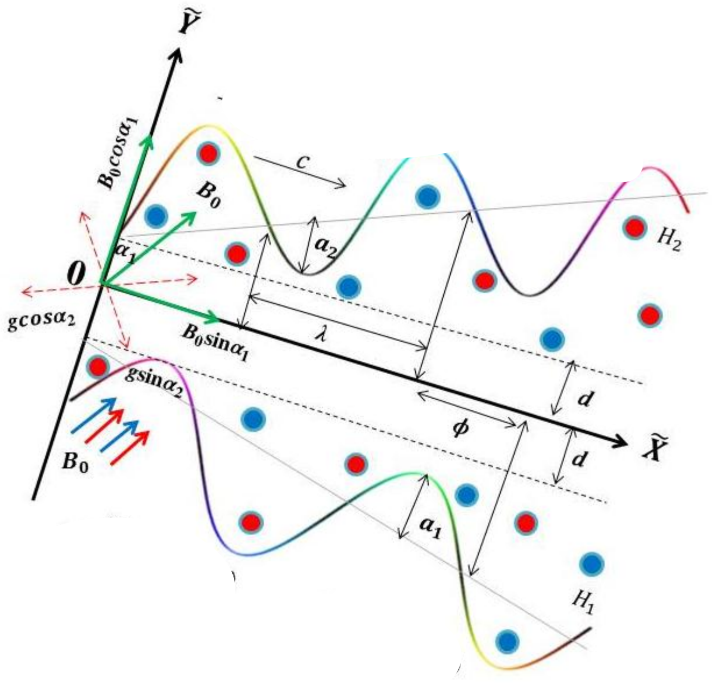


Fig. (4-1): physical structure of the problem

4-2 The Governing Equations

The constitutive equations for a Williamson fluid is given by: [75]

$$\tau = -[\mu_\infty + (\mu_0 + \mu_\infty)(1 - \Gamma \overline{\dot{y}})^{-1}]\overline{\dot{y}} \qquad \text{......(4-1)}$$

where τ is the extra stress tensor, μ_∞ is the infinite shear rate viscosity, μ_0 is the zero shear rate viscosity, Γ is the time constant and $\overline{\dot{y}}$ is defined as :

$$\overline{\dot{y}} = \sqrt{\frac{1}{2}\sum_i \sum_j \overline{\dot{y}}_{ij}\, \overline{\dot{y}}_{ji}} = \sqrt{\frac{1}{2}\Pi} \qquad \text{......(4-2)}$$

Here $\prod$ is the second invariant strain tensor, which is given by $\prod = Tr(A_1^2)$ where

$$A_1 = \nabla\overline{W} + (\nabla\overline{W})^T \qquad ......(4\text{-}3)$$

We consider the constitutive equation (4-3), the case for which $\mu_\infty = 0$ and $\Gamma\dot{\overline{y}} < 1$. Thus, the component of extra stress tensor therefore can be written as

$$\tau = -\mu_0[(1-\Gamma\dot{\overline{y}})^{-1}]\dot{\overline{y}} = -\mu_0[(1+\Gamma\dot{\overline{y}})]\dot{\overline{y}} \qquad ......(4\text{-}4)$$

The above model is reduced to the Newtonian model if $\Gamma = 0$.

Let $\overline{V} = (\overline{u}, \overline{\text{v}})$ be the velocity vector in the Cartesian coordinates in the two-dimension $(\overline{X}, \overline{Y})$.

The strain is defined by:

$$e = \tfrac{1}{2}[(\nabla\overline{W}) + (\nabla\overline{W})^{\mathrm{T}}] = \begin{pmatrix} \dfrac{\partial\overline{U}}{\partial\overline{X}} & \dfrac{1}{2}\left(\dfrac{\partial\overline{U}}{\partial\overline{Y}} + \dfrac{\partial\overline{V}}{\partial\overline{X}}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial\overline{U}}{\partial\overline{Y}} + \dfrac{\partial\overline{V}}{\partial\overline{X}}\right) & \dfrac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ......(4\text{-}5)$$

The shear strain is defined by:

$$\dot{\overline{\gamma}} = 2e = \begin{pmatrix} 2\dfrac{\partial\overline{U}}{\partial\overline{X}} & \dfrac{\partial\overline{U}}{\partial\overline{Y}} + \dfrac{\partial\overline{V}}{\partial\overline{X}} \\ \dfrac{\partial\overline{U}}{\partial\overline{Y}} + \dfrac{\partial\overline{V}}{\partial\overline{X}} & 2\dfrac{\partial\overline{V}}{\partial\overline{Y}} \end{pmatrix} \qquad ......(4\text{-}6)$$

and $(\dot{\overline{y}})^2 = \dfrac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}\dot{\overline{y}}_{ij}\dot{\overline{y}}_{ji}$

Thus:

$$\begin{aligned}(\dot{\overline{y}})^2 &= \frac{1}{2}\sum_{i=1}^{2}(\dot{\overline{y}}_{i1}\dot{\overline{y}}_{1i} + \dot{\overline{y}}_{i2}\dot{\overline{y}}_{2i}) \\ &= \frac{1}{2}((\dot{\overline{y}}_{11}\dot{\overline{y}}_{11} + \dot{\overline{y}}_{12}\dot{\overline{y}}_{21}) + (\dot{\overline{y}}_{21}\dot{\overline{y}}_{12} + \dot{\overline{y}}_{22}\dot{\overline{y}}_{22})) \\ &= \frac{1}{2}((\dot{\overline{y}}_{11})^2 + (\dot{\overline{y}}_{12})^2 + (\dot{\overline{y}}_{12})^2 + (\dot{\overline{y}}_{22})^2) \\ &= \frac{1}{2}((\dot{\overline{y}}_{11})^2 + 2(\dot{\overline{y}}_{12})^2 + (\dot{\overline{y}}_{22})^2)\end{aligned}$$

$$=\frac{1}{2}(4)(\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2+\frac{1}{2}(4)(\frac{\partial\overline{V}}{\partial\overline{Y}})^2$$

$$=2(\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2+2(\frac{\partial\overline{V}}{\partial\overline{Y}})^2$$

$$=2((\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{V}}{\partial\overline{Y}})^2)+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2$$

Hence, we have:

$$\overline{\dot{y}}=\sqrt{2((\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{V}}{\partial\overline{Y}})^2)+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2} \qquad ......(4-7)$$

$$\bar{\tau}_{\overline{XX}}=-2\mu_0[(1+\Gamma\overline{\dot{y}})\frac{\partial\overline{U}}{\partial\overline{X}} \qquad .....(4-7a)$$

$$\bar{\tau}_{\overline{XY}}=-\mu_0[(1+\Gamma\overline{\dot{y}}).(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}}) \qquad .....(4-7b)$$

$$\bar{\tau}_{\overline{YY}}=-\mu_0[(1+\Gamma\overline{\dot{y}}).2\frac{\partial\overline{V}}{\partial\overline{Y}} \qquad .....(4-7c)$$

4-3 Calculation of Lorentz Force [59]

To calculate the Lorentz force $(\overline{J}\times\overline{B})$, we will apply a magnetic field in the $\overline{XY}$ -direction. The effect of this force on the fluid flow, will be analyzed. Now, apply magnetic field in $\overline{XY}$ -direction $(B_0\sin\alpha_1, B_0\cos\alpha_1, 0)$ and to calculate Lorentz force we start with:

$$\overline{V}\times\overline{B}=\begin{vmatrix} e_i & e_j & e_k \\ \overline{U} & \overline{V} & 0 \\ B_0\sin\alpha_1 & B_0\cos\alpha_1 & 0 \end{vmatrix}=B_0(\overline{U}\cos\alpha_1-\overline{V}\sin\alpha_1)e_k \qquad ......(4-8a)$$

Let $\overline{J}=\sigma(\overline{V}\times\overline{B})=\sigma B_0(\overline{U}\cos\alpha_1-\overline{V}\sin\alpha_1)e_k \qquad ......(4-8b)$

Then by Ohm's law one has:

$$\overline{J}\times\overline{B}=\begin{vmatrix} e_i & e_j & e_k \\ 0 & 0 & \sigma B_0(\overline{U}\cos\alpha_1-\overline{V}\sin\alpha_1) \\ B_0\sin\alpha_1 & B_0\cos\alpha_1 & 0 \end{vmatrix}$$

$$=-\sigma B_0^2\cos\alpha_1(\overline{U}\cos\alpha_1-\overline{V}\sin\alpha_1)e_i+\sigma B_0^2\sin\alpha_1(\overline{U}\cos\alpha_1-\overline{V}\sin\alpha_1)e_j \qquad ......(4-8c)$$

Where (e_i, e_j, e_k) are the unit vectors, $\bar{J}$ is the induced current density. We observed that the effect of magnetic field is appear on the flow in the $\overline{XY}$ -direction due to the inclination angle α_1 of magnetic field.

<u>4-4 Basic Equations of the Problem</u>

The basic equations governing the non Newtonian incompressible Williamson fluid under the effect of MHD flow and heat transfer in the inclined direction through porous medium in the laboratory frame $(\bar{X}, \bar{Y})$

The continuity equation is given by:

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \qquad ......(4-9)$$

The momentum equations are:

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial}{\partial \bar{X}}\bar{t}_{\overline{XX}} - \frac{\partial}{\partial \bar{Y}}\bar{t}_{\overline{XY}}$$

$$-\sigma B_0^2 \cos\beta(\bar{U}\cos\beta - \bar{V}\sin\beta) - \frac{\mu_0}{k_0}\bar{U} - \rho g \sin\alpha \qquad ......(4-10)$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial}{\partial \bar{X}}\bar{t}_{\overline{XY}} - \frac{\partial}{\partial \bar{Y}}\bar{t}_{\overline{YY}}$$

$$+\sigma B_0^2 \sin\beta(\bar{U}\cos\beta - \bar{V}\sin\beta) - \frac{\mu_0}{k_0}\bar{V} + \rho g \cos\alpha\,. \qquad ......(4-11)$$

The temperature equation is given by:

$$\rho C_\rho\left(\frac{\partial T}{\partial \bar{t}} + \bar{U}\frac{\partial T}{\partial \bar{X}} + \bar{V}\frac{\partial T}{\partial \bar{Y}}\right) = k_1\left[\frac{\partial^2 T}{\partial(\bar{X})^2} + \frac{\partial^2 T}{\partial(\bar{Y})^2}\right] + 2\mu_0\left[\left(\frac{\partial \bar{U}}{\partial \bar{X}}\right)^2 + \left(\frac{\partial \bar{V}}{\partial \bar{Y}}\right)^2\right] + \mu_0$$

$$\left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}}\right)^2 + \sigma B_0^2(\bar{U}\cos\beta - \bar{V}\sin\beta)^2 + \frac{\mu_0}{k_0}(\bar{U})^2 \qquad ......(4-12)$$

Where $\bar{U}$ is the axial velocity, $\bar{V}$ is transverse velocity, $\bar{Y}$ is transverse coordinates, α is the inclination angle of channel, β is the inclination angle of magnetic field.

<u>4-5 Method of Solution of the Problem</u>

In order to simplify the governing equations of motion, temperature, we may introduce the following dimensionless transformation:

$$x=\frac{\overline{X}}{\lambda},\ y=\frac{\overline{Y}}{d},\ t=\frac{c\bar{t}}{\lambda},\ u=\frac{\overline{U}}{c},\ v=\frac{\overline{V}}{\delta c},\quad h_1=\frac{\overline{H_1}}{d},\quad h_2=\frac{\overline{H_2}}{d},\ p=\frac{d^2\overline{P}}{c\lambda\mu_0},\quad a=\frac{a_1}{d},\quad b=\frac{a_2}{d},$$

$$m=\frac{m'\lambda}{d},\quad \theta=\frac{T-T_0}{T_1-T_0},\ \delta=\frac{d}{\lambda},\quad \mathrm{Re}=\frac{\rho c d}{\mu_0},\ M=\sqrt{\frac{\sigma}{\mu_0}}B_0 d,\ k^2=\frac{d^2}{k_0},\quad P\mathrm{r}=\frac{\mu_0 C\rho}{k_1},$$

$$Ec=\frac{c^2}{C_\rho(T_1-T_0)},\quad \mathrm{t}_{xx}=\frac{\lambda}{\mu_0 c}\bar{t}_{\overline{XX}},\quad \mathrm{t}_{xy}=\frac{d}{\mu_0 c}\bar{t}_{\overline{XY}},\quad \mathrm{t}_{yy}=\frac{d}{\mu_0 c}\bar{t}_{\overline{YY}},\quad \eta=\frac{-d^2\rho g}{c\mu_0}=\frac{-\mathrm{Re}}{Fr},$$

$$\dot{y}=\frac{d}{c}\dot{\overline{y}},\ \mathrm{We}=\frac{\Gamma c}{d},\ \mathrm{Fr}=\frac{c^2}{gd},\ \mathrm{Br}=\mathrm{Pr\,Ec} \qquad \text{......(4-13)}$$

Where T_1 is the temperature of the upper wall, T_0 is the temperature of the lower wall, we is Deisenberg number.

Substituting (4-13) into equations (4-9)-(4-12) we get:

Eq. (4-9) is transformed automatically.

From eq. (4-10) we have:

$$\rho\left(\frac{\partial\overline{U}}{\partial\bar{t}}+\overline{U}\frac{\partial\overline{U}}{\partial\overline{X}}+\overline{V}\frac{\partial\overline{U}}{\partial\overline{Y}}\right)=-\frac{\partial\overline{P}}{\partial\overline{X}}-\frac{\partial}{\partial\overline{X}}\bar{t}_{\overline{XX}}-\frac{\partial}{\partial\overline{Y}}\bar{t}_{\overline{XY}}-\sigma B_0^2\cos\beta(\overline{U}\cos\beta-\overline{V}\sin\beta)$$

$$-\frac{\mu_0}{k_0}\overline{U}-\rho g\sin\alpha.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+Cu\frac{C}{\lambda}\frac{\partial u}{\partial x}+C\delta v\frac{C}{d}\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}-\frac{\mu_0 C}{\lambda^2}\frac{\partial}{\partial x}t_{xx}-\frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{xy}-\sigma\mathrm{B}_0^{\,2}Cos\,\beta$$

$$(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{k_0}Cu-\rho g\sin\alpha.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}u\frac{\partial u}{\partial x}+\frac{C^2}{\lambda}v\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}-\frac{C\mu_0}{\lambda^2}\frac{\partial}{\partial x}t_{xx}-\frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{xy}-\sigma\mathrm{B}_0^{\,2}Cos\,\beta$$

$$(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{k_0}Cu-\rho g\sin\alpha.$$

$$\rho\frac{C^2}{\lambda}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{C\mu_0}{d^2}\frac{\partial P}{\partial x}-\frac{C\mu_0}{\lambda^2}\frac{\partial}{\partial x}t_{xx}-\frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{xy}-\sigma\mathrm{B}_0^{\,2}Cos\,\beta$$

$$(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{k_0}Cu-\rho g\sin\alpha. \qquad \text{........(4-14)}$$

Multiplying both sides of eq. (4-14) by $\left(\frac{d^2}{C\mu_0}\right)$ we obtain:

$$\rho\frac{C^2}{\lambda}\frac{d^2}{C\mu_0}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}-\frac{C\mu_0}{\lambda^2}\frac{d^2}{C\mu_0}\frac{\partial}{\partial x}t_{xx}-\frac{C\mu_0}{d^2}\frac{d^2}{C\mu_0}\frac{\partial}{\partial y}t_{xy}$$

$$-\sigma B_0^{\,2}Cos\,\beta\frac{d^2}{C\,\mu_0}(Cu\cos\beta - C\,\delta v\,\sin\beta) - \frac{\mu_0}{k_0}Cu\,\frac{d^2}{C\,\mu_0} - \rho g\,\sin\alpha\,\frac{d^2}{C\,\mu_0}.$$

$$\frac{\rho Cd}{\mu_0}\frac{d}{\lambda}(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} - \delta^2\,\frac{\partial}{\partial x}t_{xx} - \frac{\partial}{\partial y}t_{xy} - \sigma B_0^{\,2}Cos\,\beta\frac{d^2}{C\,\mu_0}Cu\cos\beta +$$

$$\sigma B_0^{\,2}Cos\,\beta\frac{d^2}{C\,\mu_0}C\,\delta v\,\sin\beta) - \frac{d^2}{k_0}u - \frac{\rho g d^2}{C\,\mu_0}\sin\alpha.$$

$$\text{Re}\,\delta(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} - \delta^2\,\frac{\partial}{\partial x}t_{xx} - \frac{\partial}{\partial y}t_{xy} - \frac{\sigma B_0^{\,2}d^2}{\mu_0}\cos^2\beta u + \frac{\sigma B_0^{\,2}d^2}{\mu_0}\cos\beta$$

$$\sin\beta\delta v - \frac{d^2}{k_0}u - \frac{\rho g d^2}{C\,\mu_0}\sin\alpha.$$

That is

$$\text{Re}\,\delta(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} - \delta^2\,\frac{\partial}{\partial x}t_{xx} - \frac{\partial}{\partial y}t_{xy} - M^2\cos^2\beta u + M^2\cos\beta\sin\beta\delta v$$

$$-K^2u + \eta\sin\alpha.$$

Which can be written as

$$\text{Re}\,\delta(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} - \delta^2\,\frac{\partial}{\partial x}t_{xx} - \frac{\partial}{\partial y}t_{xy} - (M^2\cos^2\beta + K^2)u + M^2\cos\beta$$

$$\sin\beta\delta v + \eta\sin\alpha. \qquad ....(4\text{-}15)$$

From eq. (4-11) we have:

$$\rho(\frac{\partial\overline{V}}{\partial\overline{t}} + \overline{U}\,\frac{\partial\overline{V}}{\partial\overline{X}} + \overline{V}\,\frac{\partial\overline{V}}{\partial\overline{Y}}) = -\frac{\partial\overline{P}}{\partial\overline{Y}} - \frac{\partial}{\partial\overline{X}}\bar{t}_{\overline{XY}} - \frac{\partial}{\partial\overline{Y}}\bar{t}_{\overline{YY}} + \sigma B_0^2\sin\beta(\overline{U}\cos\beta - \overline{V}\sin\beta)$$

$$-\frac{\mu_0}{k_0}\overline{V} + \rho g\,\cos\alpha\,.$$

$$\rho(\frac{C^2d}{\lambda^2}\frac{\partial v}{\partial t} + Cu\,\frac{Cd}{\lambda^2}\frac{\partial v}{\partial x} + C\,\delta v\,.\frac{C}{\lambda}\frac{\partial v}{\partial y}) = -\frac{C\,\lambda\mu_0}{d^3}\frac{\partial P}{\partial y} - \frac{\mu_0 C}{\lambda d}\frac{\partial}{\partial x}t_{xy} - \frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{yy} + \sigma B_0^2$$

$$\sin\beta(Cu\cos\beta - C\,\delta v\,\sin\beta) - \frac{\mu_0}{k_0}C\,\delta v + \rho g\,\cos\alpha\,.$$

$$\rho(\frac{C^2d}{\lambda^2}\frac{\partial v}{\partial t} + \frac{C^2d}{\lambda^2}u\,\frac{\partial v}{\partial x} + \frac{C^2}{\lambda}\frac{d}{\lambda}v\,\frac{\partial v}{\partial y}) = -\frac{C\,\lambda\mu_0}{d^3}\frac{\partial P}{\partial y} - \frac{\mu_0 C}{\lambda d}\frac{\partial}{\partial x}t_{xy} - \frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{yy} + \sigma B_0^2$$

$$\sin\beta(Cu\cos\beta - C\,\delta v\,\sin\beta) - \frac{\mu_0}{k_0}C\,\delta v + \rho g\,\cos\alpha\,.$$

$$\rho(\frac{C^2 d}{\lambda^2}\frac{\partial v}{\partial t}+\frac{C^2 d}{\lambda^2}u\frac{\partial v}{\partial x}+\frac{C^2 d}{\lambda^2}v\frac{\partial v}{\partial y})=-\frac{C\lambda\mu_0}{d^3}\frac{\partial P}{\partial y}-\frac{\mu_0 C}{\lambda d}\frac{\partial}{\partial x}t_{xy}-\frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{yy}+\sigma B_0^2$$

$$\sin\beta(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{k_0}C\delta v+\rho g\cos\alpha\,.$$

$$\rho\frac{C^2 d}{\lambda^2}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{C\lambda\mu_0}{d^3}\frac{\partial P}{\partial y}-\frac{\mu_0 C}{\lambda d}\frac{\partial}{\partial x}t_{xy}-\frac{\mu_0 C}{d^2}\frac{\partial}{\partial y}t_{yy}+\sigma B_0^2\sin\beta\cos\beta$$

$$Cu-\sigma B_0^2\sin^2\beta C\delta v)-\frac{\mu_0}{k_0}C\delta v+\rho g\cos\alpha\,. \qquad ......(4\text{-}16)$$

Now, multiplying both sides of (4-16) by $(\frac{d^3}{C\lambda\mu_0})$ we get:

$$\frac{\rho C^2 d}{\lambda^2}\frac{d^3}{\lambda C\mu_0}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}-\frac{C\mu_0}{\lambda d}\frac{d^3}{\lambda C\mu_0}\frac{\partial}{\partial x}t_{xy}-\frac{C\mu_0}{d^2}\frac{d^3}{C\lambda\mu_0}\frac{\partial}{\partial y}t_{yy}$$

$$+\sigma B_0^2\sin\beta\cos\beta Cu\frac{d^3}{C\lambda\mu_0}-\sigma B_0^2\sin^2\beta C\delta v\frac{d^3}{C\lambda\mu_0}-\mu_0 C\delta v\frac{d^3}{\lambda C\mu_0}+\rho g\cos\alpha$$

$$\frac{d^3}{C\lambda\mu_0}.$$

$$\frac{\rho Cd}{\mu_0}\frac{d^3}{\lambda^3}(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}-\frac{d^2}{\lambda^2}\frac{\partial}{\partial x}t_{xy}-\frac{d}{\lambda}\frac{\partial}{\partial y}t_{yy}+\frac{\sigma B_0^2 d^2}{\mu_0}\frac{d}{\lambda}\sin\beta\cos\beta u-$$

$$\frac{\sigma B_0^2 d^2}{\mu_0}\frac{d}{\lambda}\delta v\sin^2\beta-\frac{d^2}{k_0}\frac{d}{\lambda}\delta v+\frac{\rho g d^2}{C\mu_0}\frac{d}{\lambda}\cos\alpha.$$

That is:

$$\mathrm{Re}\,\delta^3(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}-\delta^2\frac{\partial}{\partial x}t_{xy}-\delta\frac{\partial}{\partial y}t_{yy}+M^2\delta\sin\beta\cos\beta u-M^2\delta^2$$

$$\sin^2\beta\,\mathrm{v}-k^2\delta^2 v-\eta\delta\cos\alpha.$$

Which can be written by the form:

$$\mathrm{Re}\,\delta^3(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}-\delta^2\frac{\partial}{\partial x}t_{xy}-\delta\frac{\partial}{\partial y}t_{yy}+M^2\delta\sin\beta\cos\beta u-(M^2$$

$$\sin^2\beta+k^2)\delta^2 v-\eta\delta\cos\alpha. \qquad .....(4\text{-}17)$$

From eq. (4-12):

$$\rho C_\rho(\frac{\partial T}{\partial \bar{t}}+\bar{U}\frac{\partial T}{\partial \bar{X}}+\bar{V}\frac{\partial T}{\partial \bar{Y}})=k_1[\frac{\partial^2 T}{\partial(\bar{X})^2}+\frac{\partial^2 T}{\partial(\bar{Y})^2}]+2\mu_0[(\frac{\partial\bar{U}}{\partial\bar{X}})^2+(\frac{\partial\bar{V}}{\partial\bar{Y}})^2]+\mu_0$$

$$(\frac{\partial\bar{U}}{\partial\bar{Y}}+\frac{\partial\bar{V}}{\partial\bar{X}})^2+\sigma B_0^2(\bar{U}\cos\beta-\bar{V}\sin\beta)^2+\frac{\mu_0}{k_0}(\bar{U})^2.$$

$$\rho C_p(\frac{C}{\lambda}\frac{\partial T}{\partial t}+Cu\frac{1}{\lambda}\frac{\partial T}{\partial x}+C\delta v\frac{1}{d}\frac{\partial T}{\partial y})=k_1[\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}]+2\mu_0(\frac{C^2}{\lambda^2}(\frac{\partial u}{\partial x})^2+$$

$$\frac{C^2\delta^2}{d^2}(\frac{\partial v}{\partial y})^2)+\mu_0(\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{\partial u}{\partial y})^2+\sigma B_0^2(Cu\cos\beta-C\delta v\sin\beta)^2+\frac{\mu_0}{k_0}C^2u^2.$$

$$\rho C_p(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\frac{\partial T}{\partial x}+C\frac{d}{\lambda}\frac{1}{d}v\frac{\partial T}{\partial y})=k_1[\frac{1}{\lambda^2}\frac{d^2}{d^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}]+2\mu_0(\frac{C^2}{\lambda^2}(\frac{\partial u}{\partial x})^2+$$

$$\frac{C^2}{d^2}\frac{d^2}{\lambda^2}(\frac{\partial v}{\partial y})^2)+\mu_0(\frac{Cd}{\lambda^2}\frac{d}{d}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{\partial u}{\partial y})^2+\sigma B_0^2(C^2u^2\cos^2\beta-2C^2\delta uv\cos\beta\sin\beta+$$

$$C^2\delta^2v^2\sin^2\beta+\frac{\mu_0}{k_0}C^2u^2.$$

$$\rho C_\rho(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\frac{\partial T}{\partial x}+\frac{C}{\lambda}v\frac{\partial T}{\partial y})=\frac{k_1}{d^2}[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}]+\frac{2\mu_0C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+$$

$$\frac{\mu_0C^2}{d^2}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+\sigma B_0^2C^2u^2\cos^2\beta-2\sigma B_0^2C^2\delta uv\cos\beta\sin\beta+\sigma B_0^2C^2\delta^2v^2$$

$$\sin^2\beta+\frac{\mu_0}{k_0}C^2u^2.$$

$$\rho C_\rho\frac{C}{\lambda}(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\frac{k_1}{d^2}[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}]+\frac{2\mu_0C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+$$

$$\frac{\mu_0C^2}{d^2}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+\sigma B_0^2C^2u^2\cos^2\beta-2\sigma B_0^2C^2\delta uv\cos\beta\sin\beta+\sigma B_0^2C^2\delta^2v^2$$

$$\sin^2\beta+\frac{\mu_0}{k_0}C^2u^2.$$

Now, since $\theta=\dfrac{T-T_0}{T_1-T_0}$ therefore $\partial T=(T_1-T_0)\partial\theta$ ......(4−18)

Thus we obtain:

$$\rho C_\rho\frac{C}{\lambda}((T_1-T_0)\frac{\partial\theta}{\partial t}+u(T_1-T_0)\frac{\partial\theta}{\partial x}+v(T_1-T_0)\frac{\partial\theta}{\partial y})=\frac{k_1}{d^2}[\delta^2(T_1-T_0)\frac{\partial^2\theta}{\partial x^2}+$$

$$(T_1-T_0)\frac{\partial^2\theta}{\partial y^2}]+\frac{2\mu_0C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+\frac{\mu_0C^2}{d^2}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+\sigma B_0^2C^2u^2\cos^2\beta$$

$$-2\sigma B_0^2C^2\delta uv\cos\beta\sin\beta+\sigma B_0^2C^2\delta^2v^2\sin^2\beta+\frac{\mu_0}{k_0}C^2u^2.$$

$$\rho C_\rho\frac{C}{\lambda}((T_1-T_0)(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=\frac{k_1}{d^2}(T_1-T_0)[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+\frac{2\mu_0C^2}{\lambda^2}$$

$$((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+\frac{\mu_0 C^2}{d^2}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+\sigma B_0^2 C^2 u^2\cos^2\beta-2\sigma B_0^2 C^2\delta uv\cos\beta$$

$$\sin\beta+\sigma B_0^2 C^2\delta^2 v^2\sin^2\beta+\frac{\mu_0}{k_0}C^2u^2. \qquad ......(4\text{-}19)$$

Multiplying both sides of eq. (4-19) by $(\frac{d^2}{k_1(T_1-T_0)})$ implies to:

$$\rho C_\rho\frac{C}{\lambda}((T_1-T_0)\frac{d^2}{k_1(T_1-T_0)}(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+\frac{2\mu_0 C^2}{\lambda^2}$$

$$\frac{d^2}{k_1(T_1-T_0)}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+\frac{\mu_0 C^2}{d^2}\frac{d^2}{k_1(T_1-T_0)}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+(\sigma B_0^2 C^2 u^2\cos^2\beta$$

$$-2\sigma B_0^2 C^2\delta uv\cos\beta\sin\beta+\sigma B_0^2 C^2\delta^2 v^2\sin^2\beta)\frac{d^2}{k_1(T_1-T_0)}+\frac{\mu_0}{k_0}C^2u^2\frac{d^2}{k_1(T_1-T_0)}.$$

$$\rho C_\rho\frac{C}{\lambda}\frac{d^2}{k_1}\frac{\mu_0}{\mu_0}(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+2\frac{d^2}{\lambda^2}\frac{C_\rho}{C_\rho}\frac{\mu_0 C^2}{k_1}\frac{1}{(T_1-T_0)}((\frac{\partial u}{\partial x})^2$$

$$+(\frac{\partial v}{\partial y})^2)+\frac{\mu_0 C^2}{k_1}\frac{C_\rho}{C_\rho}\frac{1}{(T_1-T_0)}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+\sigma B_0^2 C^2(u^2\cos^2\beta-2\delta uv\cos\beta\sin\beta$$

$$+\delta^2 v^2\sin^2\beta)\frac{d^2}{k_1(T_1-T_0)}\frac{\mu_0}{\mu_0}\frac{C_\rho}{C_\rho}+\frac{d^2}{k_0}\frac{C_\rho}{C_\rho}\frac{\mu_0 C^2}{k_1}\frac{1}{(T_1-T_0)}.$$

Thus we have:

$$\frac{\rho Cd}{\mu_0}\frac{\mu_0 C_\rho}{k_1}\frac{d}{\lambda}(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+2\delta^2\frac{C^2}{C_\rho(T_1-T_0)}\frac{\mu_0 C_\rho}{k_1}((\frac{\partial u}{\partial x})^2$$

$$+(\frac{\partial v}{\partial y})^2)+\frac{\mu_0 C_\rho}{k_1}\frac{C^2}{C_\rho(T_1-T_0)}(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+k^2\frac{C^2}{C_\rho(T_1-T_0)}\frac{\mu_0 C_\rho}{k_1}u^2+\frac{\sigma B_0^2 d^2}{\mu_0}\frac{\mu_0 C_\rho}{k_1}$$

$$\frac{C^2}{C_\rho(T_1-T_0)}(u^2\cos^2\beta-2\delta uv\cos\beta\sin\beta+\delta^2 v^2\sin^2\beta).$$

Which can be written as the form;

$$\operatorname{Re}\Pr\delta(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+2\delta^2 Ec\Pr((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+Ec\Pr$$

$$(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})^2+k^2 Ec\Pr u^2+M^2 Ec\Pr(u^2\cos^2\beta-2\delta uv\cos\beta\sin\beta+\delta^2 v^2\sin^2\beta).$$

That is:

$$\operatorname{Re}\Pr\delta(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})=[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}]+2\delta^2 Br((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+Br(\delta^2\frac{\partial v}{\partial x}$$
$$+\frac{\partial u}{\partial y})^2+Br(k^2+M^2\cos^2\beta)u^2-2uvM^2Br\delta\cos\beta\sin\beta+M^2Br\delta^2v^2\sin^2\beta)$$

......(4-20)

From eq. (4-7) we have:

$$\overline{\dot{y}}=\sqrt{2((\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{V}}{\partial\overline{Y}})^2)+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2}$$

That is:

$$(\overline{\dot{y}})^2=2((\frac{\partial\overline{U}}{\partial\overline{X}})^2+(\frac{\partial\overline{V}}{\partial\overline{Y}})^2)+(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})^2$$

$$(\overline{\dot{y}})^2=2(\frac{C^2}{\lambda^2}(\frac{\partial u}{\partial x})^2+\frac{C^2}{\lambda^2}(\frac{\partial v}{\partial y})^2)+(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{Cd}{\lambda^2}\frac{\partial v}{\partial x})^2$$

$$\frac{C^2}{d^2}(\dot{y})^2=2\frac{C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{Cd}{\lambda^2}\frac{d}{d}\frac{\partial v}{\partial x})^2$$

$$\frac{C^2}{d^2}(\dot{y})^2=2\frac{C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+(\frac{C}{d}\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2$$

$$\frac{C^2}{d^2}(\dot{y})^2=2\frac{C^2}{\lambda^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+\frac{C^2}{d^2}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2 \qquad ....(4-21)$$

Now, multiplying both sides of eq.(4-21) by $(\frac{d^2}{C^2})$ we have:

$$(\dot{y})^2=2\frac{C^2}{\lambda^2}\frac{d^2}{C^2}((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2$$

Thus we have:

$$(\dot{y})^2=2\delta^2((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2$$

which may written as:

$$(\dot{y})=\sqrt{2\delta^2((\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2)+(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2} \qquad ......(4-22)$$

From eq. (4-7a):

$$\bar{\tau}_{\overline{XX}} = -2\mu_0[(1+\Gamma\overline{\dot{y}})\frac{\partial\overline{U}}{\partial\overline{X}},$$

$$\frac{\mu_o C}{\lambda}\tau_{xx} = -2\mu_0[(1+\Gamma\overline{\dot{y}})\frac{C}{\lambda}\frac{\partial u}{\partial x}$$

$$\frac{\mu_o C}{\lambda}\tau_{xx} = -2\frac{\mu_o C}{\lambda}[(1+\Gamma\overline{\dot{y}})\frac{\partial u}{\partial x} \qquad ......(4-23)$$

Multiplying both sides of eq. (4-23) by $(\frac{\lambda}{\mu_0 C})$ we obtain:

$$\tau_{xx} = -2[(1+\Gamma\overline{\dot{y}})\frac{\partial u}{\partial x}$$

$$\tau_{xx} = -2[(1+\frac{\Gamma C}{d}\dot{y})\frac{\partial u}{\partial x}$$

Thus we have:

$$t_{xx} = -2[(1+We\,\dot{y})]\frac{\partial u}{\partial x} \qquad ......(4-24)$$

From eq. (4-7b):

$$\bar{\tau}_{\overline{XY}} = -\mu_0[(1+\Gamma\overline{\dot{y}}).(\frac{\partial\overline{U}}{\partial\overline{Y}}+\frac{\partial\overline{V}}{\partial\overline{X}})$$

$$\frac{\mu_0 C}{d}\tau_{xy} = -\mu_0[(1+\Gamma\overline{\dot{y}}).(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{Cd}{\lambda^2}\frac{\partial v}{\partial x})$$

$$\frac{\mu_0 C}{d}\tau_{xy} = -\mu_0[(1+\Gamma\overline{\dot{y}}).(\frac{C}{d}\frac{\partial u}{\partial y}+\frac{Cd}{\lambda^2}\frac{d}{d}\frac{\partial v}{\partial x})$$

$$\frac{\mu_0 C}{d}\tau_{xy} = -\mu_0[(1+\Gamma\overline{\dot{y}}).\frac{C}{d}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})$$

$$\frac{\mu_0 C}{d}\tau_{xy} = -\frac{\mu_0 C}{d}[(1+\Gamma\overline{\dot{y}}).(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}) \qquad .....(4-25)$$

Multiplying both sides of eq. (4-25) by $(\frac{d}{\mu_0 C})$ we get:

$$\tau_{xy} = -[(1+\Gamma\overline{\dot{y}}).(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})$$

$$\tau_{xy} = -[(1+\frac{\Gamma C}{d}\dot{y}).(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})$$

Thus we have:

$$\tau_{xy} = -[(1+We\,\dot{y}).(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}) \qquad ......(4-26)$$

From eq. (4-7c):

$$\bar{\tau}_{\bar{Y}\bar{Y}} = -2\mu_0[(1+\Gamma\bar{\dot{y}}).2\frac{\partial\bar{V}}{\partial\bar{Y}}$$

$$\frac{\mu_0 C}{d}\tau_{yy} = -2\mu_0[(1+\Gamma\bar{\dot{y}}).\frac{C}{\lambda}\frac{\partial v}{\partial y} \qquad ......(4-27)$$

Multiplying both sides of (4-27) by $(\frac{d}{\mu_0 C})$ we get:

$$\tau_{yy} = -2\frac{\mu_0 C}{\lambda}\frac{d}{\mu_0 C}[(1+\Gamma\bar{\dot{y}}).\frac{\partial v}{\partial y}$$

$$\tau_{yy} = -2\delta[(1+\frac{\Gamma C}{d}\dot{y}).\frac{\partial v}{\partial y}$$

Thus we have:

$$\tau_{yy} = -2\delta[(1+We\,\dot{y}).\frac{\partial v}{\partial y} \qquad ....(4-28)$$

Now, under the assumption of long wave length $(\delta \ll 1)$ and low Reynolds number, the eqs. (4-15), (4-17), (4-20), (4-22), (4-24), (4-26) and (4-28) can be written as:

$$0 = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial y}\tau_{xy} - (M^2\cos^2\beta + k^2)u + \eta\sin\alpha \qquad ....(4-29)$$

$$0 = -\frac{\partial p}{\partial y} \qquad ....(4-30)$$

$$0 = \frac{\partial^2\theta}{\partial y^2} + \mathrm{Br}(\frac{\partial u}{\partial y})^2 + \mathrm{Br}(M^2\cos^2\beta + k^2)u^2 \qquad ....(4-31)$$

$$\dot{y} = \frac{\partial u}{\partial y} \qquad ....(4-32)$$

$$\tau_{xx} = -2[1+we\,\dot{y}]\frac{\partial u}{\partial y} \qquad ....(4-33)$$

$$\tau_{xy} = -[1+we\,\dot{y}]\frac{\partial u}{\partial y} \qquad ....(4-34)$$

$$\tau_{yy} = 0 \qquad ....(4-35)$$

Introducing the stream functions $(u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x})$ in eqs. (4-29)- (4-34) implies to:

$$0 = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial y}\tau_{xy} - (M^2 \cos^2\beta + k^2)\frac{\partial \psi}{\partial y} + \eta \sin\alpha \qquad \text{........(4-36)}$$

$$0 = -\frac{\partial^2 \theta}{\partial y^2} + \text{Br}(\frac{\partial^2 \psi}{\partial y^2})^2 + \text{Br}(M^2 \cos^2\beta + k^2)(\frac{\partial \psi}{\partial y})^2 \qquad \text{.........(4-37)}$$

$$\dot{y} = \frac{\partial^2 \psi}{\partial y^2} \qquad \text{.........(4-38)}$$

$$\tau_{xx} = \text{-}2[1 + \text{we}\frac{\partial^2 \psi}{\partial y^2}]\frac{\partial^2 \psi}{\partial x \partial y} \qquad \text{.........(4-39)}$$

$$\tau_{xy} = \text{-}[1 + \text{we}\frac{\partial^2 \psi}{\partial y^2}]\frac{\partial^2 \psi}{\partial y^2} \qquad \text{.........(4-40)}$$

4-6 Rate of Volume Flow and Boundary Conditions

In order to discuss the results quantitatively we assume that the instantaneous volume rate of the flow F(x, t), is periodic in (x-t), as: [58]

$$F(x,t) = \theta' + a\sin(2\pi(x-t)+\phi) + b\sin(2\pi(x-t)) \qquad \text{......(4-41)}$$

In which θ' is the mean flow rate in the wave frame, F is the mean flow rate in the laboratory frame:

$$F = \int_{h_1}^{h_2} u\,dy$$

$$= \int_{h_1}^{h_2} \frac{\partial \psi}{\partial y} dy = \psi(h_2) - \psi(h_1)$$

Selecting $\psi(h_2) = \frac{F}{2}$, then implies $\psi(h_1) = \frac{-F}{2}$

The boundary conditions in dimensionless stream function will now take the following form:

$$\left.\begin{array}{l} \psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} = 0 \ \text{ and } \theta = 1 \ at\ (y = h_2) \\ \psi = \frac{-F}{2}, \frac{\partial \psi}{\partial y} = 0 \ \text{ and } \frac{\partial \theta}{\partial y} = 0 \ at\ (y = h_1) \end{array}\right] \qquad \text{......(4-42)}$$

In which

$$h_2 = 1 + mx + b\sin(2\pi(x - t))$$
$$h_1 = -1 - mx - a\sin(2\pi(x - t) + \phi)$$

The non-dimensional expression for the average rise pressure Δp is given as in eq. (2-47):

The frictional force F_1' at the lower wall $y = h_1$ across the wave length is given by:

$$F_1' = \int_0^1 (h_1^2(-\frac{\partial P}{\partial x})dx)_{t=0.5} \qquad ......(4-43)$$

The coefficient of heat transfer at the upper wall is given by:

$$Z = (h_2)_x(\theta y)_{y=h_2} \qquad ......(4-44)$$

4-7 Perturbation Analysis of the Problem

It is clear that the resulting equation of motion Eq.(4-40) and equation of heat which is expressed by the eq.(4-41) are not linear because it contains unknown ψ of some powers which must be solved to yield the desired stream function of fluid and the heat transfer of fluid. Due to that non linearity it is difficult to solve it. Thus we use the perturbation technique to find the solution. We expand ψ, F, P and θ for series of small weissenberg number, thus we write:

$$\psi = \psi_0 + We\psi_1 + ...$$
$$F = F_0 + WeF_1 + ...$$
$$p = p_0 + Wep_1 + ...$$
$$\theta = \theta_0 + We\theta_1 + ... \qquad ....(4-45)$$

Now substituting Eq.(4-45) into Eq. (4-36),(4-37),(4-38),(4-39) and (4-40) , thus we get:

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}[\frac{\partial^2\psi}{\partial y^2} + We(\frac{\partial^2\psi}{\partial y^2})^2] - (M^2\cos^2\beta + K^2)\frac{\partial\psi}{\partial y} + \eta\sin\alpha$$

$$\frac{\partial}{\partial x}(p_0 + Wep_1) = \frac{\partial}{\partial y}[\frac{\partial^2}{\partial y^2}(\psi_0 + We\psi_1) + We(\frac{\partial^2}{\partial y^2}(\psi_0 + We\psi_1))^2] - (M^2\cos^2\beta + K^2)$$

$$\frac{\partial}{\partial y}(\psi_0 + We\psi_1) + \eta\sin\alpha.$$

$$\frac{\partial}{\partial x}(p_0 + Wep_1) = \frac{\partial}{\partial y}[\frac{\partial^2\psi_0}{\partial y^2} + We\frac{\partial^2\psi_1}{\partial y^2} + We(\frac{\partial^2\psi_0}{\partial y^2} + We\frac{\partial^2\psi_1}{\partial y^2})^2] - (M^2\cos^2\beta + K^2)$$

$$(\frac{\partial\psi_0}{\partial y}+We\frac{\partial\psi_1}{\partial y})+\eta\sin\alpha.$$

$$\frac{\partial}{\partial x}(p_0+Wep_1)=\frac{\partial}{\partial y}[\frac{\partial^2\psi_0}{\partial y^2}+We\frac{\partial^2\psi_1}{\partial y^2}+We((\frac{\partial^2\psi_0}{\partial y^2})^2+2We\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}+(We)^2(\frac{\partial^2\psi_1}{\partial y^2})^2)]-$$
$$(M^2\cos^2\beta+K^2)(\frac{\partial\psi_0}{\partial y}+We\frac{\partial\psi_1}{\partial y})+\eta\sin\alpha.$$

$$\frac{\partial}{\partial x}(p_0+Wep_1)=\frac{\partial}{\partial y}[\frac{\partial^2\psi_0}{\partial y^2}+We\frac{\partial^2\psi_1}{\partial y^2}+We(\frac{\partial^2\psi_0}{\partial y^2})^2+2(We)^2\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}+(We)^3(\frac{\partial^2\psi_1}{\partial y^2})^2]-$$
$$(M^2\cos^2\beta+K^2)(\frac{\partial\psi_0}{\partial y}+We\frac{\partial\psi_1}{\partial y})+\eta\sin\alpha. \qquad ....(4-46)$$

Also, we have from eq. (4-37):

$$0=\frac{\partial^2\theta}{\partial y^2}+\mathrm{Br}[(\frac{\partial^2\psi}{\partial y^2})^2+\mathrm{Br}(M^2\cos^2\beta+K^2)(\frac{\partial\psi}{\partial y})^2$$

$$0=\frac{\partial^2}{\partial y^2}(\theta_0+We\theta_1)+\mathrm{Br}(\frac{\partial^2}{\partial y^2}(\psi_0+We\psi))^2+\mathrm{Br}(M^2\cos^2\beta+K^2)(\frac{\partial}{\partial y}(\psi_0+We\psi))^2$$

$$0=\frac{\partial^2\theta_0}{\partial y^2}+We\frac{\partial^2\theta_1}{\partial y^2}+\mathrm{Br}(\frac{\partial^2\psi_0}{\partial y^2}+We\frac{\partial^2\psi_1}{\partial y^2})^2+\mathrm{Br}(M^2\cos^2\beta+K^2)(\frac{\partial\psi_0}{\partial y}+We\frac{\partial\psi_1}{\partial y})^2$$

$$0=\frac{\partial^2\theta_0}{\partial y^2}+We\frac{\partial^2\theta_1}{\partial y^2}+\mathrm{Br}[(\frac{\partial^2\psi_0}{\partial y^2})^2+2We\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}+(We)^2(\frac{\partial^2\psi_1}{\partial y^2})^2]+\mathrm{Br}(M^2\cos^2\beta$$

$$+K^2)[(\frac{\partial\psi_0}{\partial y})^2+2We\frac{\partial\psi_0}{\partial y}\frac{\partial\psi_1}{\partial y}+(We)^2(\frac{\partial\psi_1}{\partial y})^2] \qquad ......(4-47)$$

Now, collecting the coefficient of like powers of We, thus one can gets the zeroth and first order equations as:

<u>4-7-1 Zero's- order system</u> $(We^{(0)})$

$$\frac{\partial p_0}{\partial x}=\frac{\partial}{\partial y}(\frac{\partial^2\psi_0}{\partial y^2})-N_1\frac{\partial\psi_0}{\partial y}+\eta\sin\alpha$$

where $N_1=(M^2\cos^2\beta+k^2)$ ....(4-48)

Differentiating eq. (4-53) with respect to y we have:

$$0=\frac{\partial^2}{\partial y^2}(\frac{\partial^2\psi_0}{\partial y^2})-N_1\frac{\partial^2\psi_0}{\partial y^2} \qquad ......(4-49)$$

which can be written as:

$$0=(\frac{\partial^4\psi_0}{\partial y^4})-N_1\frac{\partial^2\psi_0}{\partial y^2} \qquad ......(4-50)$$

Also we have:

$$0=\frac{\partial^2\theta_0}{\partial y^2}+\mathrm{Br}(\frac{\partial^2\psi_0}{\partial y^2})^2+\mathrm{Br}N_1(\frac{\partial\psi_0}{\partial y})^2 \qquad ......(4-51)$$

Along with the corresponding boundary conditions:

$$\psi_0=\frac{F_0}{2},\frac{\partial\psi_0}{\partial y}=0,\ \theta_0=1,at\left(y=h_2\right)$$

$$\psi_0=\frac{-F_0}{2},\frac{\partial\psi_0}{\partial y}=0,\frac{\partial\theta_0}{\partial y}=0,at\left(y=h_1\right) \qquad ......(4-52)$$

4-7-2 First order system $(We^{(1)})$

$$\frac{\partial p_1}{\partial x}=\frac{\partial}{\partial y}(\frac{\partial^2\psi_1}{\partial y^2}+(\frac{\partial^2\psi_0}{\partial y^2})^2-N_1\frac{\partial\psi_1}{\partial y} \qquad .....(4-53)$$

Differentiable eq. (4-58) with respect to y we have:

$$0=\frac{\partial^2}{\partial y^2}(\frac{\partial^2\psi_1}{\partial y^2}+(\frac{\partial^2\psi_0}{\partial y^2})^2)-N_1\frac{\partial^2\psi_1}{\partial y^2} \qquad ......(4-54)$$

Also we have:

$$0=\frac{\partial^2\theta_1}{\partial y^2}+2\mathrm{Br}(\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2})+2\mathrm{Br}N_1(\frac{\partial\psi_0}{\partial y}\frac{\partial\psi_1}{\partial y}) \qquad ....(4-55)$$

The corresponding boundary conditions are:

$$\psi_1=\frac{F_1}{2},\frac{\partial\psi_1}{\partial y}=0,\ \theta_1=0\quad at\left(y=h_2\right)$$

$$\psi_1=\frac{-F_1}{2},\frac{\partial\psi_1}{\partial y}=0,\frac{\partial\theta_1}{\partial y}=0\ \ at\left(y=h_1\right) \qquad ......(4-56)$$

4-8 solution of the problem

4-8-1 Solution for the zeroth order system $(We^{(0)})$

We solve Eq. (4-50) and we can find the solution of the zeroth order system which is:

$$\psi_0=a_3+a_2e^{-n_1y}n_2+a_1e^{n_1y}n_2+a_4y; \qquad ......(4-57)$$

$$where(n_1=\sqrt{N_1};n_2=\frac{1}{N_1});$$

Also, if we solve the equation (4-51), we can obtain the solution of temperature of the zeroth order system which is:

$$\theta_0 = 2a_2a_4Bre^{-n_1y}n_1n_2 - 2a_1a_4Bre^{n_1y}n_1n_2 - \frac{1}{2}a_2{}^2Bre^{-2n_1y}n_1^2n_2^2 - \frac{1}{2}a_1{}^2Bre^{2n_1y}n_1^2n_2^2 - \frac{1}{2}a_4{}^2Br\,n_1^2\,\mathrm{y}^2 + \mathrm{c}_1 + \mathrm{yc}_2; \qquad ......(4-58)$$

$a_i,(i=1,2,3,4)$ and $C_j,(j=1,2)$ are constants can be obtained by using the boundary conditions in Eq.(4-52) that is:

$$a_1 = \frac{f_0}{(e^{h_1n_1}(-2+h_1n_1-h_2n_1)+e^{h_2n_1}(2+h_1n_1-h_2n_1))n_2};$$

$$a_2 = -\frac{e^{(h_1+h_2)n_1}f_0}{(e^{h_1n_1}(-2+h_1n_1-h_2n_1)+e^{h_2n_1}(2+h_1n_1-h_2n_1))n_2};$$

$$a_3 = \frac{(e^{h_1n_1}+e^{h_2n_1})f_0(h_1+h_2)n_1}{2(e^{h_1n_1}(-2+h_1n_1-h_2n_1)+e^{h_2n_1}(2+h_1n_1-h_2n_1))};$$

$$a_4 = \frac{(e^{h_1n_1}+e^{h_2n_1})f_0(h_1+h_2)n_1}{(e^{h_1n_1}(-2+h_1n_1-h_2n_1)+e^{h_2n_1}(2+h_1n_1-h_2n_1))};$$

$$c_1 = \frac{1}{2}(2+a_4{}^2Brh_2(-2h_1+h_2)n_1^2 - 4a_4Bre^{-(h_1+h_2)n_1}n_1(-a_1e^{(h_1+2h_2)n_1}+a_1e^{(2h_1+h_2)n_1}h_2n_1 + a_2(e^{h_1n_1}+e^{h_2n_1}h_2n_1))n_2 + a_1^2Bre^{2h_2n_1}n_1^2n_2^2 - 2a_1^2Bre^{2h_1n_1}h_2n_1^3n_2^2 + a_2^2Brn_1^2(e^{-2h_2n_1}+2e^{-2h_1n_1}h_2n_1)n_2^2;$$

$$c_2 = Bre^{-2h_1n_1}n_1^2(a_4^2e^{2h_1n_1}h_1 + 2a_4e^{h_1n_1}(a_2+a_1e^{2h_1n_1})n_2 - (a_2{}^2-a_1^2e^{4h_1n_1})n_1n_2{}^2);$$

<u>**4-8-2 Solution of the first order system**</u> $(We^{(1)})$

If we solve the equation (4-54) we can find the solution of the first order system which is:

$$\psi_1 = \frac{-e^{-2n_1y}(a_2^2n_1^4n_2^2 + a_1^2e^{4n_1y}n_1^4n_2^2 - 3e^{n_1y}(e^{2n_1y}b_1+b_2))}{3n_1^2} + b_3 + yb_4; \qquad ....(4-59)$$

Also, if we solve the equation (4-55) we can find the solution of temperature of the first order system which is:

$$\theta_1 = -\frac{1}{9n_1}Br\,(4a_2{}^3e^{-3n_1y}n_1^5n_2^3 + 4a_1^3e^{3n_1y}n_1^5n_2^3 + 3\mathrm{a}_1e^{2n_1y}n_1n_2(-3b_1+a_1a_4n_1^3n_2) - 3\mathrm{a}_2 e^{-2n_1y}n_1n_2(-3b_2+a_2a_4n_1^3n_2) - 6e^{n_1y}(3a_4b_1+a_1n_1^2n_2(3b_4-2\mathrm{a}_1\mathrm{a}_2n_1^3n_2^2)) + 6e^{-n_1y}(3a_4b_2 + a_2n_1^2n_2(3b_4-2\mathrm{a}_1\mathrm{a}_2n_1^3n_2^2)) - 9a_4b_4n_1^3\mathrm{y}^2) + \mathrm{c}_3 + \mathrm{yc}_4 \qquad ......(4-60)$$

$b_i,(i=1,2,3,4)$ and $\mathrm{C}_j,(j=3,4)$ are constants can be determinates by using the boundary conditions in Eq.(4-56).

4-8-3 Solution of the heat transfer coefficient z(x)

If we solve the equation (4-44), we obtain the solution of heat transfer coefficient z(x) at the upper wall $y = h_2$, which is:

$$
\begin{aligned}
z(x) = \frac{1}{3} e^{-3n_1 y} & (3C_2 e^{3n_1 y} - 4a_2{}^3 Br\, n_1^5 n_2^3 we - a_2^2 Bre^{n_1 y} n_1^3 n_2{}^2(-3 - 2a_4 n_1 we + 4a_1 e^{n_1 y} \\
& n_1^2 n_2 we) + 2a_2 Bre^{n_1 y} n_1 n_2(-3a_4 e^{n_1 y} n_1 + (3b_2 - 3b_4 e^{n_1 y} n_1 + 2a_1^2 e^{3n_1 y} n_1^4 n_2^2) we) + e^{2n_1 y} \\
& (4a_1^3 Bre^{4n_1 y} n_1^5 n_2^3 we + a_1^2 Bre^{3n_1 y} n_1^3 n_2^2)(-3 + 2a_4 n_1 we - 6a_1 Bre^{2n_1 y} n_1 n_2 (a_4 n_1 + b_1 e^{n_1 y} we \\
& + b_4 n_1 we) - 3(-C_4 e^{n_1 y} we + a_4{}^2 Bre^{n_1 y} n_1^2 y + 2a_4 Br\, we(b_2 + b_1 e^{2n_1 y} + b_4 e^{n_1 y} n_1^2 y)))) \\
& (m + 2b\pi \cos(2\pi(t - x)); \qquad \ldots\ldots(4-61)
\end{aligned}
$$

4-9 Results and Discussion

In this section, the numerical and computational results are discussed for the problem of peristaltic transport of incompressible Non Newtonian Williamson fluid under the effect of inclined magnetic field through porous medium in an inclined tapered asymmetric channel with help of using heat transfer and non-slip conditions. The numerical evaluations of the analytical results which is showed by using the perturbation technique for small values of wiessenberg number under the assumption of long wave length and low Reynolds number approximation. The effect of some important parameters are displayed graphically.

4-9-1 Pumping Characteristic

Figure (4-2)-(4-5) shows the variation of Δp against time mean flow rate θ'. The whole region is considered into five parts (1) peristaltic pumping region where $(\Delta p > 0,\ \theta' > 0)$, (2) augmented pumping (co-pumping) region where $(\Delta p < 0,\ \theta' > 0)$, (3) when $(\Delta p > 0,\ \theta' < 0)$, then it is retrograde pumping region. There is a co-pumping region where $(\Delta p < 0,\ \theta' < 0)$. (5) $(\Delta p = 0)$ Corresponds to the free pumping region. The expression for Δp via θ' is showed in eq. (2-47). The effects of sundry parameters on Δp have been evaluated numerically using (MATHIMATICA) program and the results are presented graphically impact of Hartmann number (M), the inclination angle of channel (α), inclination of magnetic field β and the parameter (η) have been come out. Figure (4-2) shows the impact of M on pressure rise Δp, it can be seen from the graph that in the retrograde region of pumping $(\Delta p > 0,\ \theta' < 0)$. The pumping rate increase and the case is conversed in the co-pumping $(\Delta p < 0,\ \theta' > 0)$ and free pumping as well as when $\theta' \in (-2,0)$. Figure (4-3) displayed the effect of β on pressure rise, it is noticed that the greater influence of β is showed in the augmented region and free region of pumping and the pumping rate is increased in these regions. Figures (4-4) and (4-5) illustrated the effects of α and (η) on Δp respectively, it is seen that the relation between pressure rise Δp and flow rate θ' is linear and the rate of pumping is enhanced in all regions with an increase of these parameters.

4-9-2 Frictional force characteristic

We found the expression for frictional force F_1' at lower wall across one wave length is given by eq. (4-43) against θ' for various values of parameters of interest in figures (4-6)-(4-14). The effects of these parameters on F_1' have been evaluated numerically using (MATHEMATICA)program and the results are presented graphically impact of Hartmann number (M), non-uniform parameter (m), the phase difference (ϕ), the porosity parameter(k), the amplitudes of upper and lower walls of the channel (a &b), the inclination angle of the channel (α), the inclination of magnetic field (β) and the parameter (η) have been carried out. Frictional force regions can be divided in three types which are $(F_1' > 0,\ \theta' > 0)$, $(F_1' < 0,\ \theta' < 0)$ and $(F_1' = 0)$. In fig. (4-6), the effects of non-uniform parameter (m) on F_1' are seen, observed that frictional force decrease in both of regions $(F_1' > 0,\ \theta' > 0)$ and $(F_1' < 0,\ \theta' < 0)$ increase by clear way in the region $(F_1' = 0)$. Figures (4-7) and (4-8) illustrated the influence of (ϕ) and b respectively, it is noticed that the frictional force in the regions $(F_1' < 0,\ \theta' < 0)$ and $(F_1' = 0)$. The effect of (a) is displayed in figure (4-9), it is observed that an increase in this parameter lead to decreasing in frictional force in the region $(F_1' > 0,\ \theta' > 0)$ and it is increasing in the regions of $(F_1' < 0,\ \theta' < 0)$ and $(F_1' = 0)$ which has similar influence of (b) and (ϕ) .the effects of M and k are illustrated in figure (4-10) and (4-11) respectively, it is noted that an increase in these parameters lead to rise up in frictional force at the region $(F_1' = 0)$ and it is reduced at the regions of $(F_1' > 0,\ \theta' > 0)$ and $(F_1' < 0,\ \theta' < 0)$ figures (4-12), (4-13) and (4-14) displayed the effects of $(\eta, \alpha$ and $\beta)$ respectively and we observed that if we increase these parameters then the frictional force increase at the regions $(F_1' > 0,\ \theta' > 0)$ and $(F_1' < 0,\ \theta' < 0)$ decrease in the region $(F_1' = 0)$.

4-9-3 Velocity distribution

Influences of various parameters on the velocity distribution have been illustrated in Fig. (4-15)- (4-20). These figures are scratched at the fixed values of x=0.3, t=0.5. The change in values of (m and ϕ) on the axial velocity u is shown in fig. (4-15), it can be found that the axial velocity u decrease at the central line and increase at the edges of the walls and the flow of fluid is reflected at two points which are (0.5981, 0.3765) and (-0.3194, 0.5093). Fig. (4-16) shows the influence of $(a\&b)$ on the axial velocity u, it observed that an increase in previous parameters causes reduce in velocity at the central line and the walls. The effects of M and k on velocity distribution are plotted in figures (4-17), it is noticed that an increase in these parameters lead to decrease in velocity at the central line and increase at the ends of the walls of the channel and through this effect of these parameters, there are two points of inflexion of flow which are (0.513, 0.4355) and (-0.447, 0.4355). Fig.(4-18) showed the effect of β on the axial velocity u which is noticed that its behavior is opposite of behavior of M and k on velocity and the flow has two points of inflexion which are (0.4839, 0.4371) and (-0.434, 0.44). The impact of θ', a and b are displayed in figure (4-19), it examined that the axial velocity is increase at the center and the walls of the channel and then taken to be decrease at the edges of the walls. Figure (4-20) displayed the influence of perturbation parameter (We) on u it is show that the velocity is rise up at the upper wall and decrease at the lower wall of the channel and the flow are reversal points at the central region which is pointed at (0.008681, 0.5594). The graphs of velocity distribution of all parameters can be described by parabolic paths.

4-9-4 Trapping phenomenon

The trapping for different values of m, ϕ, a, b, M, K, θ' and β are shown in Figs.(4-21)-(4-29) at fixed values of (t=0.5). The stream lines and different circulation bolus are seen for different values of parameters of interesting by various graphs. The effect of non- uniform parameter on the trapping are shown in fig. (4-21), it is examined that the size of trapped bolus increase but whenever we raise the values of m more than the previous the size of bolus began to reduce but increase in number. Fig. (4-22) shows the stream lines pattern for different values of phase ϕ, we observed that the size of the trapped bolus increase by increasing ϕ. The influence of upper and lower amplitudes of channel $(a\&b)$ as well as the Williamson parameter (we) are shown in figures (4-23), (4-24) and (4-25) respectively, it is found that the bolus is taken to decrease in size in both sides of channel with an increase of these parameters. The stream lines for the different values of Hartmann number M are plotted in fig. (4-26) for the fixed values of all other parameters. One could observe that the volume of the bolus decreases along wave length. The stream lines for different values of k are shown in fig. (4-27), it is also observed that the size and number of circulation bolus decrease as (k) increased. Figures (4-28) and (4-29) are shown the impacts of inclination angle of magnetic field (β) and the mean of the flow rate on the pattern of the stream lines, we found that an increase in these parameters results that the volume and number of bolus can be growing in the upper and lower walls of the channel.

4-9-5 Temperature characteristics

The expressions for temperature of the fluid under the effect of peristaltic is illustrated in figures. (4-30)- (4-38) for the fixed values of t=0.5. The effects of non-uniform parameter (m) on the temperature are shown in fig. (4-30), we note that the magnitude of temperature decrease at the center and lower wall of the channel, but the flow is reflected at a point of inflexion in the upper wall and the temperature will be increased. The temperature distribution for ϕ is plotted in fig (4-31), it is seen that the temperature enhanced with an increase in ϕ. Figure (4-32) displayed the effect of a on temperature and it is observed that the temperature is decreased at the central region and the walls, but there is reflection in the flow at the upper wall of channel and then the temperature is rise up at this point. The effect of parameter (b) on temperature is illustrated in figure (4-33) which is noted that an increase in this parameter causes in the value of temperature at the center and walls of channel. The effects of parameters M and k are shown in figures (4-34) and (4-35) respectively, it is examined that an increase in these parameters lead to increase in temperature distribution at the center and walls of channel, but there is two points which the flow is conversed at the upper wall which made the temperature will be reduced. The influence of θ' and Br are plotted in figs. (4-36) and (4-37) respectively, it is found the temperature will be raise up with an increase of previous parameter. Fig.(4-38) is made to study the impact of(β) On temperature distribution, it is noticed that the magnitude of temperature decrease at the core and the walls of channel but at the ends of upper wall of the channel, the temperature distribution will be taken to increase with increase in β .

4-9-6 Heat transfer coefficient

In fig. (4-39)- (4-46), the variation of heat transfer coefficient z(x) for fixed values of (t=0.5) and for variations values of emerging parameter is analyzed. The heat transfer is actually defines the rate of heat transfer or heat flux at the upper wall. It is oscillatory. This is expected due to propagation of sinusoidal waves along the channel walls. The effect of (m) on heat transfer coefficient is shown in figure (4-39), it is noted that the heat coefficient is decreasing with an increase of m. figure (4-40) displayed the effect of a on z(x), which is observed that an increase in this parameter lead to increase in heat coefficient in the portions of $0.8 \le x < 1$ and $0.2 \le x < 0.6$ and it is decreasing in the regions of $0.6 < x < 0.8$ and $1 \le x < 1.2$. The effects of b and θ' are shown in figures (4-41) and (4-42), we noticed that heat coefficient decrease in the portion of $0.4 < x < 0.8$ and increase in the region of $0.8 < x < 1.2$ with an increase of these above parameters. The influence of M, k, Br are illustrated in figures (4-43) and (4-44) and (4-45) respectively. It is observed that their manners is similar to effect of (b) and θ' .In the sometime the behavior of β on heat coefficient is opposite to behavior of M and k and it is displayed in figure (4-46).

4-9-7 Pressure gradient distribution

Effect of various parameters on the pressure gradient versus x have been illustrated in fig. (4-47)- (4-56). These figures are scratched at the fixed values of (t=0.5). From figure (4-47) displays the effect of parameter (m) on pressure gradient, it is noticed that an increase in m leads to reduce in pressure gradient in the portion of $-0.4 < x < 0.4$ and increase in the region of $0.4 < x < 0.8$. Figure (4-48) illustrated the effect of the parameter ϕ, it is observed that the pressure gradient decrease in the region of $0 < x < 0.6$ and rise up at the regions of $0.6 < x < 0.8$ and $-0.2 < x < 0$. The effects of (a & b) are shown in figures (4-49) and (4-50) respectively, it is noticed that pressure increase in the region of $0 < x < 0.4$ and decrease in the regions of $-0.4 < x < 0$ and $0.6 < x < 0.8$. Figure (4-51) and (4-52) illustrates the impacts of M and k on pressure gradient, it is observed that pressure increase in the portion $0 < x < 0.4$ and decrease in the portions of $-0.4 < x < 0$ and $0.4 < x < 0.8$. The effect of β on pressure is displayed in figure (4-53), which is behaved opposite to behavior of M on pressure. The influence of η and α are shown in figures (4-54) and (4-55) respectively, it is noticed that an increase in these parameters causes an increase in pressure in all regions of flow. The effect of θ' is plotted in fig. (4-56), it is observed that pressure is reduced with an increase of θ'.

4-10 Concluding Remarks

In this chapter , we investigated the peristaltic transport of Williamson fluid under the influence of inclined magnetic field through porous medium as well as effects of non-slip conditions and heat transfer are considered in an inclined tapered asymmetric channel. Along wave length and low Reynolds number approximations are adopted. A regular perturbation method for small values of weissenberg number is employed to obtain the expression for stream function, axial velocity, temperature and pressure rise. The effects of Hartmann number (M), porosity parameter (k), wave amplitudes (a& b), channel non- uniform parameter (m), phase difference (ϕ), inclination angle of channel (α), inclination angle of magnetic field β and others are also investigated in details, it found that:

1. The pressure rise Δp against θ' increase in the pumping $(\Delta p > 0,\ \theta' < 0)$ with an increase of M, η and α.
2. The pressure rise Δp against θ' increase in the pumping $(\Delta p < 0,\ \theta' > 0)$ with an increase of β, η and α and decrease with an increase of M.
3. The pressure rise Δp against θ' increase in the pumping $(\Delta p = 0)$ with an increase of α, η and β and decrease with an increase of M.
4. The relation between pressure rise Δp and mean flow rate θ' is linear with an increase of η and α and the curves of pumping is parallel.
5. The relation between pressure rise Δp and mean flow rate θ' is nonlinear with an increase of β and M and the curves of pumping is intersected.
6. The frictional force F_1' at lower wall of channel across one wave length against mean flow θ' increase in the region $(F_1' > 0,\ \theta' > 0)$ with an increase of $\phi, b, \eta, \alpha, \beta$ and decrease with an increase of m, a, M, k.
7. The frictional force F_1' at lower wall of channel across one wave length against mean flow θ' increase in the region $(F_1' < 0,\ \theta' < 0)$ with an increase of a, η, α, β and decrease with an increase of m, ϕ, b, M, k.
8. The frictional force F_1' at lower wall of channel across one wave length against mean flow θ' increase in the region $(F_1' = 0)$ with an increase of m, a, M, k and decrease with an increase of $\phi, \eta, \alpha, \beta$, b.
9. The relation between F_1' and mean flow rate θ' is linear with an increase of $m, M, k, \eta, \alpha, \beta$ and the graphs are parallel curves or lines.
10. The relation between F_1' and mean flow rate θ' is non-linear with an increase of ϕ, a, b and the graphs are intersected curves or lines.
11. The graphs of frictional force F_1' and pressure rise Δp against mean flow rate θ' across one wave length are converse in direction.

12. The pressure gradient $(\frac{dP}{dx})$ increase in magnitude with an increase of η and α and decrease with an increase of θ', but the impact of other partient parameters is oscillatory or wobbling.
13. The axial velocity increase at central region of channel with an increase of β, θ', a, b and decrease with an increase of (m, ϕ),(a, b) and (M, k).
14. The axial velocity is rise up at the upper wall with an increase of (We)and decrease at the lower wall.
15. There are some inflexion points that objections the flow of fluid with an increase of (m, ϕ), (M, k), We and β.
16. The profiles of velocity are parabolic.
17. The temperature distribution is rise up at the center region or core of the channel with an increase of m, a and β.
18. There are some points of deviation that change the flow of the fluid and it's temperature can be change as we have seen with an increase of (m, ϕ , a) by clear way at the upper wall of channel.
19. The profiles of temperature distribution are parabolic under the impact of b, M, k, Br, β and θ' .
20. Heat transfer coefficient z(x) at the upper wall of channel is decreasing function of m.
21. At the region, $0.8 < x < 1.2$, we observed that the temperature coefficient is increased with an increase of a, b, M, k, Br, θ' and decrease with an increase of β.
22. At the region, $0.4 < x < 0.8$, z(x) will be we increase at the increasing of β and decrease with an increase of b, M, k, Br, θ'.
23. The size of trapped bolus increase with an increase of ϕ and decrease with an increase of a, We, b, M .
24. The size and number of trapped bolus increase with an increase of θ' and β and decrease with an increase of k.
25. The influence of m on size and number of circulation bolus is an even or irregular.

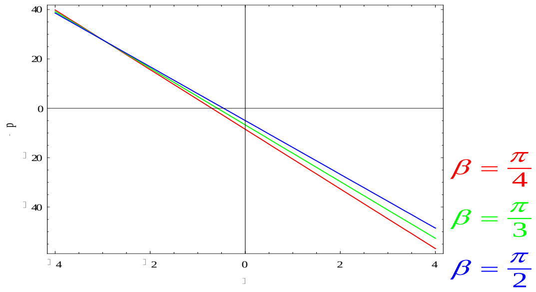


Fig.(4-3) effect of β on Δp

$m = 0.4, \phi = \pi/6, a = 0.3, b = 0.2,$

$K = 1, \alpha = \pi/3, M = 2, \eta = 6$

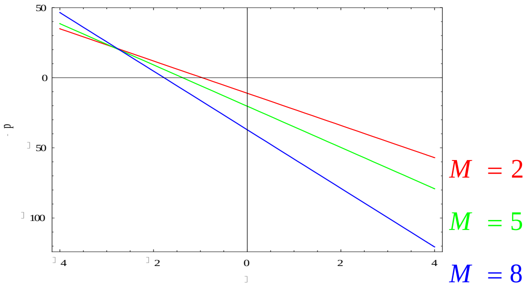


Fig.(4-2) effect of M on Δp

$m = 0.4, \phi = \pi/6, a = 0.3, b = 0.2,$

$K = 1, \alpha = \pi/3, \beta = \pi/3, \eta = 1$

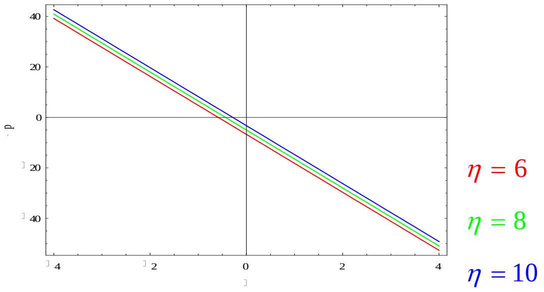


Fig.(4-5) effect of η on Δp

$m = 0.4, \phi = \pi/6, a = 0.3, b = 0.2,$

$K = 1, \alpha = \pi/3, \beta = \pi/3, M = 2$

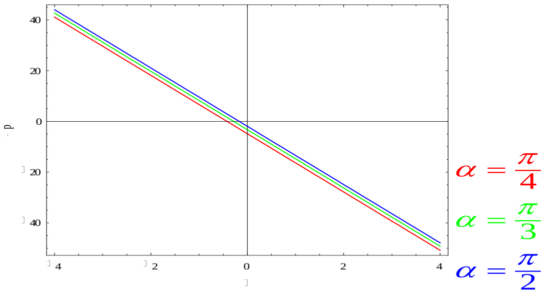


Fig.(4-4) effect of α on Δp

$m = 0.4, \phi = \pi/6, a = 0.3, b = 0.2,$

$K = 1, \beta = \pi/3, M = 2, \eta = 10$

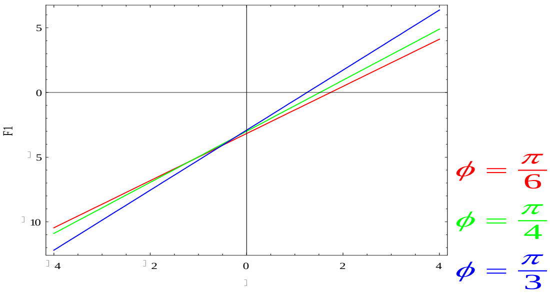


Fig.(4-7) effect of ϕ on friction

$m = 0.2, t = 0.5, a = 0.3, b = 0.2,$

$K = 1, \alpha = \pi/3, \beta = \pi/3, M = 2, \eta = 1$

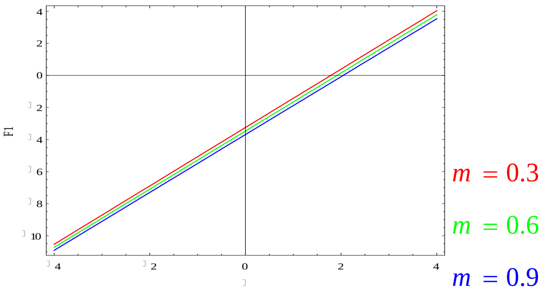


Fig.(4-6) effect of m on friction

$t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2,$

$K = 1, \alpha = \pi/3, \beta = \pi/3, M = 2, \eta = 1$

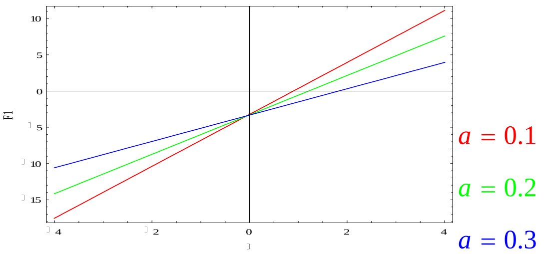


Fig.(4-9) effect of (a) on friction.

$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.2$

$K = 1, \alpha = \pi/3, \beta = \pi/3, M = 2, \eta = 1$

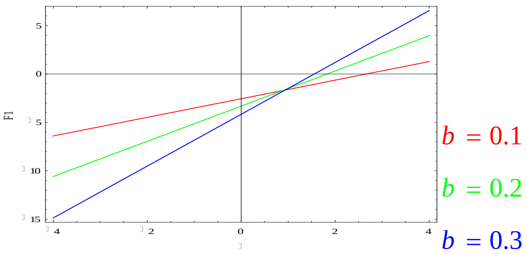


Fig.(4-8) effect of (b) on friction.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$K = 1, \alpha = \pi/3, \beta = \pi/3, M = 2, \eta = 1$

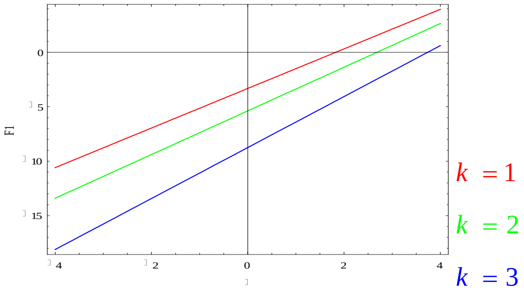


Fig.(4-11) effect of (k) on friction.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, \alpha = \pi/3, \beta = \pi/3, \eta = 1, M = 2$

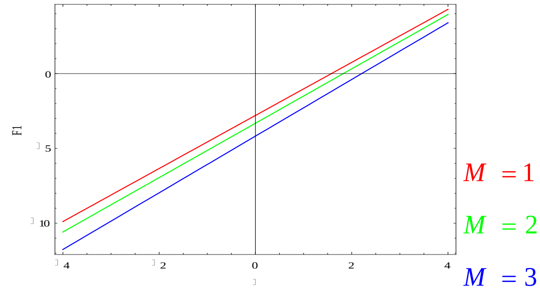


Fig.(4-10) effect of (M) on friction.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, K = 1, \alpha = \pi/3, \beta = \pi/3, \eta = 1$

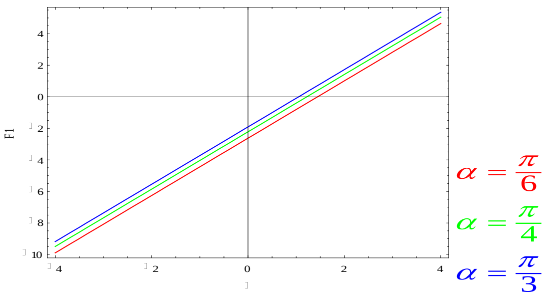


Fig.(4-13) effect of α on friction

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, \beta = \pi/3, M = 2, k = 1, \eta = 6$

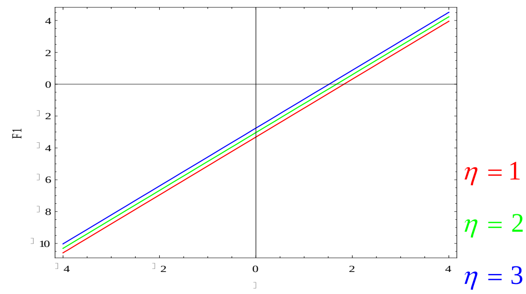


Fig.(4-12) effect of (η) on friction.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, \alpha = \pi/3, \beta = \pi/3, M = 2, k = 1$

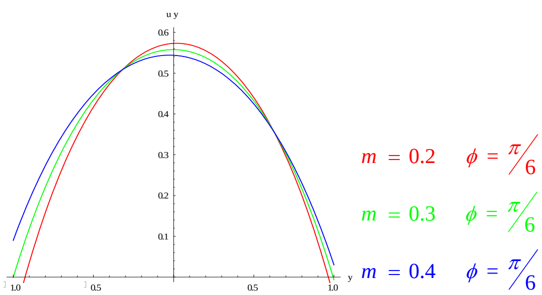


Fig.(4-15) effect of $(m \,\&\, \phi)$ on velocity

$t = 0.5, a = 0.2, b = 0.1, M = 2,$

$We = 0.0001, k = 1, \beta = \pi/3, \theta = 1, x = 0.3$

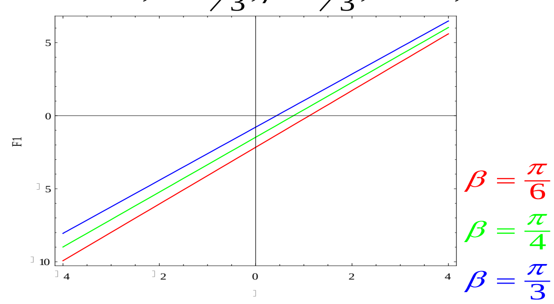


Fig.(4-14) effect of β on friction

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3,$

$b = 0.2, \alpha = \pi/3, M = 2, k = 1, \eta = 10$

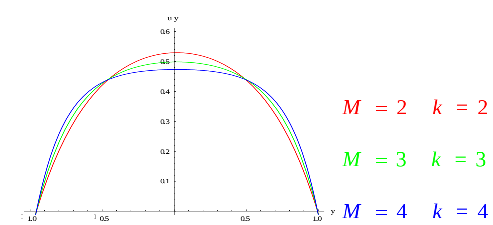


Fig.(4-17) effect of $(M \,\&\, k)$ on velocity

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1$

$We = 0.0001, \beta = \pi/3, \theta = 1, x = 0.3$

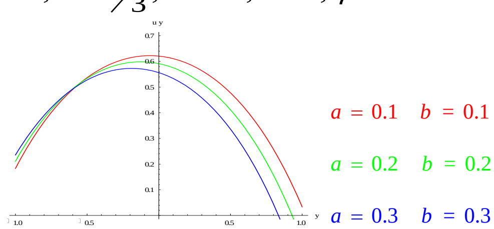


Fig.(4-16) effect of $(a \,\&\, b)$ on velocity

$m = 0.4, t = 0.5, \phi = \pi/2, M = 2,$

$We = 0.0001, k = 1, \beta = \pi/3, \theta = 1, x = 0.3$

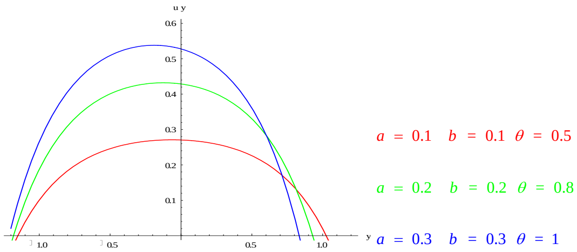


Fig.(4-19) effect of $(a \,\&\, b \,\&\, \theta)$ on velocity

$m = 0.4, t = 0.5, \phi = \pi/6, M = 5,$

$We = 0.0001, \beta = \pi/3, x = 0.3, k = 1$

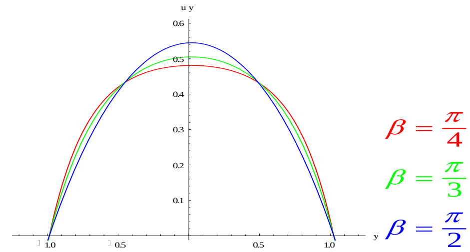


Fig.(4-18) effect of β on velocity

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1$

$M = 5, We = 0.0001, k = 1, \theta = 1, x = 0.3$

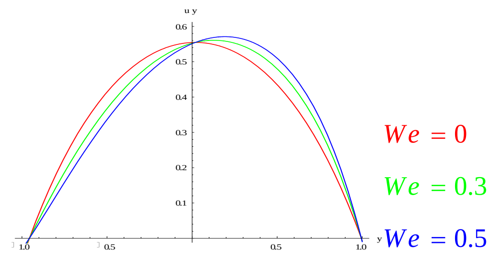
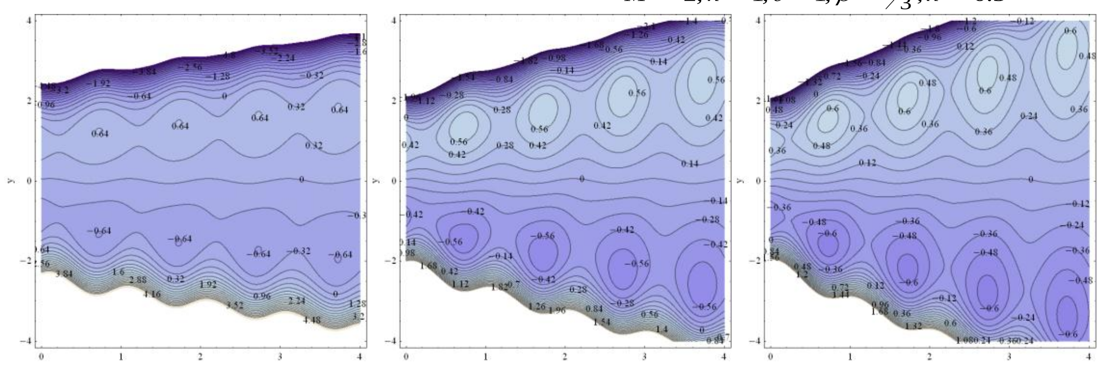


Fig.(4-20) effect of We on velocity

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1$

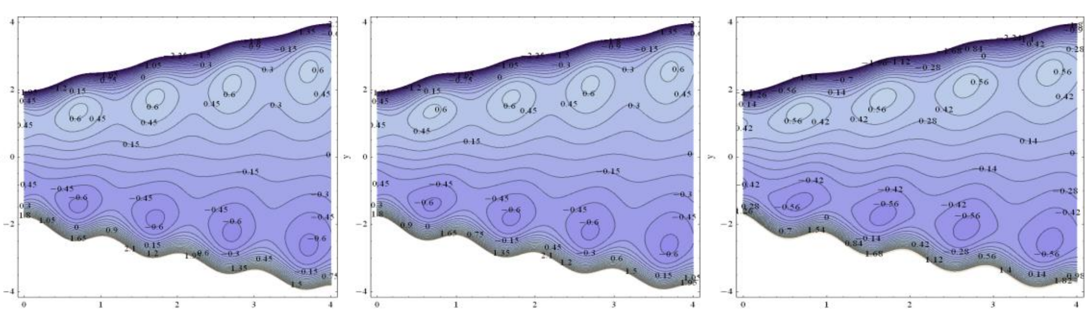
$M = 2, k = 1, \theta = 1, \beta = \pi/3, x = 0.3$



(4-21) Stream lines for

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, M = 2, k = 1, \theta = 1, \beta = \pi/3$

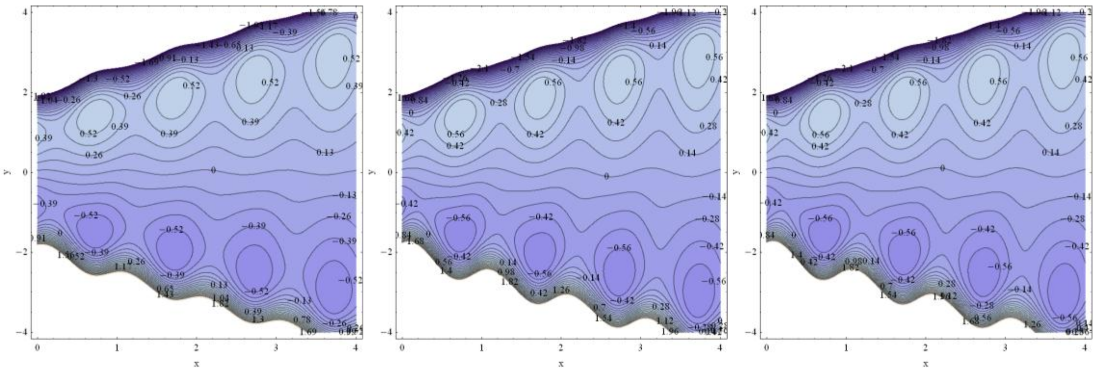
$(a)\ m = 0.2$, (b) $m = 0.4$, (c) $m = 0.6$



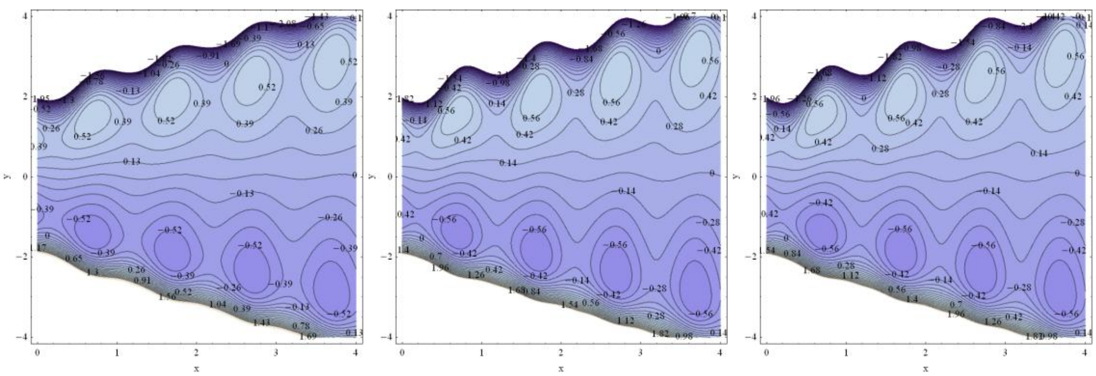
(4-22) Stream lines for

$m = 0.4, t = 0.5, a = 0.2, b = 0.1, We = 0.0001, M = 5, k = 1, \theta = 1, \beta = \pi/3$

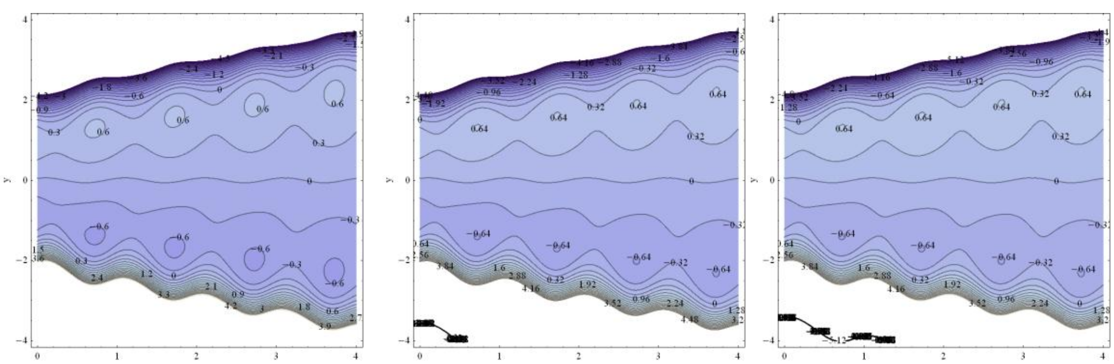
$(a)\ \phi = \pi/4$, (b) $\phi = \pi/3$ $(c)\ \phi = \pi/2$



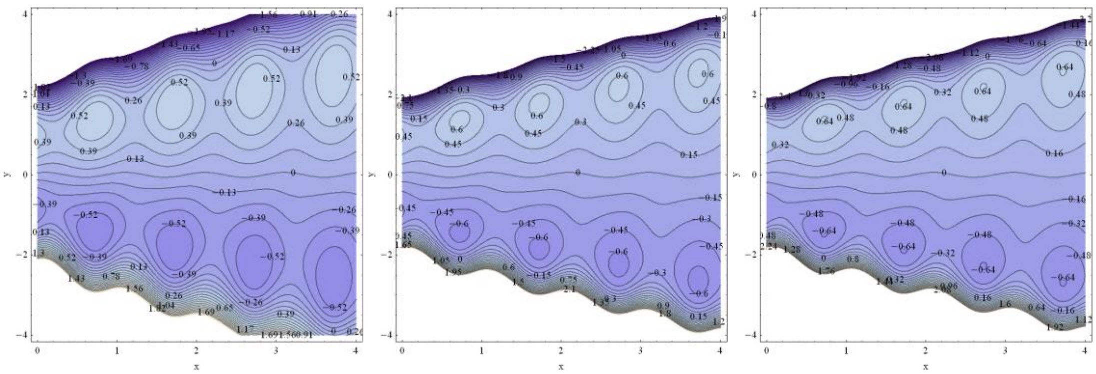
(4-23) Stream lines for
$m = 0.5, t = 0.5, \phi = \pi/6, We = 0.0001, b = 0.1, M = 5, k = 1, \theta = 1, \beta = \pi/3$
$(a)\, a = 0.2$, (b) $a = 0.3$ $(c)\ a = 0.31$



(4-24) Stream lines for
$m = 0.5, t = 0.5, \phi = \pi/6, We = 0.0001, a = 0.1, M = 5, k = 1, \theta = 1, \beta = \pi/3$
$(a)\, b = 0.2$, (b) $b = 0.3$ $(c)\ b = 0.31$



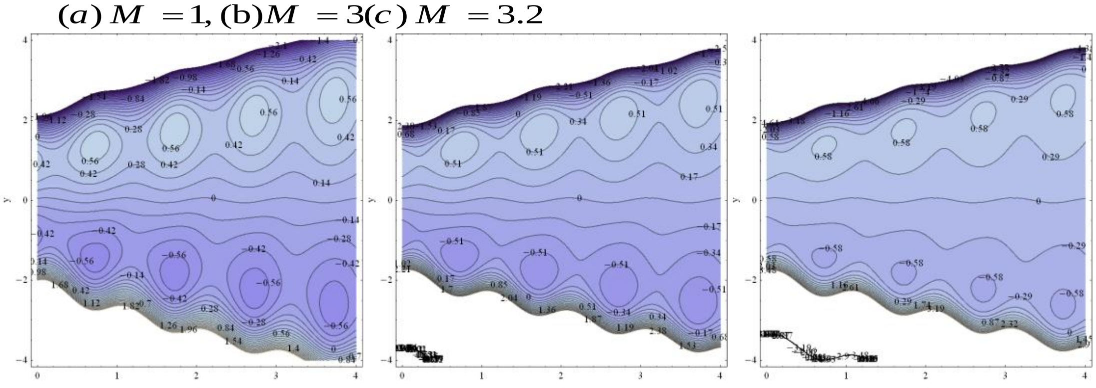
(4-25) Stream lines for
$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, M = 5, k = 1, \theta = 1, \beta = \pi/3$
$(a) We = 0$, (b) $We = 0.001$ $(c)\ We = 0.0015$



(4-26) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, We = 0.0001, k = 1, \theta = 1, \beta = \pi/6$

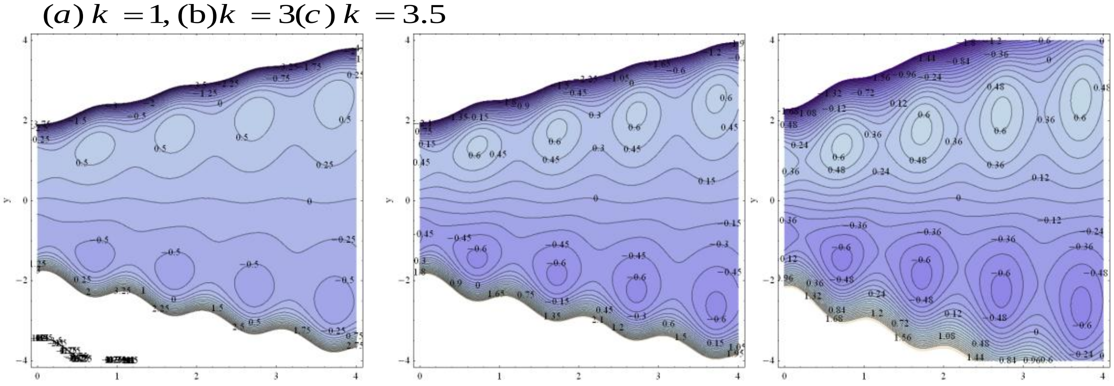
$(a)\ M = 1,\ (\mathrm{b}) M = 3 (c)\ M = 3.2$



(4-27) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, We = 0.0001, k = 1, \theta = 1, \beta = \pi/3, M = 5$

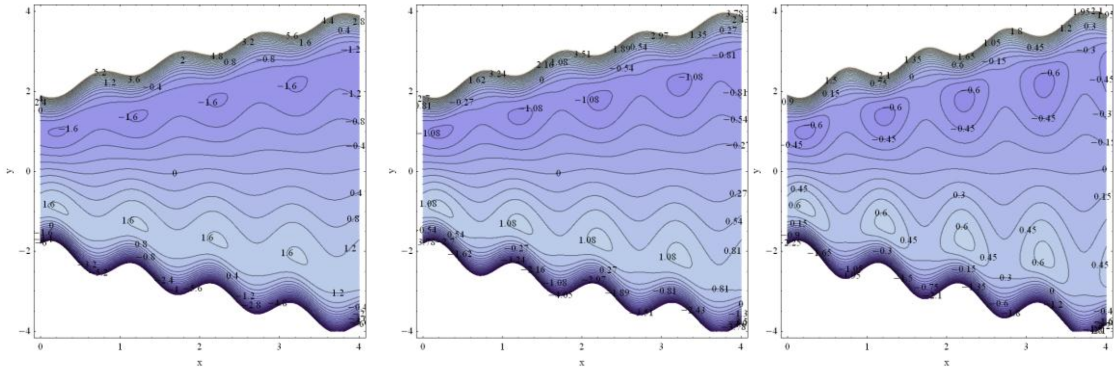
$(a)\ k = 1,\ (\mathrm{b}) k = 3 (c)\ k = 3.5$



(4-28) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, We = 0.0001, k = 1, \theta = 1, M = 5$

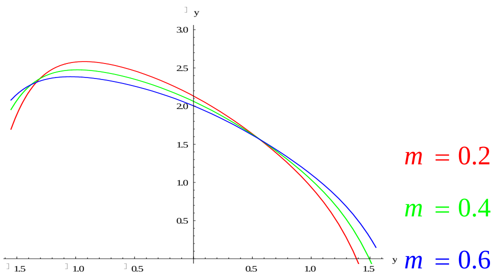
$(a)\ \beta = \pi/4,\ (\mathrm{b}) \beta = \pi/3 (c)\ \beta = \pi/2$



(4-29) Stream lines for

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, M = 5, We = 0.0001, k = 1, \beta = \pi/3$

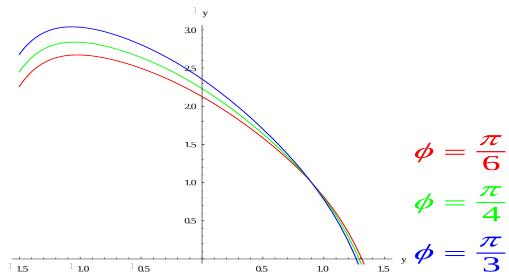
$(a)\ \theta = -3$, (b) $\theta = -2$, $(c)\ \theta = -1$,



(4-30) Effect of m on temperature

$t = 0.5, \phi = \pi/6, a = 0.1, b = 0.2, We = 0.0001$

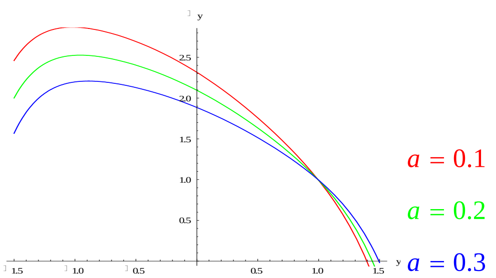
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.3$



(4-31) Effect of ϕ on temperature

$m = 0.3, t = 0.5, a = 0.1, b = 0.2, We = 0.0001$

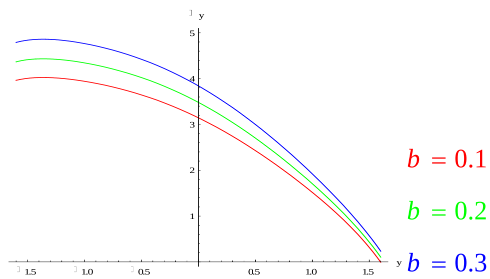
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.3$



(4-32) Effect of (a) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, b = 0.1, We = 0.0001$

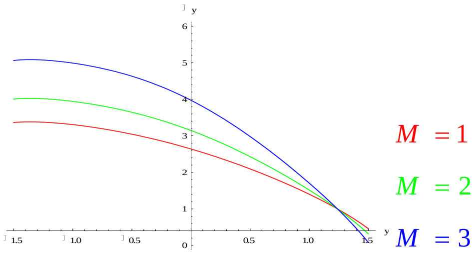
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.3$



(4-33) Effect of (b) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, We = 0.0001$

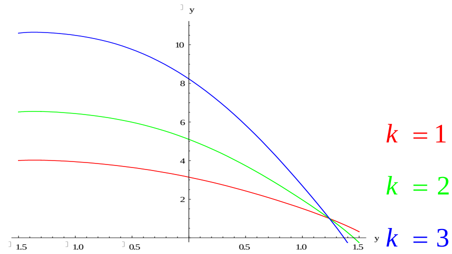
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.6$



(4-34) Effect of (M) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

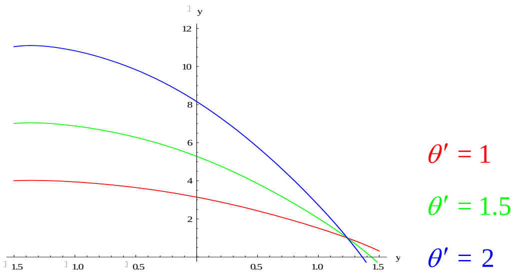
$We = 0.0001, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.6$



(4-35) Effect of (k) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, We = 0.0001$

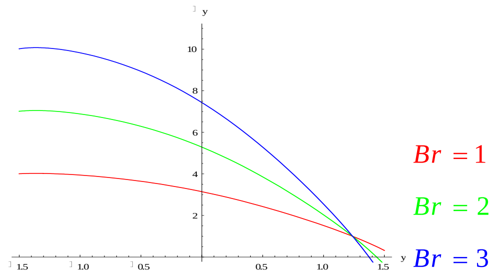
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.6$



(4-36) Effect of (θ') on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, b = 0.1, We = 0.0001$

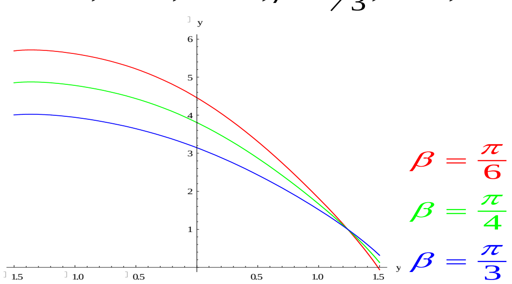
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1, x = 0.6$



(4-37) Effect of (Br) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

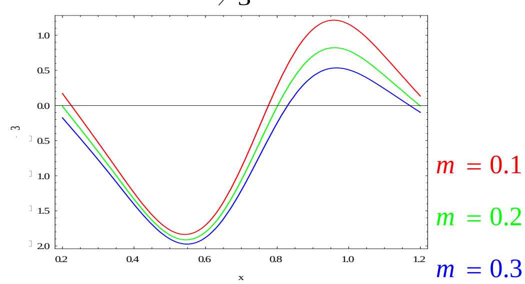
$M = 2, K = 1, \beta = \pi/3, \theta' = 1, x = 0.6, We = 0.0001$



(4-38) Effect of (β) on temperature

$m = 0.3, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

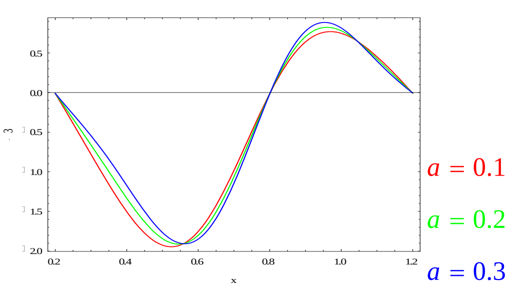
$M = 2, K = 1, Br = 1, \theta' = 1, x = 0.6, We = 0.0001$



(4-39) Effect of m on heat transfer

$t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1, We = 0.0001$

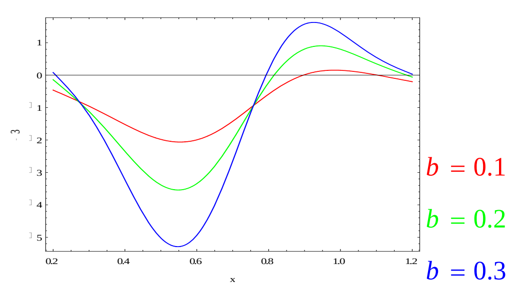
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1$



(4-40) Effect of (a) on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, b = 0.1, We = 0.0001$

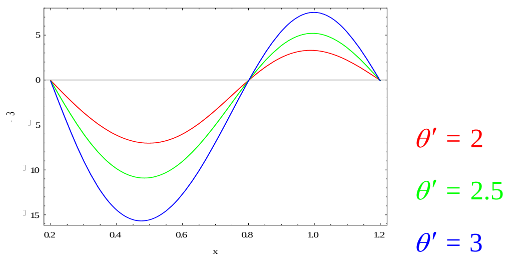
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1$



(4-41) Effect of (b) on heat transfer

$m = 0.5, t = 0.5, \phi = \pi/6, a = 0.2, We = 0.0001$

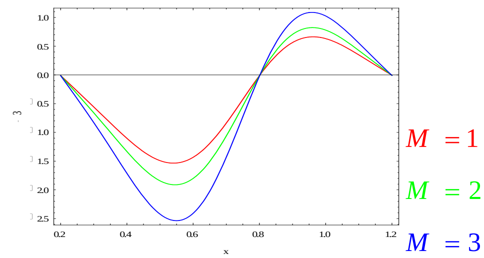
$M = 2, K = 1, Br = 1, \beta = \pi/3, \theta' = 1$



(4-42) Effect of (θ') on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

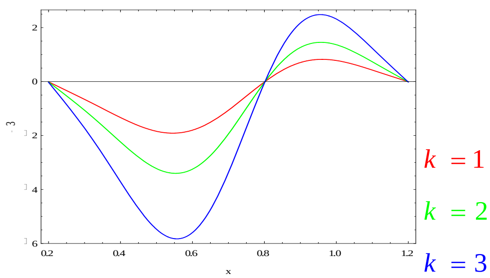
$We = 0.0001, M = 2, K = 1, Br = 1, \beta = \pi/3$



(4-43) Effect of M on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

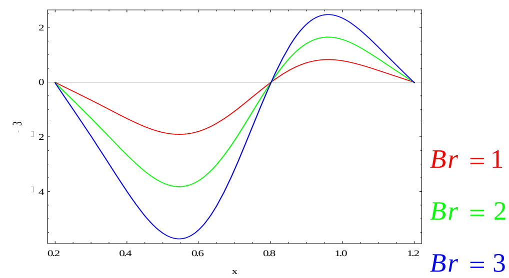
$We = 0.0001, K = 1, Br = 1, \theta' = 1, \beta = \pi/3$



(4-44) Effect of K on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

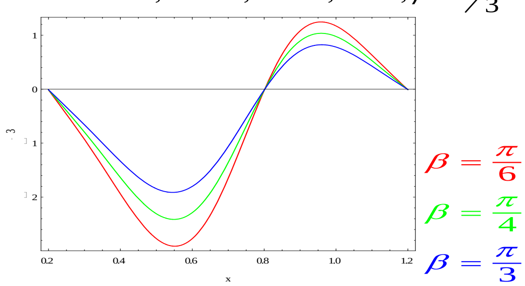
$We = 0.0001, M = 2, Br = 1, \theta' = 1, \beta = \pi/3$



(4-45) Effect of Br on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

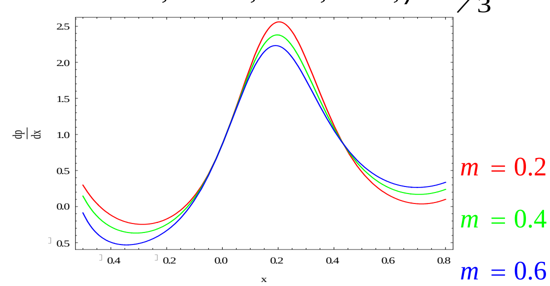
$We = 0.0001, M = 2, k = 1, \theta' = 1, \beta = \pi/3$



(4-46) Effect of β on heat transfer

$m = 0.2, t = 0.5, \phi = \pi/6, a = 0.2, b = 0.1,$

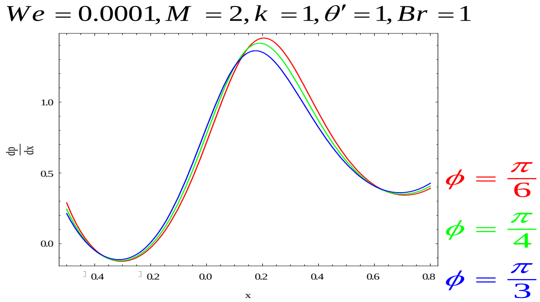
$We = 0.0001, M = 2, k = 1, \theta' = 1, Br = 1$



(4-47) Effect of m on gradient.

$t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, M = 2, \theta' = 0.1$

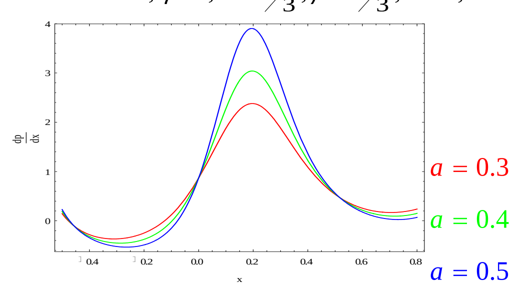
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1,$



(4-48) Effect of ϕ on gradient.

$m = 0.4, t = 0.5, a = 0.2, b = 0.1, M = 2, \theta' = 0.1$

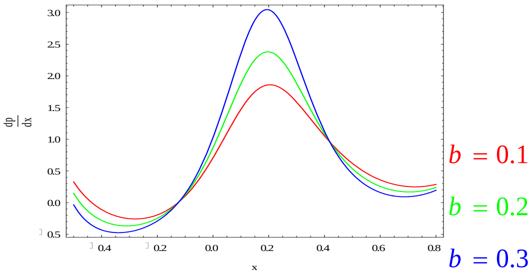
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1,$



(4-49) Effect of a on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, b = 0.2, M = 2, \theta' = 0.1$

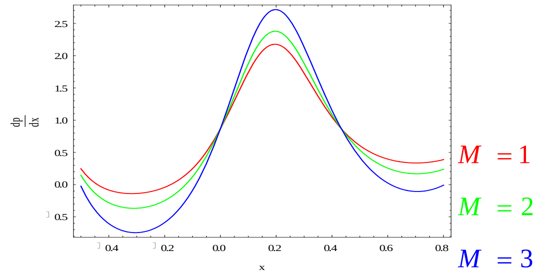
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1,$



(4-50) Effect of b on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, M = 2, \theta' = 0.1$

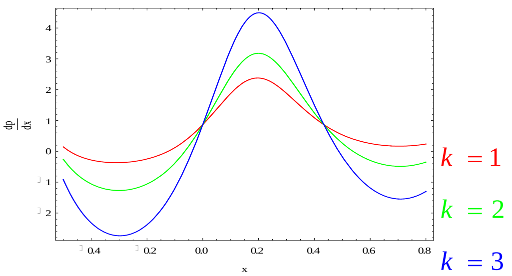
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1,$



(4-51) Effect of M on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \theta' = 0.1$

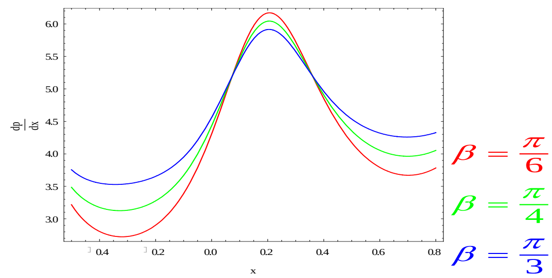
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1,$



(4-52) Effect of k on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \theta' = 0.1$

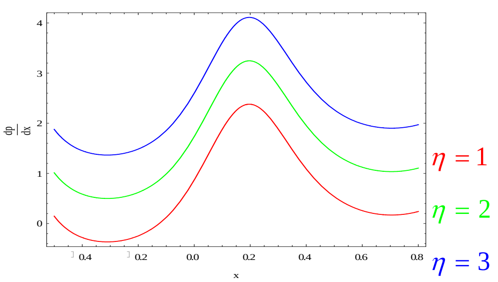
$We = 0.0001, \eta = 1, \alpha = \pi/3, \beta = \pi/3, M = 2$



(4-53) Effect of β on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \theta' = 0.3$

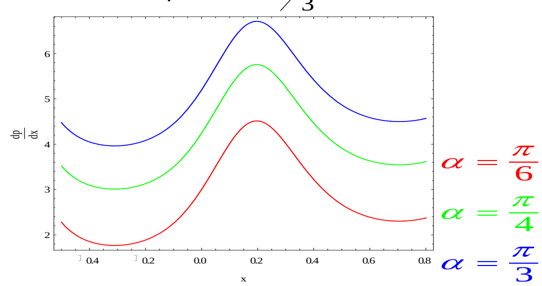
$We = 0.0001, \eta = 6, \alpha = \pi/3, M = 2, k = 1$



(4-54) Effect of η on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \theta' = 0.1$

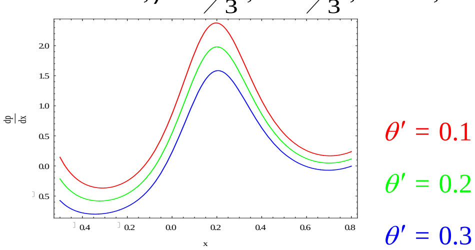
$We = 0.0001, \beta = \pi/3, \alpha = \pi/3, M = 2, k = 1$



(4-55) Effect of α on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \theta' = 0.1$

$We = 0.0001, \eta = 6, M = 2, \beta = \pi/3, k = 1$



(4-56) Effect of θ' on gradient.

$m = 0.4, t = 0.5, \phi = \pi/6, a = 0.3, b = 0.2, \eta = 1,$

$We = 0.0001, \beta = \pi/3, \alpha = \pi/3, M = 2, k = 1$

Chapter Five

Effects of Inclined Magnetic Field and Wall Properties on the Peristaltic Transport of Jeffrey Fluid Through Porous Medium in an Inclined Symmetric Channel.

Introduction

Fluid transport subject to the sinusoidal waves travelling on the walls of the channel/ tube. Motivations about the peristalsis is due to its vast occurring in many physiological mechanisms such as passage of urine from kidney to bladder, spermatozoa transport in the duct us efferent of the male reproductive tract, blood circulation in the small blood vessels, food stuffs through esophagus and alimentary canal etc. utility of such flows persuaded engineers to exploit these in many industrial applications. These include roller finger pumps, heart lung machines and corrosive fluids transport in nuclear industry. It is now well established fact that most of the fluids occurring in physiology and in industry are of non Newtonian type. Blood, bile, chyme, cosmetic products, mud at low shear rate etc, are examples of non Newtonian fluids. There are numerous studies available now on the peristaltic motion of viscous and non Newtonian fluids in a planar channel (see [1, 2, 3, 4, 5, 9]) and many refs. There in. little attention has been given to the peristalsis in an inclined channel, for example [40, 41]. The porous medium and heat transfer effects are quite important in the biological tissue. Especially such considerations are significant in blood flow simulation related to tumors and muscles, drugs transport, production of osteo inductive material, nutrients to brain cells etc. MHD peristaltic flows have acquired a lot of credence due to their applications. The effects of MHD on the peristaltic flow of Newtonian and non Newtonian fluids for different geometries have been discussed by many researches ([6], [48], [65] and [100]) with a view to understand some practical phenomena such as blood pump machine and Magnetic Resonance Imaging (MRI) which is used for diagnosis of brain, vascular diseases and all the human body. In the studies ([6], [48], [65] and [100]), the uniform MHD has been used. There are a few attempts in which induced magnetic field is used. They are mentioned in the works of ([26], [43], [50], and [39]). Rathad et al. [84] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. A new model for study the effect of wall properties on peristaltic transport of a viscous fluid has been investigated by Mokhtar and Haroun [10], Srnivas et al. [100] studied the effect of slip, wall properties and heat transfer on MHD peristaltic transport. Sreenadh et al. [99] studied the effects of wall properties and heat transfer on the peristaltic transport of food bolus through esophagus. The purpose of this chapter is to examine the effects of heat transfer on the peristaltic transport of an incompressible Jeffrey fluid with constant viscosity in an inclined non uniform planar channel under the assumptions of long wave length and low Reynolds number and by helping of wall properties and slip conditions inclined magnetic field is consider through porous medium. The flow is investigated in a wave frame of reference moving with velocity of wave. The governing momentum and temperature is solved by exact way using "MATHIMATICA"software program. Numerical results are obtained for stream trapping bolus, velocity, temperature and pressure gradient and illustrated by graph by using different parameters.

5.1 The Mathematical model of the Problem

Let us consider the inclined magnetic field and heat transfer of an incompressible Jeffrey fluid with constant viscosity in a flexible inclined planar channel with flexible induced by sinusoidal waves trains propagating with constant speed C along the channel walls through a porous medium of two-dimensional symmetric channel. We assume that infinite wave train traveling along the non-uniform walls. We choose a rectangular coordinate system for the channel with $\overline{X}$ along the direction of wave propagation and parallel to the center line and $\overline{Y}$ transverse to it. The lower and upper walls of the channel have the same temperature (T_0).

The wall deformation is given by

$$\overline{H}(\overline{x},\overline{t}) = \mp(d + m'\overline{x} + a\sin[\frac{2\pi}{\lambda}(\overline{x} - c\overline{t})]) \qquad ......(5.1)$$

Where a is the amplitudes of the waves, λ is the wave length, 2d is the width of the channel at the inlet, $m'(m' << 1)$ is the non-uniform parameters, $\overline{X}$ is the axial coordinates, $\bar{t}$ is the time (see fig. (5-1)).

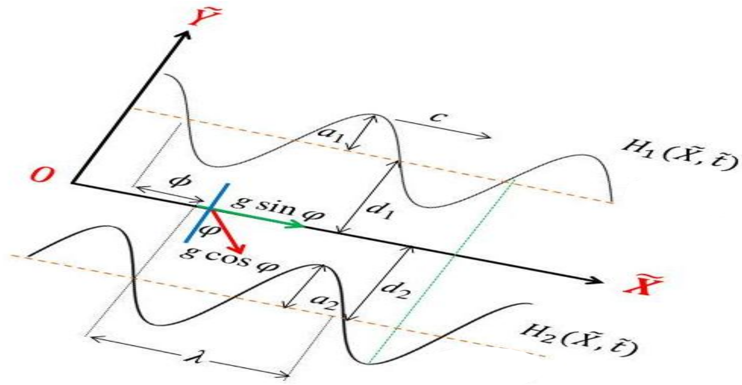


Figure 5-1 Diagrammatic of the problem

5-2-1 Basic Equations of the Problem

The basic equations governing the non Newtonian incompressible Jeffrey fluid are given by:

The continuity equation is given by:

$$\frac{\partial \bar{U}}{\partial \bar{X}}+\frac{\partial \bar{V}}{\partial \bar{Y}}=0 \qquad ......(5-2)$$

The momentum equations are:

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}}+\bar{U}\frac{\partial \bar{U}}{\partial \bar{X}}+\bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right)=-\frac{\partial \bar{P}}{\partial \bar{X}}+\frac{\mu_0}{1+\lambda_1}\left(\frac{\partial^2 \bar{U}}{\partial \bar{X}^2}+\frac{\partial^2 \bar{U}}{\partial \bar{Y}^2}\right)-\sigma B_0^2\cos\beta(\bar{U}\cos\beta-\bar{V}\sin\beta)$$

$$-\frac{\mu_0}{K_0}\bar{U}+\rho g\sin\alpha. \qquad ......(5-3)$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}}+\bar{U}\frac{\partial \bar{V}}{\partial \bar{X}}+\bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right)=-\frac{\partial \bar{P}}{\partial \bar{Y}}+\frac{\mu_0}{1+\lambda_1}\left(\frac{\partial^2 \bar{V}}{\partial \bar{X}^2}+\frac{\partial^2 \bar{V}}{\partial \bar{Y}^2}\right)+\sigma B_0^2\sin\beta(\bar{U}\cos\beta-\bar{V}\sin\beta)$$

$$-\frac{\mu_0}{K_0}\bar{V}-\rho g\cos\alpha \qquad ......(5-4)$$

The temperature equation is given by:

$$\rho C_\rho\left(\frac{\partial T}{\partial \bar{t}}+\bar{U}\frac{\partial T}{\partial \bar{X}}+\bar{V}\frac{\partial T}{\partial \bar{Y}}\right)=k_1\left[\frac{\partial^2 T}{\partial(\bar{X})^2}+\frac{\partial^2 T}{\partial(\bar{Y})^2}\right]+2\mu_0\left[\left(\frac{\partial \bar{U}}{\partial \bar{X}}\right)^2+\left(\frac{\partial \bar{V}}{\partial \bar{Y}}\right)^2\right]+\mu_0\left(\frac{\partial \bar{V}}{\partial \bar{X}}+\frac{\partial \bar{U}}{\partial \bar{Y}}\right)^2$$

$$+\frac{\mu_0}{K_0}\bar{U}^2+\sigma B_0^2(\bar{U}\cos\beta-\bar{V}\sin\beta)^2 \qquad ......(5-5)$$

Where β is the inclination angle of magnetic field, α inclination angle of channel.

5-2-2 Flexible wall

The governing equation of motion of the flexible wall may be expressed as: [71]

$$L^{*} = \overline{P} - \overline{P_0} \qquad ......(5\text{-}6)$$

Where L^{*} is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that:

$$L^{*} = -\tau \frac{\partial^2}{\partial \overline{X}^2} + m_1' \frac{\partial^2}{\partial \overline{t}^2} + C' \frac{\partial}{\partial \overline{t}} \qquad ......(5\text{-}7)$$

Where τ is the elastic tension in the membrane, m_1' is the mass per unit area, C is the coefficient of viscous damping forces.

Continuity of stress at $\overline{Y} = \pm \overline{H}$ and using momentum equation, yield:

$$\frac{\partial}{\partial \overline{X}} L^{*}(\overline{H}) = \frac{\partial \overline{P}}{\partial \overline{X}} = \frac{\mu_0}{1+\lambda_1}\left(\frac{\partial^2 \overline{U}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{U}}{\partial \overline{Y}^2}\right) - \rho\left(\frac{\partial \overline{U}}{\partial \overline{t}} + \overline{U}\frac{\partial \overline{U}}{\partial \overline{X}} + \overline{V}\frac{\partial \overline{U}}{\partial \overline{Y}}\right) - \sigma B_0^2 \cos\beta$$

$$(\overline{U}\cos\beta - \overline{V}\sin\beta) - \frac{\mu_0}{K_0}\overline{U} + \rho g \sin\alpha. \qquad ......(5-8)$$

5-3 Method of solution of the Problem

In order to simplify the governing equations of motion and temperature, we may introduce the following dimensionless transformations as follows:

$$x = \frac{\overline{X}}{\lambda},\ y = \frac{\overline{Y}}{d}, \delta = \frac{d}{\lambda},\ u = \frac{\overline{U}}{c},\ v = \frac{\overline{V}}{\delta c},\ p = \frac{d^2 \overline{P}}{\mu \lambda c}, m = \frac{m'\lambda}{d}, t = \frac{c\overline{t}}{\lambda}\ , b = \frac{a}{d}, \mathrm{Re} = \frac{\rho c d}{\mu},$$

$$\mathrm{Fr} = \frac{c^2}{gd}, \mathrm{Pr} = \frac{\mu C_p}{k_1}, k^2 = \frac{d^2}{k_0}, h = \frac{\overline{H}}{d}, \theta = \frac{T - T_0}{T_0},\ M^2 = \frac{\sigma B_0^2 d^2}{\mu_0},\ \mathrm{Br} = \alpha_1 = Ec\,\mathrm{Pr},$$

$$\eta = \frac{\mathrm{Re}}{Fr} = \frac{\rho g d^2}{c\mu}, \mathrm{u} = \frac{\partial \varphi}{\partial y}, \mathrm{v} = -\frac{\partial \varphi}{\partial x}, \mathrm{Ec} = \frac{c^2}{C_\rho T_0}, \qquad ......(5\text{-}9)$$

Substituting (5-9) into equations (5-1)-(5-8), we have:

From equation (5-3):

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t} + Cu\frac{C}{\lambda}\frac{\partial u}{\partial x} + C\delta v\,\frac{C}{d}\frac{\partial u}{\partial y}\right) = -\frac{\mu_0 C}{d^2}\frac{\partial P}{\partial x} + \frac{\mu_0}{1+\lambda_1}\left(\frac{C}{\lambda^2}\frac{\partial^2 u}{\partial x^2} + \frac{C}{d^2}\frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 \cos\beta$$

$$(Cu\cos\beta - C\,\delta v\sin\beta) - \frac{\mu_0}{K_0}Cu + \rho g\sin\alpha.$$

$$\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t} + \frac{C^2}{\lambda}u\frac{\partial u}{\partial x} + C\frac{d}{\lambda}\frac{C}{d}v\frac{\partial u}{\partial y}\right) = -\frac{\mu_0 C}{d^2}\frac{\partial P}{\partial x} + \frac{\mu_0}{1+\lambda_1}\left(\frac{C}{\lambda^2}\frac{\partial^2 u}{\partial x^2}\frac{d^2}{d^2} + \frac{C}{d^2}\frac{\partial^2 u}{\partial y^2}\right)$$

$$-\sigma B_0^2\cos^2\beta Cu + \sigma B_0^2\cos\beta\sin\beta C\,\delta v - \frac{\mu_0}{K_0}Cu + \rho g\sin\alpha.$$

$$\rho\frac{C^2}{\lambda}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\mu_0 C}{d^2}\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\frac{\mu_0 C}{d^2}\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$-\sigma B_0^2\cos^2\beta Cu + \sigma B_0^2\cos\beta\sin\beta C\,\delta v - \frac{\mu_0}{K_0}Cu + \rho g\sin\alpha \qquad \text{......(5-10)}$$

Multiplying both sides of (5-10) by $\left(\frac{d^2}{\mu_0 C}\right)$ we get:

$$\rho\frac{C^2}{\lambda}\frac{d^2}{\mu_0 C}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\,.\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\frac{\mu_0 C}{d^2}\frac{d^2}{\mu_0 C}\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma$$

$$B_0^2\cos^2\beta Cu\frac{d^2}{\mu_0 C} + \sigma B_0^2\cos\beta\sin\beta C\,\delta v\frac{d^2}{\mu_0 C} - \frac{\mu}{K_0}Cu\frac{d^2}{\mu_0 C} + \rho g\sin\alpha.\frac{d^2}{\mu_0 C}$$

$$\frac{\rho C d}{\mu_0}.\frac{d}{\lambda}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\,.\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma}{\mu_0}B_0^2 d^2\cos^2\beta u + \frac{\sigma}{\mu_0}$$

$$B_0^2 d^2\cos\beta\sin\beta\delta v - \frac{d^2}{K_0}u + \frac{\rho g d^2}{\mu C}\sin\alpha.$$

Thus we have:

$$\operatorname{Re}\delta\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \mu^2\cos^2\beta u + \mu^2\delta\cos\beta$$

$$\sin\beta v - K^2 u + \eta\sin\alpha$$

which can be written as:

$$\operatorname{Re}\delta\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - (\mu^2\cos^2\beta + K^2)u + \mu^2\delta\cos\beta$$

$$\sin\beta v + \eta\sin\alpha \qquad \text{......(5-11)}$$

From equation (5-4):

$$\rho\left(\frac{\partial\overline{V}}{\partial\bar{t}} + \overline{U}\frac{\partial\overline{V}}{\partial\overline{X}} + \overline{V}\frac{\partial\overline{V}}{\partial\overline{Y}}\right) = -\frac{\partial\overline{P}}{\partial\overline{Y}} + \frac{\mu_0}{1+\lambda_1}\left(\frac{\partial^2\overline{V}}{\partial\overline{X}^2} + \frac{\partial^2\overline{V}}{\partial\overline{Y}^2}\right) + \sigma B_0^2\sin\beta(\overline{U}\cos\beta - \overline{V}\sin\beta)$$

$$-\frac{\mu}{K_0}\overline{V} - \rho g \cos\alpha$$

$$\rho\left(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t} + Cu\,\frac{C\delta}{\lambda}\frac{\partial v}{\partial x} + C\delta v\,\frac{C\delta}{d}\frac{\partial v}{\partial y}\right) = -\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y} + \frac{\mu_0}{1+\lambda_1}\left(\frac{C\delta}{\lambda^2}\frac{\partial^2 v}{\partial x^2} + \frac{C\delta}{d^2}\frac{\partial^2 v}{\partial y^2}\right) +$$

$$\sigma B_0^2 \sin\beta(Cu\cos\beta - C\delta v\sin\beta) - \frac{\mu_0}{K_0}C\delta v - \rho g\cos\alpha$$

$$\rho\left(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t} + \frac{C^2\delta}{\lambda}u\,\frac{\partial v}{\partial x} + \frac{C^2\delta^2}{d}v\,\frac{\partial v}{\partial y}\right) = -\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y} + \frac{\mu_0}{1+\lambda_1}\left(\frac{C\delta}{\lambda^2}\frac{d^2}{d^2}\frac{\partial^2 v}{\partial x^2} + \frac{C\delta}{d^2}\frac{\partial^2 v}{\partial y^2}\right) +$$

$$\sigma B_0^2 \sin\beta(Cu\cos\beta - C\delta v\sin\beta) - \frac{\mu_0}{K_0}C\delta v - \rho g\cos\alpha$$

$$\rho\left(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t} + \frac{C^2\delta}{\lambda}u\,\frac{\partial v}{\partial x} + \frac{C^2\delta}{d}\frac{d}{\lambda}v\,\frac{\partial v}{\partial y}\right) = -\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y} + \frac{\mu_0}{1+\lambda_1}\frac{C\delta}{d^2}\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) +$$

$$\sigma B_0^2 \sin\beta(Cu\cos\beta) - \sigma B_0^2\sin^2\beta C\delta v - \frac{\mu_0}{K_0}C\delta v - \rho g\cos\alpha$$

$$\rho\frac{C^2\delta}{\lambda}\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = -\frac{\lambda\mu_0 C}{d^3}\frac{\partial P}{\partial y} + \frac{\mu_0}{1+\lambda_1}\frac{C\delta}{d^2}\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \sigma B_0^2\sin\beta\cos\beta Cu$$

$$-\sigma B_0^2\sin^2\beta C\delta v - \frac{\mu_0}{K_0}C\delta v - \rho g\cos\alpha. \qquad ......(5\text{-}12)$$

Multiplying both sides of (5-12) by $\left(\frac{d^3}{\lambda\mu_0 C}\right)$ we get:

$$\frac{\rho \cancel{C^2}\delta}{\lambda}\frac{d^3}{\lambda\mu_0\cancel{C}}\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \frac{\cancel{\mu_0}}{1+\lambda_1}\frac{\cancel{C}\delta}{\cancel{d^2}}\frac{\cancel{d^3}}{\lambda\mu_0\cancel{C}}\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \sigma B_0^2$$

$$\sin\beta\cos\beta\cancel{C}u\,\frac{d^3}{\lambda\mu_0\cancel{C}} - \sigma B_0^2\sin^2\beta\cancel{C}\delta v\,\frac{d^3}{\lambda\mu\cancel{C}} - \frac{\cancel{\mu_0}}{K_0}\cancel{C}\delta v\,\frac{d^3}{\lambda\cancel{\mu_0}\cancel{C}} - \rho g\cos\alpha\frac{d^3}{\lambda\mu_0 C}.$$

$$\frac{\rho Cd}{\mu_0}\frac{d^2}{\lambda^2}\delta\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \frac{\delta^2}{1+\lambda_1}\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\sigma B_0^2 d^2}{\mu_0}\frac{d}{\lambda}\sin\beta\cos\beta u$$

$$-\frac{\sigma B_0^2 d^2}{\mu_0}\delta\frac{d}{\lambda}\sin^2\beta v - \frac{d^2}{K_0}\delta v\,\frac{d}{\lambda} - \frac{\rho g d^2}{\mu_0 C}\frac{d}{\lambda}\cos\alpha.$$

$$\mathrm{Re}\,\delta^3\left(\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \frac{\delta^2}{1+\lambda_1}\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + {\mu_0}^2\delta\sin\beta\cos\beta u - {\mu_0}^2\delta^2$$

$$\sin^2\beta v - K^2\delta^2 v - \eta\delta\cos\alpha.$$

$$\operatorname{Re}\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\frac{\delta^2}{1+\lambda_1}\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\mu_0{}^2\delta\sin\beta\cos\beta u-$$

$$(\mu_0{}^2\sin^2\beta+K^2)\delta^2 v-\eta\delta\cos\alpha. \qquad ....(5\text{-}13)$$

From Eq. (5-5) we have:

$$\rho C_\rho\left(\frac{\partial \overline{T}}{\partial \overline{t}}+\overline{U}\frac{\partial T}{\partial \overline{X}}+\overline{V}\frac{\partial T}{\partial \overline{Y}}\right)=k_1\left[\frac{\partial^2 T}{\partial \overline{X}^2}+\frac{\partial^2 T}{\partial \overline{Y}^2}\right]+2\mu_0\left[\left(\frac{\partial \overline{U}}{\partial \overline{X}}\right)^2+\left(\frac{\partial \overline{V}}{\partial \overline{Y}}\right)^2\right]+\mu_0\left(\frac{\partial \overline{V}}{\partial \overline{X}}+\frac{\partial \overline{U}}{\partial \overline{Y}}\right)^2$$

$$+\frac{\mu_0}{K_0}\overline{U}^2+\sigma B_0^2(\overline{U}\cos\beta-\overline{V}\sin\beta)^2$$

$$\rho C_\rho\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+Cu\frac{1}{\lambda}\frac{\partial T}{\partial x}+C\delta v\frac{1}{d}\frac{\partial T}{\partial y}\right)=k_1\left[\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}\right]+2\mu_0\left[\frac{C^2}{\lambda^2}\left(\frac{\partial u}{\partial x}\right)^2+\right.$$

$$\left.\frac{C^2\delta^2}{d^2}\left(\frac{\partial v}{\partial y}\right)^2\right]+\mu_0\left(\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{\partial u}{\partial y}\right)^2+\frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2+\sigma B_0^2(Cu\cos\beta-C\delta v\sin\beta)^2.$$

$$\rho C_\rho\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\frac{\partial T}{\partial x}+C\frac{d}{\lambda}\frac{1}{d}v\frac{\partial T}{\partial y}\right)=k_1\left[\frac{1}{\lambda^2}\frac{d^2}{d^2}\frac{\partial^2 T}{\partial x^2}+\frac{1}{d^2}\frac{\partial^2 T}{\partial y^2}\right]+2\mu_0\left[\frac{C^2}{\lambda^2}\left(\frac{\partial u}{\partial x}\right)^2\right.$$

$$\left.+\frac{C^2}{d^2}\frac{d^2}{\lambda^2}\left(\frac{\partial v}{\partial y}\right)^2\right]+\mu_0\left(\frac{Cd}{\lambda^2}\frac{d}{d}\frac{\partial v}{\partial x}+\frac{C}{d}\frac{\partial u}{\partial y}\right)^2+\frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2+\sigma B_0^2(C^2u^2\cos^2\beta-2C^2\delta uv$$

$$\cos\beta\sin\beta+C^2\delta^2 v^2\sin^2\beta)$$

$$\rho C_\rho\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\lambda}u\frac{\partial T}{\partial x}+\frac{C}{\lambda}v\frac{\partial T}{\partial y}\right)=\frac{k_1}{d^2}\left[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\frac{2\mu_0 C^2}{\lambda^2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+$$

$$\frac{\mu_0 C^2}{d^2}\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2+\sigma B_0^2C^2u^2\cos^2\beta-2\sigma B_0^2\delta C^2uv\cos\beta\sin\beta+\sigma B_0^2C^2$$

$$\delta^2 v^2\sin^2\beta.$$

$$\rho C_\rho\frac{C}{\lambda}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k_1}{d^2}\left[\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\frac{2\mu_0 C^2}{\lambda^2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+$$

$$\frac{\mu_0 C^2}{d^2}\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2+\sigma B_0^2C^2u^2\cos^2\beta-2\sigma B_0^2\delta C^2uv\cos\beta\sin\beta+\sigma B_0^2C^2$$

$$\delta^2 v^2\sin^2\beta.$$

$$....(5\text{-}14)$$

Now, since

$$\theta = \frac{T - T_0}{T_0} \quad then \quad \partial T = T_0 \partial\theta \qquad ....(5-15)$$

Thus we can write eq. (5-14) by:

$$\rho C_\rho \frac{C}{\lambda}((T_0)\frac{\partial\theta}{\partial t} + u(T_0)\frac{\partial\theta}{\partial x} + v(T_0)\frac{\partial\theta}{\partial y}) = \frac{k_1}{d^2}[\delta^2(T_0)\frac{\partial^2\theta}{\partial x^2} + (T_0)\frac{\partial^2\theta}{\partial y^2}] + \frac{2\mu_0 C^2}{\lambda^2}[(\frac{\partial u}{\partial x})^2 +$$

$$(\frac{\partial v}{\partial y})^2] + \frac{\mu_0 C^2}{d^2}(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 + \frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2 + \sigma B_0^2 C^2 u^2\cos^2\beta - 2\sigma B_0^2 \delta C^2 uv\cos\beta\sin\beta +$$

$$\sigma B_0^2 C^2 \delta^2 v^2 \sin^2\beta.$$

$$\rho C_\rho \frac{C}{\lambda}(T_0)(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}) = \frac{k_1}{d^2}(T_0)[\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}] + \frac{2\mu_0 C^2}{\lambda^2}[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2] +$$

$$\frac{\mu_0 C^2}{d^2}(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 + \frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2 + \sigma B_0^2 C^2 u^2\cos^2\beta - 2\sigma B_0^2 \delta C^2 uv\cos\beta\sin\beta + \sigma B_0^2 C^2$$

$$\delta^2 v^2 \sin^2\beta$$

…… (5-16)

Multiplying both sides (5-16) by $(\frac{d^2}{K_1(T_0)})$ we get:

$$\rho C_\rho \frac{C}{\lambda}(T_0)(\frac{d^2}{K_1(T_0)})(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}) = [\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}] + \frac{2\mu_0 C^2}{\lambda^2}(\frac{d^2}{K_1(T_0)})$$

$$[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial y})^2] + \frac{\mu_0 C^2}{d^2}(\frac{d^2}{K_1(T_0)})(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 + \frac{\mu_0}{K_0}\mathrm{C}^2\mathrm{u}^2(\frac{d^2}{K_1(T_0)}) + (\sigma B_0^2 C^2$$

$$u^2\cos^2\beta - 2\sigma B_0^2\delta C^2 uv\cos\beta\sin\beta + \sigma B_0^2 C^2\delta^2 v^2\sin^2\beta)(\frac{d^2}{K_1(T_0)}).$$

$$\rho C_\rho \frac{C}{\lambda}\frac{d^2}{K_1}\frac{\mu_0}{\mu_0}(\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}) = [\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}] + \frac{2d^2}{\lambda^2}\frac{C_\rho}{C_\rho}\frac{\mu_0 \mathrm{C}^2}{K_1}(\frac{1}{(T_0)})[(\frac{\partial u}{\partial x})^2$$

$$+(\frac{\partial v}{\partial y})^2] + \frac{\mu_0 C^2}{K_1}\frac{C_\rho}{C_\rho}(\frac{1}{(T_0)})(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})^2 + \frac{d^2}{K_0}\frac{C_\rho}{C_p}\frac{\mu_0 \mathrm{C}^2}{K_1}(\frac{1}{(T_0)})\mathrm{u}^2 + \sigma B_0^2 C^2(u^2$$

$$\cos^2\beta - 2uv\,\delta\cos\beta\sin\beta + \delta^2 v^2\sin^2\beta)(\frac{d^2}{K_1(T_0)})\frac{\mu_0}{\mu_0}\frac{C_\rho}{C_\rho}.$$

$$\frac{\rho C d}{\mu_0}\frac{\mu_0 C_\rho}{K_1}\frac{d}{\lambda}\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]+2\delta^2\left(\frac{C^2}{C_\rho(T_0)}\right)\frac{\mu_0 C_p}{K_1}\left[\left(\frac{\partial u}{\partial x}\right)^2\right.$$
$$\left.+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{C^2}{C_\rho(T_0)}\right)\frac{\mu_0 C_\rho}{K_1}\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+K^2\left(\frac{C^2}{C_p(T_0)}\right)\frac{\mu_0 C_p}{K_1}u^2+\frac{\sigma B_0^2 d^2}{\mu_0}\frac{\mu_0 C_p}{K_1}$$
$$\left(\frac{C^2}{C_p(T_0)}\right)(u^2\cos^2\beta-2uv\,\delta\cos\beta\sin\beta+\delta^2 v^2\sin^2\beta).$$

$$\operatorname{Re}\Pr\delta\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]+2\delta^2 Ec\Pr\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+Ec\Pr$$
$$\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+K^2 Ec\Pr u^2+\mu_0{}^2 Ec\Pr(u^2\cos^2\beta-2uv\,\delta\cos\beta\sin\beta+\delta^2 v^2\sin^2\beta)$$

$$\operatorname{Re}\Pr\delta\left(\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]+2\delta^2 Br\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+Br\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2$$
$$+K^2 Br u^2+\mu_0{}^2 Br(u^2\cos^2\beta-2uv\,\delta\cos\beta\sin\beta+\delta^2 v^2\sin^2\beta)$$

..........(5-17)

From equation (5-8):

$$\frac{\partial}{\partial\overline{X}}\left(-\tau\frac{\partial^2}{\partial\overline{X}^2}+m_1'\frac{\partial^2}{\partial\bar{t}^2}+C'\frac{\partial}{\partial\bar{t}}\right)(\overline{H})=\frac{\partial\overline{P}}{\partial\overline{X}}=\frac{\mu_0}{1+\lambda_1}\left(\frac{\partial^2\overline{U}}{\partial\overline{X}^2}+\frac{\partial^2\overline{U}}{\partial\overline{Y}^2}\right)-\rho\left(\frac{\partial\overline{U}}{\partial\bar{t}}+\overline{U}\frac{\partial\overline{U}}{\partial\overline{X}}\right.$$
$$\left.+\overline{V}\frac{\partial\overline{U}}{\partial\overline{Y}}\right)-\sigma B_0^2\cos\beta(\overline{U}\cos\beta-\overline{V}\sin\beta)-\frac{\mu_0}{K_0}\overline{U}+\rho g\sin\alpha.$$

$$\frac{1}{\lambda}\frac{\partial}{\partial x}\left(-\tau\frac{1}{\lambda^2}\frac{\partial^2}{\partial x^2}+m_1'\frac{C^2}{\lambda^2}\frac{\partial^2}{\partial t^2}+C'\frac{C}{\lambda}\frac{\partial}{\partial t}\right)(dh)=\frac{\mu_0}{1+\lambda_1}\left(\frac{C}{\lambda^2}\frac{\partial^2 u}{\partial x^2}+\frac{C}{d^2}\frac{\partial^2 u}{\partial y^2}\right)-\rho\left(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}\right.$$
$$\left.+Cu\frac{C}{\lambda}\frac{\partial u}{\partial x}+C\delta v\frac{C}{d}\frac{\partial u}{\partial y}\right)-\sigma B_0^2\cos\beta(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{K_0}Cu+\rho g\sin\alpha.$$
$$\frac{1}{\lambda}\left(-\tau\frac{1}{\lambda^2}\frac{\partial^3}{\partial x^3}+m_1'\frac{C^2}{\lambda^2}\frac{\partial^3}{\partial t^2\partial x}+C'\frac{C}{\lambda}\frac{\partial^2}{\partial x\,\partial t}\right)(dh)=\frac{\mu_0}{1+\lambda_1}\left(\frac{C}{\lambda^2}\frac{d^2}{d^2}\frac{\partial^2 u}{\partial x^2}+\frac{C}{d^2}\frac{\partial^2 u}{\partial y^2}\right)-\rho\left(\frac{C^2}{\lambda}\right.$$
$$\left.\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}u\frac{\partial u}{\partial x}+\frac{C^2}{d}\frac{d}{\lambda}v\frac{\partial u}{\partial y}\right)-\sigma B_0^2\cos\beta(Cu\cos\beta-C\delta v\sin\beta)-\frac{\mu_0}{K_0}Cu+\rho g\sin\alpha.$$
$$\left(\frac{-\tau}{\lambda^3}\frac{\partial^3}{\partial x^3}+m_1'\frac{C^2}{\lambda^3}\frac{\partial^3}{\partial t^2\partial x}+C'\frac{C}{\lambda^2}\frac{\partial^2}{\partial x\,\partial t}\right)(dh)=\frac{\mu_0}{1+\lambda_1}\frac{C}{d^2}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho\frac{C^2}{\lambda}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right.$$
$$\left.+v\frac{\partial u}{\partial y}\right)-\sigma B_0^2\cos^2\beta Cu+\sigma B_0^2\cos\beta\sin\beta C\delta v-\frac{\mu_0}{K_0}Cu+\rho g\sin\alpha \qquad ....(5\text{-}18)$$

Multiplying both sides of (5-18) by $(\frac{d^2}{\mu_0 C})$ we get:

$$\frac{d^2}{\mu_0 C}\left(\frac{-\tau}{\lambda^3}\frac{\partial^3}{\partial x^3}+m_1'\frac{C^2}{\lambda^3}\frac{\partial^3}{\partial t^2\partial x}+C'\frac{C}{\lambda^2}\frac{\partial^2}{\partial x\,\partial t}\right)(dh)=\frac{\mu_0}{1+\lambda_1}\frac{C}{d^2}\frac{d^2}{\mu_0 C}\left(\delta^2\frac{\partial^2 u}{\partial x^2}\right.$$

$$\left.+\frac{\partial^2 u}{\partial y^2}\right)-\rho\frac{C^2}{\lambda}\frac{d^2}{\mu_0 C}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\sigma B_0^2\cos^2\beta Cu\frac{d^2}{\mu_0 C}+\sigma B_0^2\cos\beta$$

$$\sin\beta\frac{d^2}{\mu_0 C}C\,\delta v-\frac{\mu_0}{K_0}Cu\frac{d^2}{\mu_0 C}+\rho g\sin\alpha\frac{d^2}{\mu_0 C}.$$

$$\left(\frac{-\tau}{\lambda^3}\frac{d^2}{\mu_0 C}\frac{\partial^3}{\partial x^3}+m_1'\frac{C^2}{\lambda^3}\frac{d^2}{\mu_0 C}\frac{\partial^3}{\partial t^2\partial x}+C'\frac{C}{\lambda^2}\frac{d^2}{\mu_0 C}\frac{\partial^2}{\partial x\,\partial t}\right)(dh)=\frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-$$

$$\rho\frac{C}{\lambda}\frac{d^2}{\mu_0}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 d^2}{\mu_0}\cos^2\beta u+\frac{\sigma B_0^2 d^2}{\mu_0}\cos\beta\sin\beta\delta v-\frac{d^2}{K_0}u+\rho g\sin\alpha\frac{d^2}{\mu_0 C}$$

$$\left(\frac{-\tau}{\lambda^3}\frac{d^3}{\mu_0 C}\frac{\partial^3}{\partial x^3}+m_1'\frac{C}{\lambda^3}\frac{d^3}{\mu_0}\frac{\partial^3}{\partial t^2\partial x}+C'\frac{1}{\lambda^2}\frac{d^3}{\mu}\frac{\partial^2}{\partial x\,\partial t}\right)(h)=\frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-$$

$$\frac{\rho C d}{\mu_0}\frac{d}{\lambda}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2 d^2}{\mu_0}\cos^2\beta u+\frac{\sigma B_0^2 d^2}{\mu_0}\cos\beta\sin\beta\delta v-\frac{d^2}{K_0}u+\frac{\rho g d^2}{\mu_0 C}\sin\alpha \qquad \text{......(5-19)}$$

Thus we can write eq. (5-20) by:

$$\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial t^2\partial x}+E_3\frac{\partial^2}{\partial x\,\partial t}\right)(h)=\frac{1}{1+\lambda_1}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\mathrm{Re}\,\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-$$

$$\mu_0^2\cos^2\beta u+\mu^2\cos\beta\sin\beta\delta v-k^2u+\eta\sin\alpha \qquad \text{........(5-20)}$$

where

$$E_1=\frac{-\tau}{\lambda^3}\frac{d^3}{\mu C},\ \mathrm{E}_2=m_1'\frac{C}{\lambda^3}\frac{d^3}{\mu},\ \mathrm{E}_3=C'\frac{1}{\lambda^2}\frac{d^3}{\mu} \qquad \text{......(5-21)}$$

and

$$h(x,t)=\mp(1+mx+b\sin(2\pi(x-t))) \qquad \text{.......(5-22)}$$

The general solution of the governing equations (5-12)-(5-21) in the general case seems to be impossible , therefore we shall confine the analysis under the assumption of small dimensionless wave length $(\delta \ll 1)$ and low Reynolds number approximation, thus we can write the above equations in the form of stream function :

$$\frac{\partial P}{\partial x}=\frac{1}{1+\lambda_1}\frac{\partial^3\varphi}{\partial y^3}-\mu^2\frac{\partial\varphi}{\partial y}\cos^2\beta-k^2\frac{\partial\varphi}{\partial y}+\eta\sin\alpha \qquad ......(5-23)$$

Which can be written as:

$$\frac{\partial P}{\partial x}=\frac{1}{1+\lambda_1}\frac{\partial^3\varphi}{\partial y^3}-(\mu_0^2\cos^2\beta+k^2)\frac{\partial\varphi}{\partial y}+\eta\sin\alpha \qquad ......(5-24)$$

$$\frac{\partial P}{\partial y}=0 \qquad ......(5-25)$$

$$0=\frac{\partial^2\theta}{\partial y^2}+Br(\frac{\partial^2\varphi}{\partial y^2})^2+Br(\mu_0^2\cos^2\beta+k^2)(\frac{\partial\varphi}{\partial y})^2 \qquad ......(5-26)$$

$$(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial t^2\partial x}+E_3\frac{\partial^2}{\partial x\,\partial t})(h)=\frac{1}{1+\lambda_1}\frac{\partial^3\varphi}{\partial y^3}-(\mu^2\cos^2\beta+k^2)\frac{\partial\varphi}{\partial y}+\eta\sin\alpha \qquad ....(5-27)$$

The corresponding dimensionless boundary conditions are given by:

$$\frac{\partial\varphi}{\partial y}=\mp\beta_1\frac{\partial^2\varphi}{\partial y^2}, \qquad \text{at } y=\mp h$$

$$(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial t^2\partial x}+E_3\frac{\partial^2}{\partial x\,\partial t})(h)=\frac{1}{1+\lambda_1}\frac{\partial^3\varphi}{\partial y^3}-(\mu^2\cos^2\beta+k^2)\frac{\partial\varphi}{\partial y}+\eta\sin\alpha,$$

$$\text{at } y=\mp h \qquad ....(5-28)$$

$$\theta=0, \quad \text{at } y=\mp h \qquad ......(5-29)$$

5-4 Solution of the problem

Equation (5-26) shows that p dependents on x only. Thus if we diff. equation (5-25) with respect to y, we have the closed form solution as follows:

$$0=\frac{1}{1+\lambda_1}\frac{\partial^4\varphi}{\partial\varphi^4}-(\mu_0^2\cos^2\beta+k^2)\frac{\partial^2\varphi}{\partial\varphi^2} \qquad ......(5-30)$$

$$\varphi=\frac{e^{\sqrt{N_1}y\sqrt{1+\lambda_1}}a_1}{N_1(1+\lambda_1)}+\frac{e^{-\sqrt{N_1}y\sqrt{1+\lambda_1}}a_2}{N_1(1+\lambda_1)}+a_3+ya_4; \qquad ......(5-31)$$

where $(N_1=k^2+m_1^2; m_1=\mu_0\cos\beta)$

a_1,a_2,a_3,a_4 are constants can be obtained by using the boundary conditions (5-28) such that:

$$a_1=\frac{\begin{array}{l}-(e^{h\sqrt{N_1}\sqrt{1+\lambda_1}}(1+\lambda_1)(-1+\sqrt{N_1}\beta_1\sqrt{1+\lambda_1}+e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}}(1+\sqrt{N_1}\beta_1\sqrt{1+\lambda_1}))\\(8b(E_1+E_2)\pi^3\cos[2\pi(-t+x)]-4bE_3\pi^2\sin[2\pi(-t+x)+\eta\sin\alpha))\end{array}}{\begin{array}{l}\sqrt{N_1}((-1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})+2(1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})\sqrt{N_1}\beta_1(1+\lambda_1)+\\(-1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})N_1\beta_1^2(1+\lambda_1)^{3/2}));\end{array}}$$

$$a_2=\frac{\begin{array}{l}(e^{h\sqrt{N_1}\sqrt{1+\lambda_1}}(1+\lambda_1)(1-2\sqrt{N_1}\beta_1\sqrt{1+\lambda_1}+N_1\beta_1^2(1+\lambda_1)+e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}}(-1+N_1\beta_1^2(1+\lambda_1)))\\(8b(E_1+E_2)\pi^3\cos[2\pi(-t+x)]-4bE_3\pi^2\sin[2\pi(-t+x)+\eta\sin\alpha))\end{array}}{\begin{array}{l}\sqrt{N_1}((-1+\sqrt{N_1}\beta_1\sqrt{1+\lambda_1})((-1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})\sqrt{N_1}\beta_1\sqrt{1+\lambda_1}+\\2(1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})\sqrt{N_1}\beta_1(1+\lambda_1)+(-1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})N_1\beta_1^2(1+\lambda_1)^{3/2}));\end{array}}$$

$$a_3=0;$$

$$a_4=\frac{8b(E_1+E_2)\pi^3\cos[2\pi(-t+x)]-4bE_3\pi^2\sin[2\pi(-t+x)+\eta\sin\alpha))}{N_1}; \qquad ....(5-32)$$

Now, if we substitute the expression for φ into eq. (5-26) and solve the resulting equation we can find the following equation solution of temperature as follows:

$$\theta=\frac{1}{2}\alpha_1(-2a_1a_2y^2-a_4N_1y^2+\frac{2a_2e^{-\sqrt{N_1}y\sqrt{1+\lambda_1}}}{\sqrt{N_1}(1+\lambda_1)^{3/2}}-\frac{2a_1e^{\sqrt{N_1}y\sqrt{1+\lambda_1}}}{\sqrt{N_1}(1+\lambda_1)^{3/2}}-\frac{a_2^2e^{-2\sqrt{N_1}y\sqrt{1+\lambda_1}}}{2N_1(1+\lambda_1)}$$
$$-\frac{a_1^2e^{2\sqrt{N_1}y\sqrt{1+\lambda_1}}}{2N_1(1+\lambda_1)}+c_1+yc_2; \qquad ......(5-33)$$

$c_i,(i=1,2)$ are constants can be obtained by using the boundary conditions (5-29) such that:

$$c_1 = \frac{1}{8N_1(1+\lambda_1)^{3/2}} e^{-2h\sqrt{N_1}\sqrt{1+\lambda_1}} \alpha_1(-4a_2 e^{h\sqrt{N_1}\sqrt{1+\lambda_1}}(1+e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}})\sqrt{N_1} + a_1^2(1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})$$
$$\sqrt{1+\lambda_1} + a_2^2(1+e^{4h\sqrt{N_1}\sqrt{1+\lambda_1}})\sqrt{1+\lambda_1} + 4a_4 e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}} h^2 N_1^{\,2}(1+\lambda_1)^{3/2} + 4a_1 e^{h\sqrt{N_1}\sqrt{1+\lambda_1}}\sqrt{N_1}$$
$$(1+e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}} + 2a_2 e^{h\sqrt{N_1}\sqrt{1+\lambda_1}} h^2\sqrt{N_1}(1+\lambda_1)^{3/2}));$$

$$c_2 = -\frac{1}{8hN_1(1+\lambda_1)^{3/2}}(a_1+a_2)e^{-2h\sqrt{N_1}\sqrt{1+\lambda_1}}(-1+e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}})\alpha_1(-4e^{h\sqrt{N_1}\sqrt{1+\lambda_1}}\sqrt{N_1} - a_1\sqrt{1+\lambda_1} +$$
$$a_2\sqrt{1+\lambda_1} - a_1 e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}}\sqrt{1+\lambda_1} + a_2 e^{2h\sqrt{N_1}\sqrt{1+\lambda_1}}\sqrt{1+\lambda_1}; \qquad ......(5\text{-}34)$$

5-5 Results and Discussion

In this section, the numerical and computational results are discussed for the problem of an incompressible Jeffrey fluid with constant viscosity in an non-uniform planner channel through porous medium with the effects of heat transfer and inclined magnetic field by helping of wall properties and slip conditions on the velocity. The numerical evaluations of the analytical results and some important results displayed graphically in figures (5-2)-(5-43). (MATHEMATICA) program is used to find out numerical results and illustrations. The analytical solutions of the momentum equations and temperature equation are found by using long wave length and low Reynolds number. The obtained solutions are discussed graphically under the variations of various pertinent parameters in the present section. The graphs for the trapping bolus, velocity distribution, temperature distribution and pressure gradient are sketching for various physical parameters.

5-5-1 Velocity Distribution

Influence of different parameters on the velocity distribution have been illustrated in figures (5-2)-(5-13). These figures are scratched at the fixed values of x= (0.8), t= (0.01). Figure (5-2) displays the effect of (E_1) on velocity distribution, it is noticed that the velocity distribution increase at the central line of channel with an increase of (E_1). The effects of (E_2) and (E_3) on velocity distribution are illustrated in figures (5-3) and (5-4) respectively, it observed that an increase in these parameters lead to increase in velocity profiles which is the same behavior of effect (E_1) on velocity. It is due to the fact that less resistance is offered to the flow because of the wall elastance and thus velocity increase. Figure (5-5) showed the impact of parameter (b) on velocity distribution, it observed that velocity increase at the core part of channel and decrease at the edges of the walls. The influences of Hartmann number M on velocity is plotted in figures (5-6), it is noticed that the velocity distribution decrease in the central region and walls of channel with an increase of M, in fact it is due that magnetic field applied in transverse direction yield resistance to fluid particles which decreases the velocity. Figure (5-7), showed the impact of porous parameter (k) on velocity which an increase in this parameter causes decreasing in value of velocity, since it forms disruption in flow of fluid which less the velocity of the fluid. The influences of η, α, m, β and β_1 are illustrated in figures (5-8), (5-9), (5-10),(5-11) and (5-12) respectively, which is noticed that an increase in these parameters lead to increase in velocity of the fluid. Figure (5-13), display the effect of Jeffrey parameter (λ_1) on axial velocity, it is observed that there is an increase in velocity distribution in the central region and walls of channel with an increase of (λ_1). It is possible only when there is increase in relaxation time and decrease in retardation time, However, in this case we can say that for Newtonian fluid ($\lambda_1 = 0$) the velocity is less than Newtonian fluid. All graphs of velocity profiles in all of its figures can be described as parabolic.

<u>5-5-2 Trapping Phenomenon</u>

The effects of various parameters like $E_1, E_2, E_3, b, M, \lambda_1, \eta, \alpha, \beta, m, k$ and β_1 on trapping can be seen through figures (5-14)-(5-25). Figure (5-14) show that the number and size of bolus trapping increasing in the upper and lower part of channel with an increase of (E_1). Figure (5-15) is plotted for the effect of (E_2) on trapping, it can be seen that there is a similar behavior of effect (E_1) on trapping with an increase of (E_2), where as the effect of (E_3) on trapping has opposite behavior of effects of (E_1) and (E_2) and it is shown in figure (5-16). However we can say that the properties of walls have oscillatory mannor. The influence of parameter (b) on trapping is illustrated in figure (5-17) and it is noticed that there is decreasing in number and size of bolus in the upper and lower parts of channel with an increase of (b). The impact of parameters M and k are seen in figures

(5-18) and (5-19) respectively and it is noticed that there is decreasing in number and size of bolus in the upper and lower parts of channel with an increase of previous parameters. It means that the bolus size gets bigger in case of flow through porous medium. Figure (5-20) displays the effect of parameter (λ_1) on trapping and it is observed that there is increase in number and size of bolus in the upper and lower walls of channel with an increase of (λ_1). That is found bolus size is small in the case Newtonian fluid ($\lambda_1 = 0$). The influence of parameter (η) on trapping are shown in figure (5-21), it is observed that an increase in (η) lead to increase in number of circulation bolus in the both walls of channel. From figure (5-22), it is observed that an increase in the channel inclination angle decreases the size of the trapped bolus. It is further noted that the effect of channel inclination angle on the size of bolus largely, depends on the value of Grashoof number for ($+\text{of Gr}$). Figure (5-23) displays the effect of magnetic field inclination angle (β) on trapping, an increase in this parameter results an increase in the size and number of trapped bolus, this is mainly due to the fact that magnetic field inclination angle when increased results in a decrease in the retarding effects of the Lorentz force and the applied magnetic field will be decrease and has small influence on the flow. Figure (5-24) shows the impact of parameter m on trapping and it is observed there is increase in size and number of bolus in the two parts of wall of channel. The effect of parameter (β_1) on trapping iis shown in figure (5-25) and it is found that there is clear increasing of number and size of bolus in the both sides of channel with an increase of previous parameter.

<u>5-5-3 Temperature characteristics</u>

The expression for temperature are given by eq. (5-33), the effects of various parameters on temperature for fixed values of (x= (0.8), t= (0.01)) are shown. The eq. (5-26) has been evaluated by using software "MATHEMATICA" and the results are presented in figures (5-26)-(5-32). As temperature is the average kinetic energy of the particles and. Kinetic energy depends on velocity, therefore increase in velocity by E_1, E_2 and E_3 leads to temperature enhancement and they are plotted in figures (5-26), (5-27) and (5-28) respectively. The influence of (b) on temperature is shown in figure (5-29) which is observed, that there is increasing in the value of temperature at the core of channel and decreasing in temperature at the edges of the walls with an increase of (b). Figures (5-30) and (5-31) display the impacts of parameters M and k respectively, it is noticed that there is decreasing in temperature profile at the central region of channel with an increase values of these parameters. It is reveals the fact that temperature is the average kinetic energy of the particles and kinetic energy depends upon the velocity. The reason behind this fact is that applied magnetic field is opposing in nature. Figure (5-32) illustrated the effect of parameter (Br) on temperature, it is noticed that an increase in (Br) causes increase in temperature distribution, it is due the fact that Brinkman number (Br) is the product of the Prandtl number (Pr) and the Eckert number (Ec), which is occurs due to the viscous dissipation effects and the temperature enhances. The effects of η, α, β and m are shown in figures (5-33), (5-34), (5-35), (5-36) respectively, it is noticed that an increase in these parameters lead to increase in value of temperature. Figure (5-37) showed the impact of λ_1 on temperature, it observed that an increase in Jeffrey parameter λ_1 yields a height magnitude of temperature that is we can say the temperature of Jeffrey fluid is larger than Newtonian fluid $(\lambda_1 = 0)$. The impact of slip-parameter on velocity (β_1) plotted in figure (5-38), it is noticed that an increase in (β_1) results increase in temperature profiles in the center of the channel and small decreasing in the value of temperature at the walls. It is interesting to mention that all graphs of temperature profiles are parabolic.

5-5-4 pressure gradient distribution

Effects of various parameters on the pressure gradient have been illustrated in figures (5-39)-(5-42). These figures are scratched at the fixed value of t= (0.01). The effects of (E_1) and (E_2) are shown in figures (5-39) and (5-40) respectively, it is observed that an increase in these parameters lead to decreasing in pressure gradient, where as the greater impact is noticed near the regions of $0.2 < x < 0.8$. The effect of parameter (b) is plotted in figure (5-41) which is noted that there is similar behavior of effects of (E_1) and (E_2) in pressure gradient. Figure (5-42) displays the impact of E_3 on pressure gradient which is observed that there is decreasing in pressure gradient at the region of $0.6 < \mathrm{x} < 1$, and increasing in pressure gradient at the region of $0 < \mathrm{x} < 0.4$. It is found that the nature of pressure gradient in all cases is oscillatory.

5-6 Concluding Remarks

The present study deals with the combined effect of inclined magnetic field and wall properties on the peristaltic transport of an incompressible Jeffrey fluid with constant viscosity through porous medium of an inclined non-uniform symmetric channel, we obtained the Exact solution of the problem under the approximation of long wave length and low Reynolds number assumptions. (MATHEMATICA PROGRAM) is used to find the solution of governing equations of momentum and temperature and find The numerical results for the streamlines, velocity, temperature and pressure gradient for different values of of pertinent parameters, and we observed the following main findings :

1. At the upper and lower part of channel, we observed that the number and size of trapping bolus increase with an increase of value $E_1, E_2, \lambda_1, \eta, \beta, m$ and β_1 but the reverse rotation obtained with an increase of E_3, b, M, k, α.
2. The axial velocity increase at the central region of channel with an increase of $E_1, E_2, E_3, b, \lambda_1, \eta, \alpha, \beta, m$ and β_1 and its decrease with an increase of M and k
3. The temperature distribution increase at the central region of channel with an increase of $E_1, E_2, E_3, b, \lambda_1, \eta, \alpha, \beta, m, Br$ and β_1 and its decrease with an increase of M, k
4. The graphs of velocity and temperature distribution noticed to be parabolic.
5. The axial pressure gradient with x increase in the regions of $x \in [0.2, 0.8]$ with an increase of E_1, E_2 and b and it is decreasing in the regions of $x \in [0, 0.2)$ and $[0.8, 1]$ with an increase of previous parameters.
6. The axial pressure gradient with x increase at the region of $x \in [0, 0.4]$ with an increase of E_3 and it is decreasing in the region of $x \in [0.6, 1]$ with an increase of previous parameter.
7. The action of pressure gradient is wobbling.

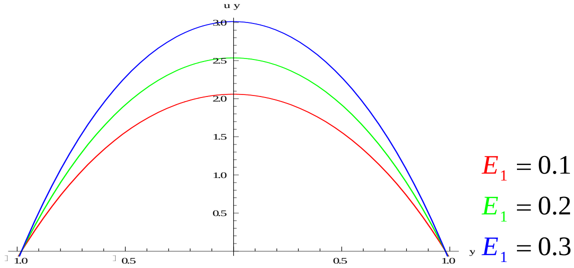


Fig.(5-2) :velocity profile for various values of E_1
$t = 0.01, E_2 = 0.2, E_3 = 0.1, b = 0.2, M = 0.9,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

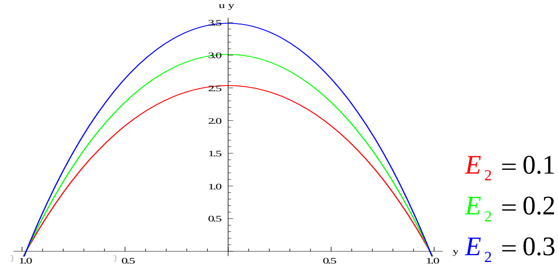


Fig.(5-3) :velocity profile for various values of E_2.
$t = 0.01, E_1 = 0.3, E_3 = 0.1, b = 0.2, M = 0.9,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

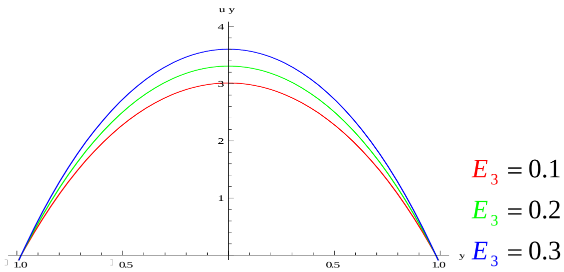


Fig.(5-4) :velocity profile for various values of E_3.
$t = 0.01, E_1 = 0.3, E_2 = 0.2, b = 0.2, M = 0.9,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

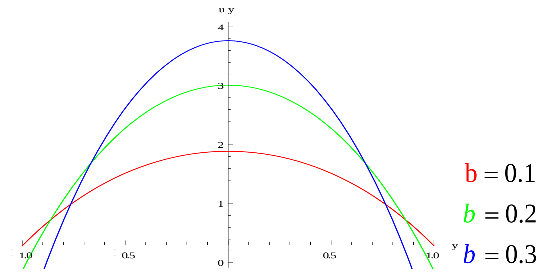


Fig.(5-5) :velocity profile for various values of b.
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, M = 0.9,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

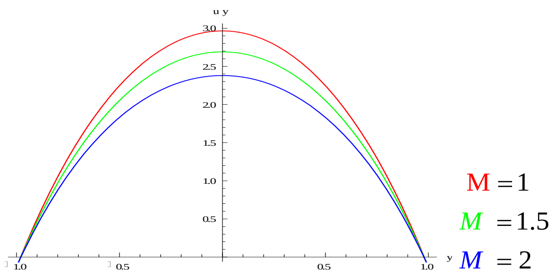


Fig.(5-6) :velocity profile for various values of M.
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

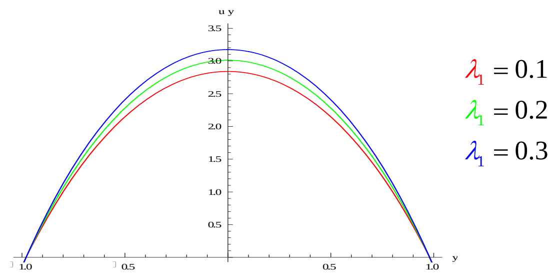


Fig.(5-7) :velocity profile for various values of λ_1.
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

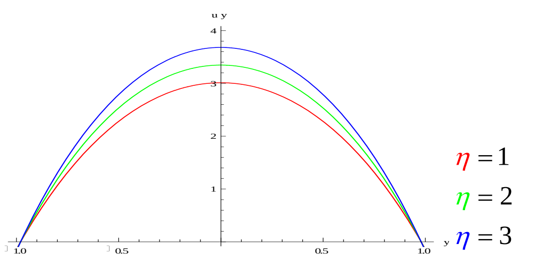


Fig.(5-8) :velocity profile for various values of η
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

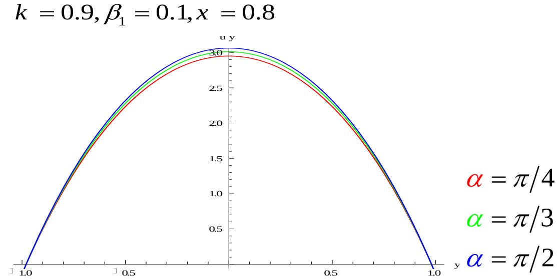


Fig.(5-9) :velocity profile for various values of α.
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \beta = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

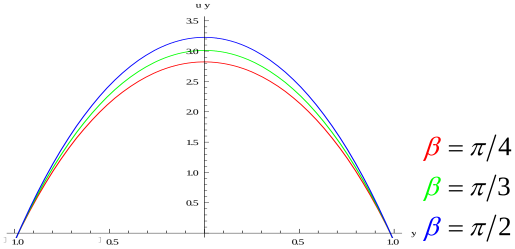


Fig.(5-10) :velocity profile for various values of β .
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, m = 0.1,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

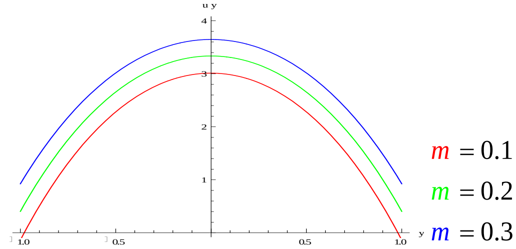


Fig.(5-11) :velocity profile for various values of m .
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$k = 0.9, \beta_1 = 0.1, x = 0.8$

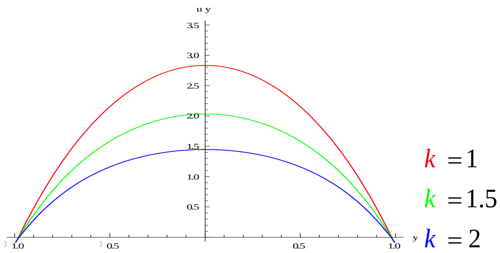


Fig.(5-12) :velocity profile for various values of k .
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$m = 0.1, \beta_1 = 0.1, x = 0.8$

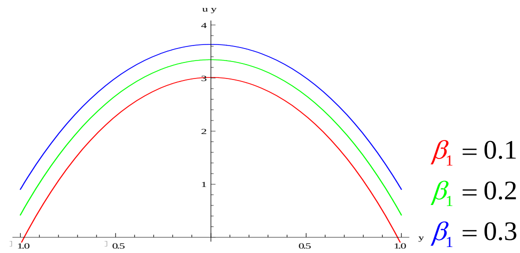


Fig.(5-13) :velocity profile for various values of β_1 .
$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$m = 0.1, k = 0.9, x = 0.8$

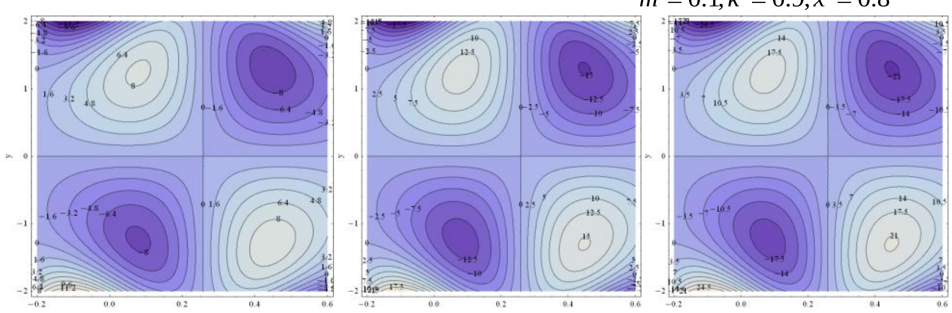


Fig.(5-14) : Stream lines in the wave frame for various values of E_1 .
$t = 0.01, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$
$(a) E_1 = 0.1, (b) E_1 = 0.3, (c) E_1 = 0.5$

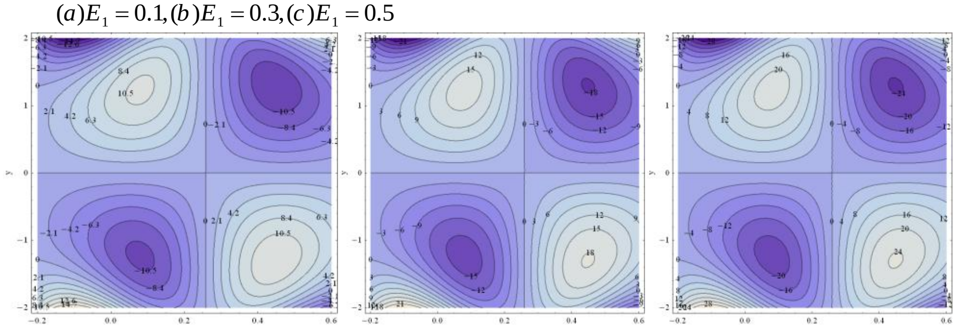


Fig.(5-15) : Stream lines in the wave frame for various values of E
$t = 0.01, E_1 = 0.3, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$
$(a) E_2 = 0.1, (b) E_2 = 0.3, (c) E_2 = 0.5$

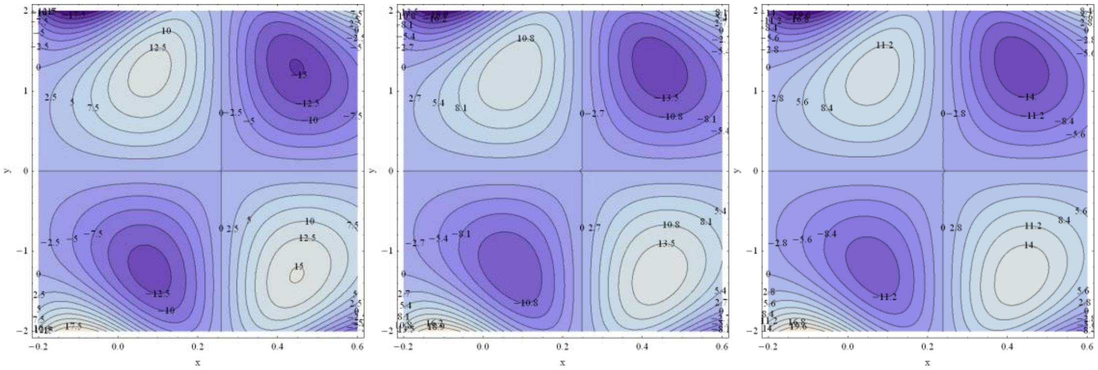


Fig.(5-16) : Stream lines in the wave frame for various values of E_3

$t = 0.01, E_1 = 0.3, E_2 = 0.2, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a) E_3 = 0.1, (b) E_3 = 0.3, (c) E_3 = 0.5$

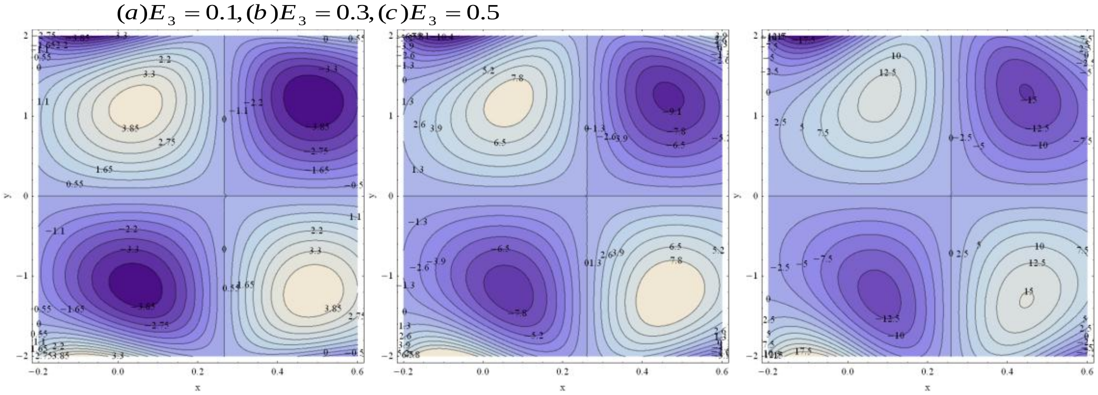


Fig.(5-17) : Stream lines in the wave frame for various values of b.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a) b = 0.1, (b) b = 0.2, (c) b = 0.3$

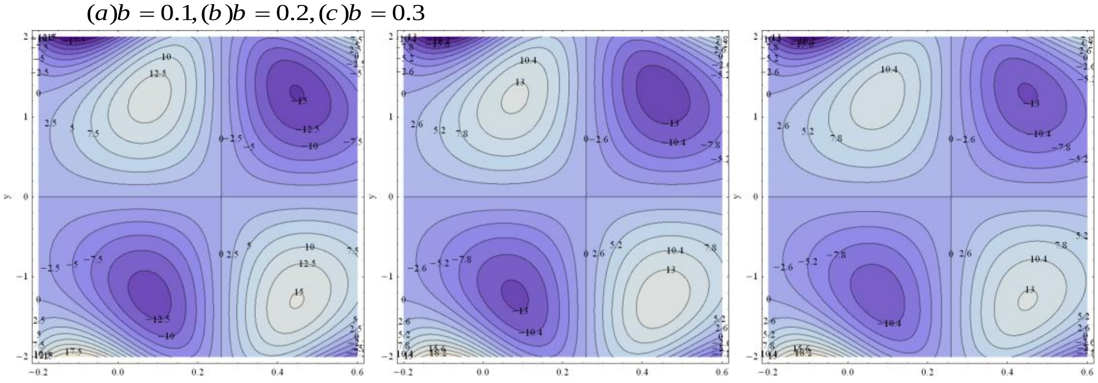


Fig.(5-18) : Stream lines in the wave frame for various values of M.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a) M = 0.9, (b) M = 1, (c) M = 1.5$

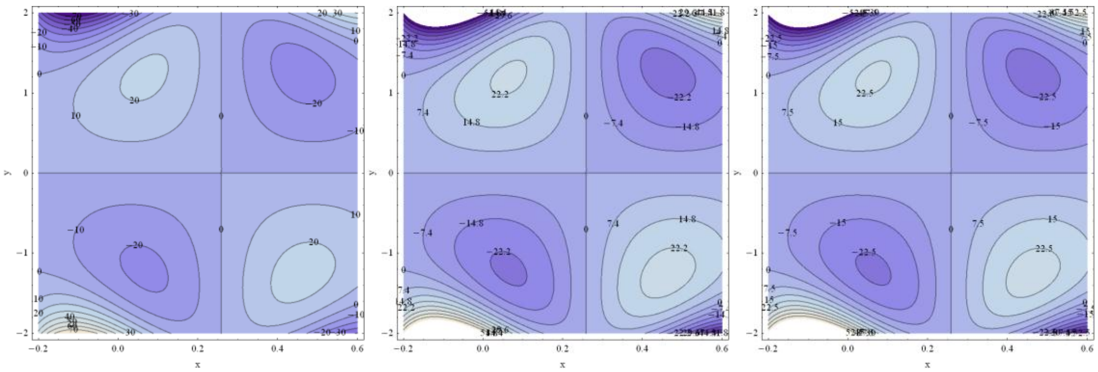


Fig.(5-19) : Stream lines in the wave frame for various values of λ_1.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a)\lambda_1 = 0.1, (b)\lambda_1 = 0.2, (c)\lambda_1 = 0.3$

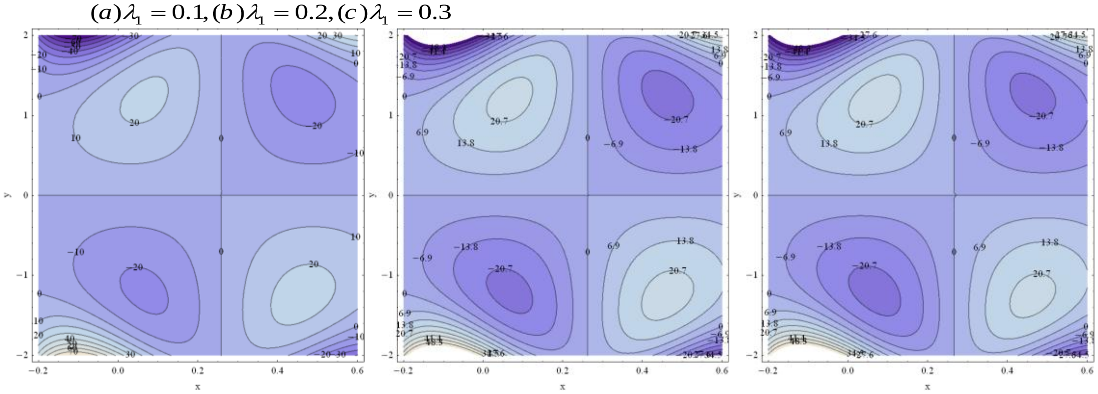


Fig.(5-20) : Stream lines in the wave frame for various values of η.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \alpha = \pi/3, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a)\eta = 1, (b)\eta = 2, (c)\eta = 3$

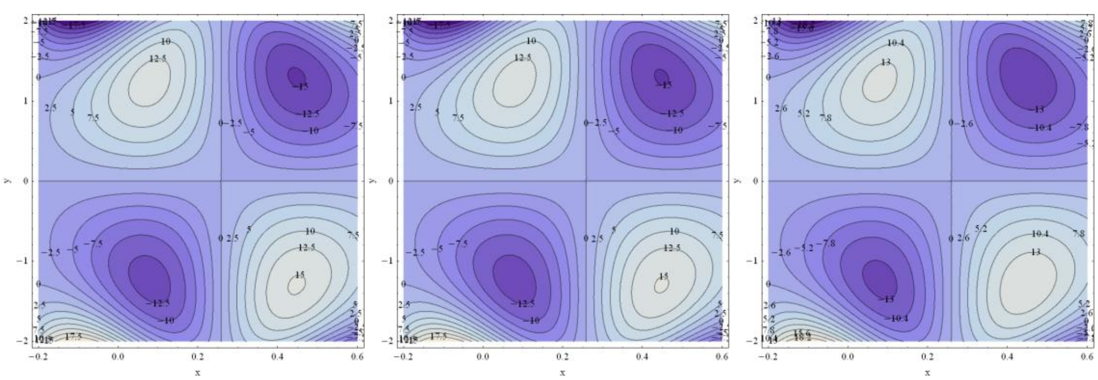


Fig.(5-21) : Stream lines in the wave frame for various values of α.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \beta = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a)\alpha = \pi/4, (b)\alpha = \pi/3, (c)\alpha = \pi/2$

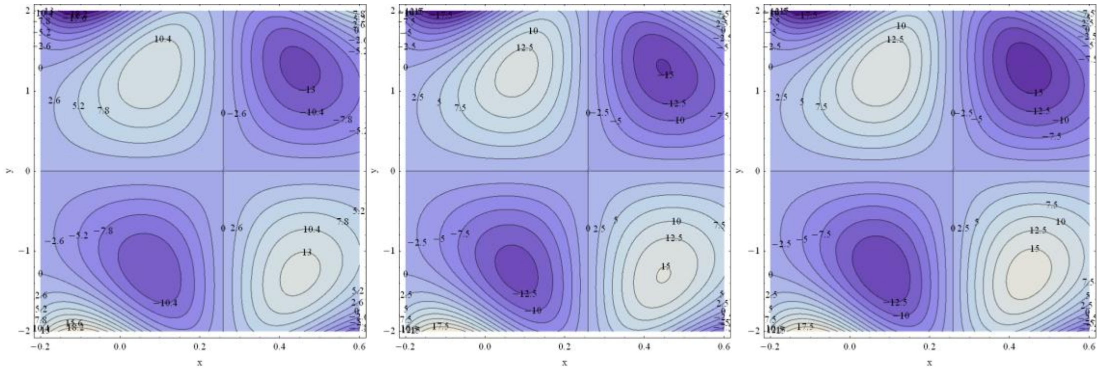


Fig.(5-22) : Stream lines in the wave frame for various values of β .

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, m = 0.2, k = 1, \beta_1 = 0.1$

$(a)\beta = \pi/4, (b)\beta = \pi/3, (c)\beta = \pi/2$

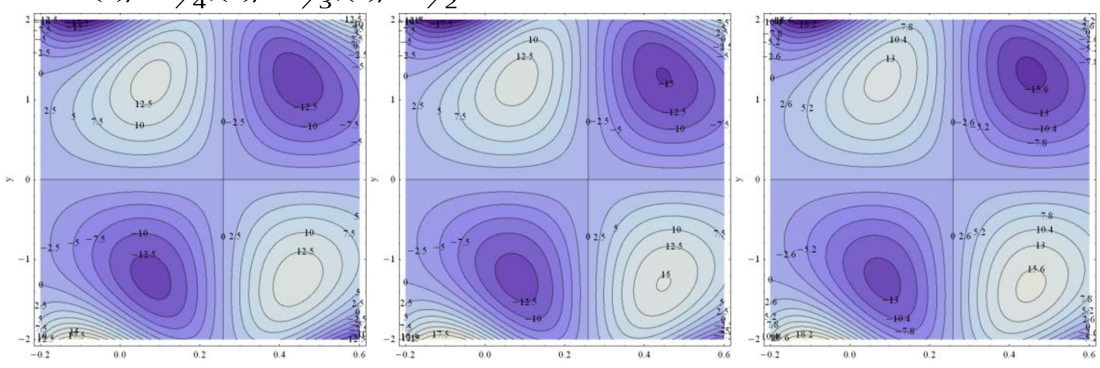


Fig.(5-23) : Stream lines in the wave frame for various values of m.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, k = 1, \beta_1 = 0.1$

$(a)m = 0.1, (b)m = 0.2, (c)m = 0.3$

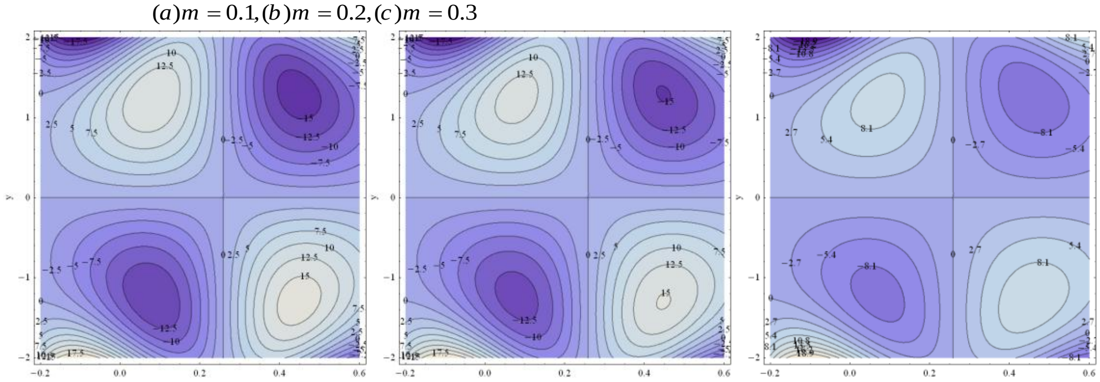


Fig.(5-24) : Stream lines in the wave frame for various values of k.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.3, M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.2, \beta_1 = 0.1$

$(a)k = 0.9, (b)k = 1, (c)k = 1.5$

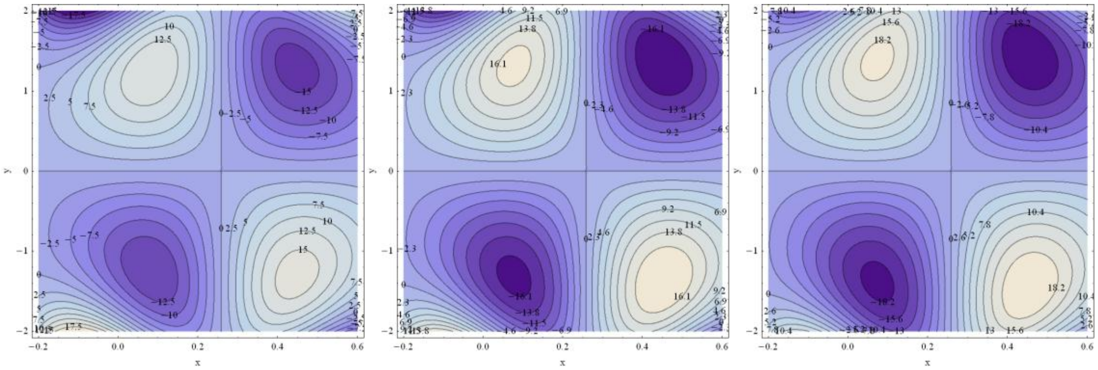


Fig.(5-25) : Stream lines in the wave frame for various values of β_1.

$t=0.01, E_1=0.3, E_2=0.2, E_3=0.1, b=0.3, M=0.9, \lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.2, k=1$

$(a)\beta_1=0.1, (b)\beta_1=0.2, (c)\beta_1=0.3$

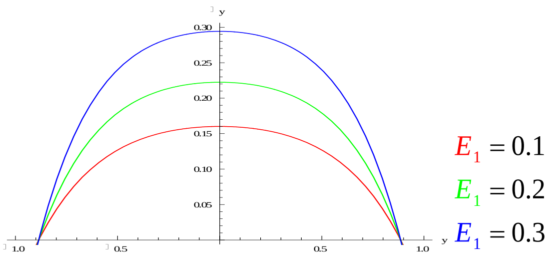


Fig.(5-26): Temperature profile for various of E_1.

$t=0.01, E_2=0.2, E_3=0.1, b=0.2, M=0.9,$

$\lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

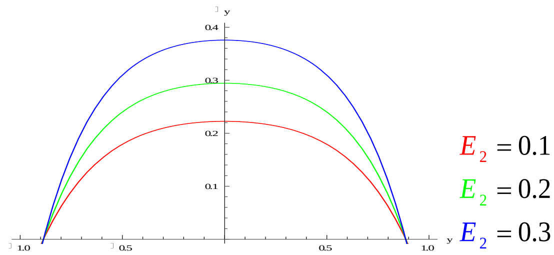


Fig.(5-27): Temperature profile for various values of E_2.

$t=0.01, E_1=0.3, E_3=0.1, b=0.2, M=0.9,$

$\lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

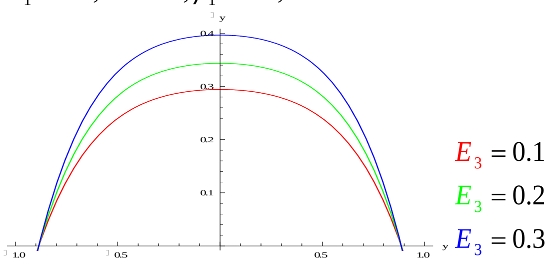


Fig.(5-28): Temperature profile for various values of E_3.

$t=0.01, E_1=0.3, E_2=0.2, b=0.2, M=0.9,$

$\lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

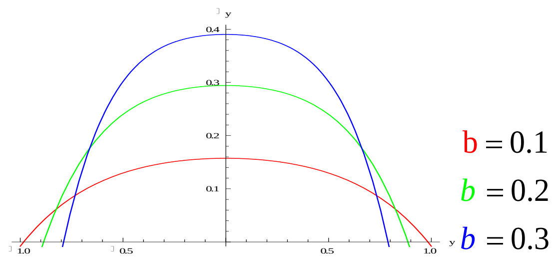


Fig.(5-29): Temperature profile for various values of b.

$t=0.01, E_1=0.3, E_2=0.2, E_3=0.1, M=0.9,$

$\lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

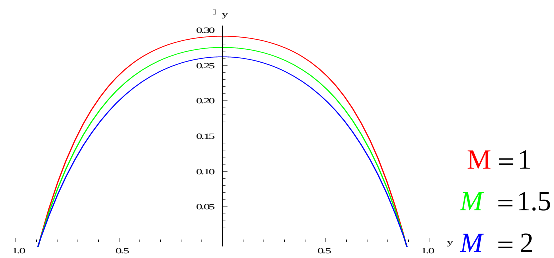


Fig.(5-30): Temperature profile for various values of M.

$t=0.01, E_1=0.3, E_2=0.2, E_3=0.1, b=0.2,$

$\lambda_1=0.2, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

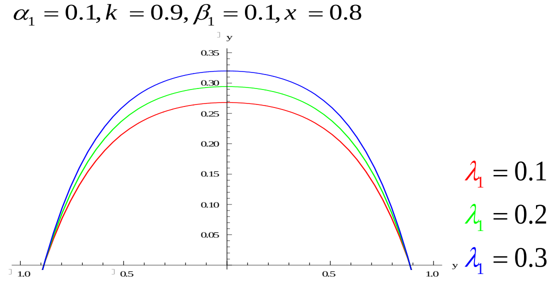


Fig.(5-31): Temperature profile for various values of λ_1.

$t=0.01, E_1=0.3, E_2=0.2, E_3=0.1, b=0.2,$

$M=0.9, \eta=1, \alpha=\pi/3, \beta=\pi/3, m=0.1,$

$\alpha_1=0.1, k=0.9, \beta_1=0.1, x=0.8$

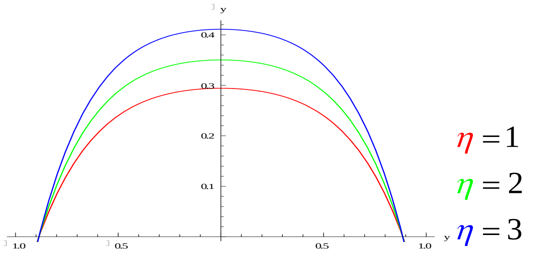


Fig.(5-32): Temperature profile for various values of η.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$\lambda_1 = 0.2, M = 0.9, \alpha = \pi/3, \beta = \pi/3, m = 0.1,$
$\alpha_1 = 0.1, k = 0.9, \beta_1 = 0.1, x = 0.8$

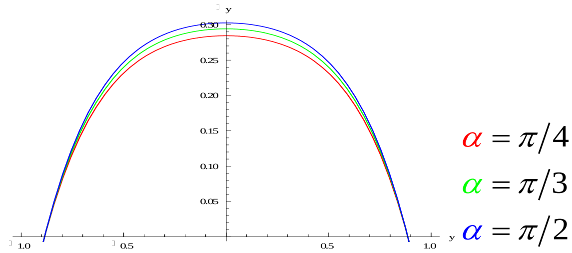


Fig.(5-33): Temperature profile for various values of α

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \beta = \pi/3, m = 0.1,$
$\alpha_1 = 0.1, k = 0.9, \beta_1 = 0.1, x = 0.8$

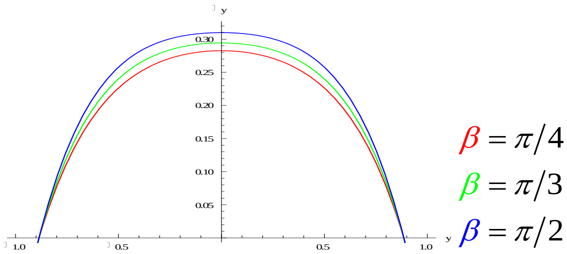


Fig.(5-34): Temperature profile for various values of β

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, m = 0.1,$
$\alpha_1 = 0.1, k = 0.9, \beta_1 = 0.1, x = 0.8$

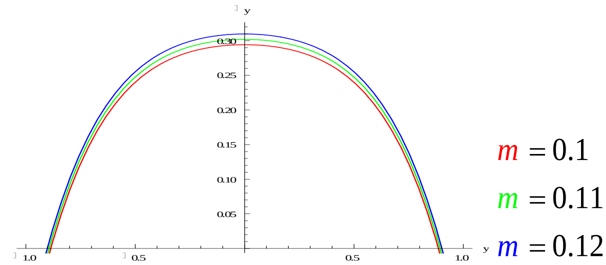


Fig.(5-35): Temperature profile for various values of m.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$\alpha_1 = 0.1, k = 0.9, \beta_1 = 0.1, x = 0.8$

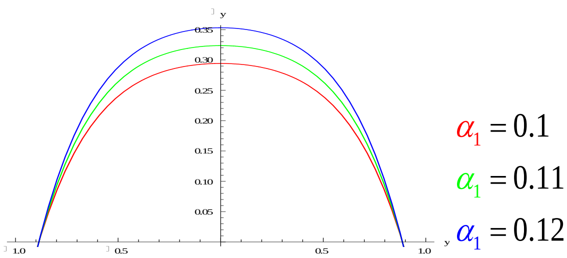


Fig.(5-36): Temperature profile for various values α_1.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 0.9, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$m = 0.1, k = 0.9, \beta_1 = 0.1, x = 0.8$

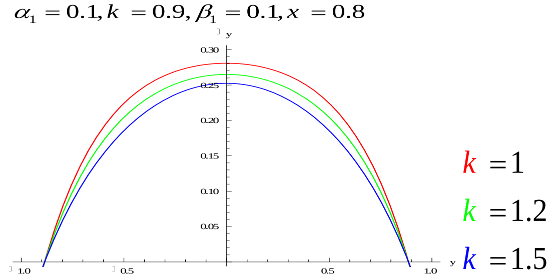


Fig.(5-37): Temperature profile for various values of k.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2,$
$M = 1, \lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3,$
$m = 0.1, \alpha_1 = 0.1, \beta_1 = 0.1, x = 0.8$

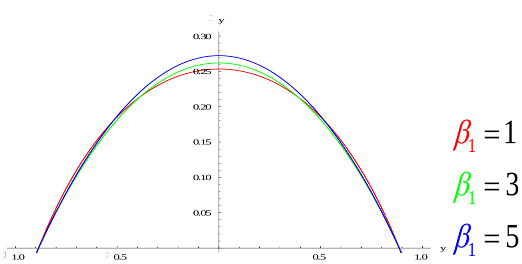


Fig.(5-38): Temperature profile for various values of β_1.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, b = 0.2, M = 0.9,$
$\lambda_1 = 0.2, \eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1, \alpha_1 = 0.1,$
$k = 0.9, x = 0.8$

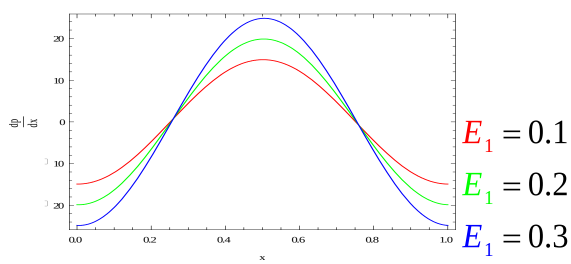


Fig.(5-39) :pressure gradient profile for various values of E_1.

$t = 0.01, E_2 = 0.2, E_3 = 0.1, b = 0.2, M = 0.9, \lambda_1 = 0.2,$
$\eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1, k = 0.9, \beta_1 = 0.1$

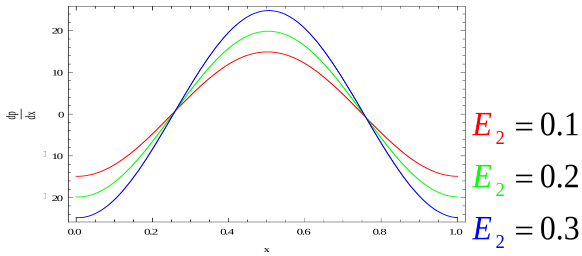


Fig.(5-40) :pressure gradient profile for various values of E_2.

$t = 0.01, E_1 = 0.3, E_3 = 0.1, b = 0.2, M = 0.9, \lambda_1 = 0.2,$
$\eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1, k = 0.9, \beta_1 = 0.1$

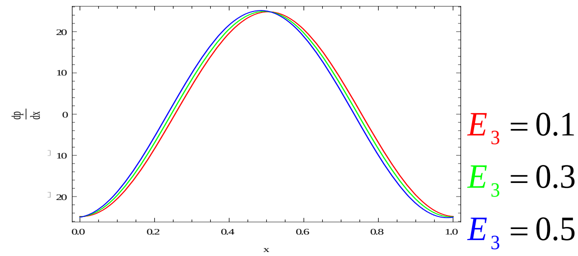


Fig.(5-41) :pressure gradient profile for various values of E_3.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, b = 0.2, M = 0.9, \lambda_1 = 0.2,$
$\eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1, k = 0.9, \beta_1 = 0.1$

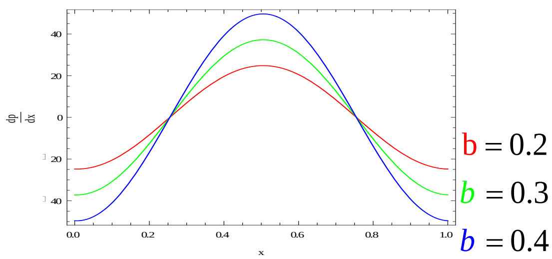


Fig.(5-42) :pressure gradient profile for various values of b.

$t = 0.01, E_1 = 0.3, E_2 = 0.2, E_3 = 0.1, M = 0.9, \lambda_1 = 0.2,$
$\eta = 1, \alpha = \pi/3, \beta = \pi/3, m = 0.1, k = 0.9, \beta_1 = 0.1$

Chapter six

Effect of Radial Magnetic Field on Peristaltic Transport of Jeffrey Fluid Variable Viscosity in Curved Channel With Heat and Mass Transfer Properties

Introduction

Peristaltic transport of fluid is quite popular topic of research amongst the mathematicians, physiologists and engineers. Such popularity of this topic is due to occurrence of peristalsis in the physiological and engineering processes. The peristaltic pumping is a mechanism for fluid transport induced by progressive wave of contraction and relaxation along the distensible tube. Fluid transport in view of peristalsis is an important biological mechanism responsible for various physiological functions of the organs in the human body. Particularly such mechanism is in urine passage from kidney to bladder through ureter, chyme movement in the gastrointestinal tract, ovum movement in the female fallopian tube, transport of spermatozoa in ducts efferent of male reproductive tract, transport of lymph in lymphatic vessels such as arterioles, capillaries, venules and in esophagus during food swallowing process. Practically the peristaltic pumps are designed by engineers for pumping corrosive fluids without contact with the walls of the pumping machinery. In nuclear industry the peristaltic pumping has been found in corrosive fluid or sensitive fluids, transport of slurries and noxious fluids. Latham [61], Jaffrin and Shapiro [96], Shapiro et al. [95] and Fung [33] were the first who made a detailed analysis on peristaltic pumping. It is also noted that initial attempts for peristalsis have been made for viscous liquids. This is not adequate since most of the materials in the physiological and engineering processes are non-Newtonian. There are three types of non-Newtonian fluids (i.e.), 1. Differential type. 2. Rate type. 3. Integral type. The non-Newtonian fluids which exhibit the characteristic of relaxation or retardation times are belong to rate type fluids. Maxwell fluid is one of the subclass of rate type fluids which contains only relaxation time behavior. The only draw back of this fluid model is that it does not explain the retardation time behavior. Therefore to fill this gap, Jeffrey fluid model is considered this model shows the behavior of linearly viscoelastic fluids due to its large number of application in polymer industries. Moreover the Jeffrey fluid model is comparatively simple linear model using time derivatives instead of convective derivatives for example the oldroyd-B fluid model does, it represents a different rheological behavior from that of the Newtonian fluid. In view of diverse characteristics of non-Newtonian materials, various constitutive equations have been suggested. Among such constitutive equations there is one for Jeffrey fluid which has been already utilized for peristaltic transport in both symmetric and asymmetric channel (see [44, 51, 27, 76, 60].

Influence of applied magnetic field on peristaltic activity is important in connection with certain problems of the movement of the conductive physiological fluids, e.g. , blood and the blood pump machines, magnetic drug targeting and relevant process of human digestive system. Such consideration is also useful in treating gastro paresis, chronic constipation and morbid obesity,

Impact of heat transfer in peristaltic transport of fluid is quite significant in food processing, oxygenation, hem dialysis, tissues conduction, heat convection for blood flow from the pores of tissues and radiation between environment and its surface. Mass transfer is useful in the a fore mentioned processes. Especially mass transfer cannot be under estimated when nutrients diffuse out from the blood to neighboring tissues. Further mass transfer involvement is quite prevalent in distillation, chemical impurities diffusion, membrane separation and combustion process. It should be noted that relationships between fluxes and driving potentials occur when both heat and mass transfer act simultaneously. Here temperature gradient generates energy flux. However mass flux and composition gradients are due to temperature gradient (which is called soret effect).

It is noted that all the a fore mentioned studies on peristaltic transport have been conducted for peristalsis in straight channels which is not realistic always since most of the pipes, arteries and glandular ducts are curved. Thus some advancements have been made for peristalsis using curvilinear coordinates. Sato et al.[94] initiated such analysis for peristaltic transport of viscous fluids. Ali et al. [16] extend the work of sato et al. in wave frame of reference. Later some attempts [17, 99, 42] have been presented to address. The curvature effects on peristalsis of fluids in a channel. In these attempts mostly the constant magnetic field are considered. Recently, Hayat et al. [45] is given in their work to explore the characteristics of radial magnetic field on peristaltic transport of Jeffrey fluid in a curved channel. Heat transfer is characterized there by utilizing convective condition. Hayat et al. [52] investigated the effect of radial magnetic field on the peristaltic flow of Jeffrey liquid in curved channel with complaint walls.

In all of previous studies, the viscosity is assumed to be constant. Now our work, we investigated the effect of radial magnetic field on the peristaltic flow of Jeffrey fluid in curved channel under the effect of variable viscosity to the temperature. We will study the effect of heat and mass transfer such as the effect of viscous dissipation and thermophoresis are considered in the transport equations which can be described by Brownian number (Br), Schmidt number (Sc) and soret number (Sr). Non-slip boundary conditions on velocity, temperature, and conservation are considered. The equations are simplified by using long wave length and low Reynolds number. The non-linear differential equations are solved analytically by using regular perturbation method for small values of Reynolds model viscosity parameter for temperature. Series solutions for stream function, axial velocity pressure gradient, temperature and conservation are given by using the regular perturbation technique. The effects of the physical parameters are considered to study the rate of temperature which is named by (heat transfer coefficient) and the effects of these parameters on above distributions are also discussed and illustrated graphically through a set of figures.

6-1 Mathematical Formulation

Consider two- dimensional motion of an viscous incompressible Jeffrey fluid in a curved channel of width $(2a)$, center at 0° and radius at R as shown in figure (6-1). The flow is generated due to the transverse deflections of sinusoidal waves of small amplitudes (b) that are imposed on the flexible walls of the channel. The inertial effects are assumed to be small. The lower and upper walls of the channel are maintained at the same temperature T_0 and concentration C_0. The equations of the walls of channel are described as follows:

$$\bar{r} = \mp H(\overline{X}, \bar{t}) = \mp a \mp b \cos\left(\frac{2\pi}{\lambda}(\overline{X} - C\bar{t})\right) \qquad \text{......(6-1)}$$

Where $\overline{X}$ is the axial distance, $\bar{r}$ is the radial distance, a is the radius of the stationary curved channel, b is the wave amplitude, λ is the wave length, $\bar{t}$ is the time and the wave length is large compared with the channel width (a) that is $(\frac{a}{\lambda} << 1)$.

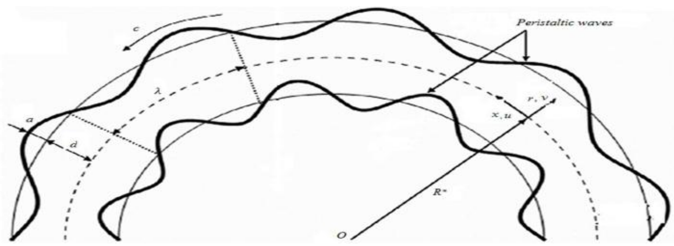


Fig (6-1) : geometry of the problem

6-2 Constitutive Equations

The constitutive equations for a Jeffrey fluid with variable viscosity of temperature can be written by : [78]

$$\bar{\tau} = -\bar{P}\,\bar{I} + \bar{S}, \qquad .....(6-2)$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\dot{\bar{\gamma}} + \lambda_2 \ddot{\bar{\gamma}}) \qquad ....(6-3)$$

Where $\bar{\tau}$ and $\bar{S}$ Cauchy stress tensor and extra stress tensor, respectively, $\bar{P}$ is the pressure, $\bar{I}$ is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\bar{\gamma}}$ is the shear rate and dots over the quantities indicate differentiation with respect to time.

Let $\bar{V} = [\bar{U}(\bar{r},\bar{X},\bar{t}), \bar{V}(\bar{r},\bar{X},\bar{t}), 0]$ be the velocity vector in the curvilinear coordinates $(\bar{r},\bar{X})$.

$$(grad\bar{V}) = (\nabla\bar{V}) = \begin{pmatrix} \dfrac{\partial\bar{V}}{\partial\bar{r}} & \dfrac{R}{\bar{r}+R}\dfrac{\partial\bar{V}}{\partial\bar{X}} - \dfrac{\bar{U}}{\bar{r}+R} \\ \dfrac{\partial\bar{U}}{\partial\bar{r}} & \dfrac{R}{\bar{r}+R}\dfrac{\partial\bar{U}}{\partial\bar{X}} + \dfrac{\bar{V}}{\bar{r}+R} \end{pmatrix} \qquad .....(6-4)$$

The strain E is defined by :

$$E = \tfrac{1}{2}[(\nabla\bar{V}) + (\nabla\bar{V})^{\mathrm{T}}] \qquad ......(6-5)$$

The shear strain or shear rate $\dot{\bar{\gamma}}$ is defined by :

$$\bar{\dot{\gamma}} = 2E = \begin{pmatrix} 2\dfrac{\partial \bar{V}}{\partial \bar{r}} & \dfrac{\partial \bar{U}}{\partial \bar{r}} + \dfrac{R}{\bar{r}+R}\dfrac{\partial \bar{V}}{\partial \bar{X}} - \dfrac{\bar{U}}{\bar{r}+R} \\ \dfrac{\partial \bar{U}}{\partial \bar{r}} + \dfrac{R}{\bar{r}+R}\dfrac{\partial \bar{V}}{\partial \bar{X}} - \dfrac{\bar{U}}{\bar{r}+R} & 2\left(\dfrac{R}{\bar{r}+R}\dfrac{\partial \bar{U}}{\partial \bar{X}} + \dfrac{\bar{V}}{\bar{r}+R}\right) \end{pmatrix} \qquad \text{.....(6-6)}$$

So, we have:

$$\bar{\dot{\gamma}}_{\bar{r}\bar{r}} = 2\frac{\partial \bar{V}}{\partial \bar{r}},\ \ \bar{\dot{\gamma}}_{\bar{r}\bar{X}} = \bar{\dot{\gamma}}_{\bar{X}\bar{r}} = \frac{\partial \bar{U}}{\partial \bar{r}} + \frac{R}{\bar{r}+R}\frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\bar{U}}{\bar{r}+R},\ \ \bar{\dot{\gamma}}_{\bar{X}\bar{X}} = 2\left(\frac{R}{\bar{r}+R}\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\bar{V}}{\bar{r}+R}\right) \quad \text{....(6-7)}$$

Now, define $\bar{\ddot{\gamma}}$ as follows:

$$\bar{\ddot{\gamma}} = \frac{D}{Dt}\bar{\dot{\gamma}} = \left(\frac{\partial}{\partial \bar{t}} + \bar{V}.\nabla\right).\bar{\dot{\gamma}} = \frac{\partial}{\partial \bar{t}}\bar{\dot{\gamma}} + (\bar{V}.\nabla)\bar{\dot{\gamma}}, \qquad \text{......(6-8)}$$

in which

$$\bar{V}.\nabla = \frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}, \qquad \text{......(6-9)}$$

Thus we have:

$$\bar{\ddot{\gamma}}_{\bar{r}\bar{r}} = \frac{\partial}{\partial \bar{t}}\bar{\dot{\gamma}}_{\bar{r}\bar{r}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\bar{\dot{\gamma}}_{\bar{r}\bar{r}}$$

$$= 2\left[\frac{\partial}{\partial \bar{t}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\right]\frac{\partial \bar{V}}{\partial \bar{r}}, \qquad \text{......(6-10)}$$

$$\bar{\ddot{\gamma}}_{\bar{r}\bar{X}} = \frac{\partial}{\partial \bar{t}}\bar{\dot{\gamma}}_{\bar{r}\bar{X}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\bar{\dot{\gamma}}_{\bar{r}\bar{X}}$$

$$= \left[\frac{\partial}{\partial \bar{t}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\right]\left(\frac{\partial \bar{U}}{\partial \bar{r}} + \frac{R}{\bar{r}+R}\frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\bar{U}}{\bar{r}+R}\right), \qquad \text{......(6-11)}$$

$$\bar{\ddot{\gamma}}_{\bar{X}\bar{X}} = \frac{\partial}{\partial \bar{t}}\bar{\dot{\gamma}}_{\bar{X}\bar{X}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\bar{\dot{\gamma}}_{\bar{X}\bar{X}}$$

$$= 2\left[\frac{\partial}{\partial \bar{t}} + \left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial \bar{X}} + \bar{V}\frac{\partial}{\partial \bar{r}}\right)\right]\left(\frac{R}{\bar{r}+R}\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\bar{V}}{\bar{r}+R}\right), \qquad \text{......(6-12)}$$

The components of shear tensor (S) are :

$$S = \begin{pmatrix} S_{\bar{r}\bar{r}} & S_{\bar{r}\bar{X}} \\ S_{\bar{X}\bar{r}} & S_{\bar{X}\bar{X}} \end{pmatrix}$$

$$S_{\bar{r}\bar{r}} = \frac{\bar{\mu}(T)}{1+\lambda_1}\left(\bar{\dot{\gamma}}_{\bar{r}\bar{r}} + \lambda_2 \bar{\ddot{\gamma}}_{\bar{r}\bar{r}}\right)$$

$$=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(2\frac{\partial\bar{V}}{\partial\bar{r}}+2\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right)\right]\frac{\partial\bar{V}}{\partial\bar{r}}\right)$$

$$=\frac{2\bar{\mu}(\bar{T})}{1+\lambda_1}\left(1+\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right)\right]\frac{\partial\bar{V}}{\partial\bar{r}}\right) \quad ......(6\text{-}13)$$

$$S_{\bar{r}\bar{X}}=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(\dot{\bar{\gamma}}_{\bar{r}\bar{X}}+\lambda_2\ddot{\bar{\gamma}}_{\bar{r}\bar{X}}\right)$$

$$=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(\left(\frac{\partial\bar{U}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\frac{\partial\bar{V}}{\partial\bar{X}}-\frac{\bar{U}}{\bar{r}+R}\right)+\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\left(\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right]\right.$$

$$=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(1+\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right]\right)\left(\frac{\partial\bar{U}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\frac{\partial\bar{V}}{\partial\bar{X}}-\frac{\bar{U}}{\bar{r}+R}\right) \quad ....(6\text{-}14)$$

$$S_{\bar{X}\bar{X}}=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(\dot{\bar{\gamma}}_{\bar{X}\bar{X}}+\lambda_2\ddot{\bar{\gamma}}_{\bar{X}\bar{X}}\right)$$

$$=\frac{\bar{\mu}(\bar{T})}{1+\lambda_1}\left(2\left(\frac{R}{\bar{r}+R}\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\bar{V}}{\bar{r}+R}\right)+\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right]\left(2\left(\frac{R}{\bar{r}+R}\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\bar{V}}{\bar{r}+R}\right)\right)\right)$$

$$=\frac{2\bar{\mu}(\bar{T})}{1+\lambda_1}\left(1+\lambda_2\left[\frac{\partial}{\partial\bar{t}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}\right]\right)\left(\frac{R}{\bar{r}+R}\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\bar{V}}{\bar{r}+R}\right), \quad ....(6\text{-}15)$$

6-3 Calculation of Lorentz Force

Fluid in this problem is flowing under the influence of radially varying magnetic field of the form [52]:

$$\bar{B}=\frac{RB_0}{R+\bar{r}}e_{\bar{r}} \quad .....(1)$$

The type of magnetic field given through eq.(1) satisfies the Maxwell equations. Velocity field for present flow configuration is taken of the form :

$$\bar{V}=[\bar{V}(\bar{r},\bar{X},\bar{t}),\bar{U}(\bar{r},\bar{X},\bar{t}),0]$$

where $\bar{U}$ and $\bar{V}$ are the axial and radial components of the velocity respectively.

The Lorentz force $\bar{F}$ in view of the magnetic and velocity fields mentioned above takes the following form:

$$\overline{J} = \overline{V} \times \overline{B} = \begin{vmatrix} e_{\bar{r}} & e_{\overline{X}} & e_{\overline{Z}} \\ \overline{V} & \overline{U} & 0 \\ \frac{R}{R+\bar{r}} B_0 & 0 & 0 \end{vmatrix} = -\frac{R}{R+\bar{r}} \overline{U} B_0 e_{\overline{Z}} \qquad \text{......(2)}$$

$$\sigma \times \overline{J} = -\frac{R}{R+\bar{r}} \sigma \overline{U} B_0 e_{\overline{Z}} \qquad \text{......(3)}$$

Utilization of ohms law gives the following expression :

$$\overline{F} = \overline{J} \times \overline{B} = \begin{vmatrix} e_{\bar{r}} & e_{\overline{X}} & e_{\overline{Z}} \\ 0 & 0 & \frac{-R}{R+\bar{r}} \sigma \overline{U} B_0 \\ \frac{R}{R+\bar{r}} B_0 & 0 & 0 \end{vmatrix} = -\left(\frac{R}{R+\bar{r}}\right)^2 \sigma \overline{U} B_0^{\,2} e_{\overline{X}} \qquad \text{......(4)}$$

That is

$$\overline{F} = \left[0, -\sigma\left(\frac{R}{R+\bar{r}}\right)^2 \overline{U} B_0^{\,2}, 0 \right] \qquad \text{.......(5)}$$

where B_0 is the strength of applied magnetic field, $e_{\bar{r}}$ is the unit vector in the radial direction, $\overline{J}$ is the current density and σ is the electric conductivity of fluid, $\overline{B}$ is the magnetic field. It is observed that the effect of magnetic field appear in the flow of axial direction.

6-4 Basic Equations

The basic equations governing the non-Newtonian in compressible viscous Jeffrey fluid are given by:

The continuity equation is given by:

$$\frac{R}{\bar{r}+R} \frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \bar{r}} + \frac{\overline{V}}{\bar{r}+R} = 0 \qquad \text{.......(6-16)}$$

The momentum equations are given by:

$$\rho\left(\frac{\partial\overline{V}}{\partial\bar{t}}+\overline{V}\frac{\partial\overline{V}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\overline{U}\frac{\partial\overline{V}}{\partial\overline{X}}-\frac{\overline{U}^{2}}{\bar{r}+R}\right)=-\frac{\partial\overline{P}}{\partial\bar{r}}+\frac{1}{\bar{r}+R}\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R)S_{\bar{r}\bar{r}}\right\}+\frac{R}{\bar{r}+R}\frac{\partial}{\partial\overline{X}}S_{\overline{X}\bar{r}}$$
$$\frac{1}{\bar{r}+R}S_{\overline{X}\overline{X}} \qquad ....(6\text{-}17)$$

$$\rho\left(\frac{\partial\overline{U}}{\partial\bar{t}}+\overline{V}\frac{\partial\overline{U}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\overline{U}\frac{\partial\overline{U}}{\partial\overline{X}}+\frac{\overline{U}\,\overline{V}}{\bar{r}+R}\right)=-\frac{R}{\bar{r}+R}\frac{\partial\overline{P}}{\partial\overline{X}}+\frac{R}{\bar{r}+R}\frac{\partial}{\partial\overline{X}}\overline{S}_{\overline{X}\overline{X}}+\frac{1}{(\bar{r}+R)^{2}}$$
$$\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R)^{2}S_{\overline{X}\bar{r}}\right\}-\sigma\left(\frac{R}{\bar{r}+R}\right)^{2}\mathrm{B}_0^{\,2}\overline{U} \qquad ....(6\text{-}18)$$

The temperature equation is given by :

$$\rho C_p\left(\frac{\partial}{\partial\bar{t}}+\overline{V}\frac{\partial}{\partial\bar{r}}+\frac{R\overline{U}}{\bar{r}+R}\frac{\partial}{\partial\overline{X}}\right)T=k_1\left[\frac{\partial^2T}{\partial\bar{r}^2}+\frac{1}{\bar{r}+R}\frac{\partial T}{\partial\bar{r}}+\left(\frac{R}{\bar{r}+R}\right)^2\frac{\partial^2T}{\partial\overline{X}^2}\right]+$$
$$(\overline{S}_{\bar{r}\bar{r}}-\overline{S}_{\overline{X}\overline{X}})\frac{\partial\overline{V}}{\partial\bar{r}}+\overline{S}_{\overline{X}\bar{r}}\left(\frac{\partial\overline{U}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\frac{\partial\overline{V}}{\partial\overline{X}}-\frac{\overline{U}}{\bar{r}+R}\right) \qquad ....(6\text{-}19)$$

The concentration equation is given by :

$$\left[\frac{\partial}{\partial\bar{t}}+\overline{V}\frac{\partial}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\overline{U}\frac{\partial}{\partial\overline{X}}\right]\overline{C}=D\left[\frac{\partial^2\overline{C}}{\partial\bar{r}^2}+\frac{1}{\bar{r}+R}\frac{\partial\overline{C}}{\partial\bar{r}}+\left(\frac{R}{\bar{r}+R}\right)^2\frac{\partial^2}{\partial\overline{X}^2}\right]\overline{C}+$$
$$\frac{DK_T}{T_m}\left[\frac{\partial^2T}{\partial\bar{r}^2}+\frac{1}{\bar{r}+R}\frac{\partial T}{\partial\bar{r}}+\left(\frac{R}{\bar{r}+R}\right)^2\frac{\partial^2T}{\partial\overline{X}^2}\right] \qquad ....(6\text{-}20)$$

Where D is the diffusion coefficient of the diffusing species, T_m the mean fluid temperature, K_T is the thermal diffusion ratio, T and $\overline{C}$ denote the fluid temperature and concentration respectively.

<u>6-5 Method of Solution</u> :

In order to simplify the governing equations of motion, temperature and concentration we may introduce the following dimensionless transformations as follows:

$$x=\frac{\overline{X}}{\lambda},\ r=\frac{\bar{r}}{a},\ u=\frac{\overline{U}}{c},\ v=\frac{\overline{V}}{\delta c},\ t=\frac{c\bar{t}}{\lambda},\ h=\frac{\overline{H}}{a},\ \phi=\frac{b}{a},\ \delta=\frac{a}{\lambda},\ K=\frac{R}{a},\ p=\frac{a^2\overline{P}}{\mu_0 c\lambda},$$
$$\mathrm{Re}=\frac{\rho ca}{\mu_0},\ M^2=\frac{\sigma B_0^2a^2}{\mu_0},\ \mathrm{S}=\frac{a\overline{S}}{\mu_0 C},\ \theta=\frac{T-T_0}{T_0},\ \varphi=\frac{\overline{C}-\overline{C}_0}{\overline{C}_0},\ \Pr=\frac{\mu_0C_\rho}{k_1},\ \mathrm{Ec}=\frac{C^2}{C_\rho T_0},$$

$$\mu(\theta)=\frac{\bar{\mu}(T)}{\mu_0},\alpha_1=\Pr.Ec=Br,\alpha_2=\mathrm{Sr}=\frac{\rho DK_T T_0}{\mu_0 T_m \bar{C}_0},\ \alpha_3=\mathrm{SC}=\frac{\mu_0}{\rho D},\mathrm{u}=-\frac{\partial\psi}{\partial r},\quad \mathrm{v}=\frac{K}{r+K}\frac{\partial\psi}{\partial x}\ ......(6-21)$$

In which ϕ is the amplitude ratio or the occlusion parameter, K is the curvature parameter, Sc is the Schmidt number, Sr is the soret number,

Now substituting (6-21) into equations (6-13)-(6-15) and into equations (6-16)-(6-20) we have:

From eq.(6-16) we have:

$$\frac{R}{\bar{r}+R}\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\partial\bar{V}}{\partial\bar{r}}+\frac{\bar{V}}{\bar{r}+R}=0$$

$$\frac{ak}{a(r+k)}.\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{C\delta}{a}\frac{\partial v}{\partial r}+\frac{C\delta v}{a(r+k)}=0$$

$$\frac{k}{(r+k)}.\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{C}{a}\frac{a}{\lambda}\frac{\partial v}{\partial r}+\frac{C}{a}\frac{a}{\lambda}\frac{v}{(r+k)}=0$$

$$\frac{k}{(r+k)}.\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{C}{\lambda}\frac{\partial v}{\partial r}+\frac{C}{\lambda}\frac{v}{(r+k)}=0 \qquad ......(6-22)$$

Multiplying both sides of (6-22) by $(\frac{\lambda}{C})$ we get :

$$\frac{k}{(r+k)}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{(r+k)}=0 \qquad ......(6-23)$$

From equation (6-17) we have:

$$\rho(\frac{\partial\bar{V}}{\partial\bar{t}}+\bar{V}\frac{\partial\bar{V}}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial\bar{V}}{\partial\bar{X}}-\frac{\bar{U}^2}{\bar{r}+R})=-\frac{\partial\bar{P}}{\partial\bar{r}}+\frac{1}{\bar{r}+R}\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R)S_{\bar{r}\bar{r}}\right\}+\frac{R}{\bar{r}+R}$$
$$\frac{\partial}{\partial\bar{X}}S_{\bar{X}\bar{r}}-\frac{1}{\bar{r}+R}S_{\bar{X}\bar{X}}$$

$$\rho(\frac{C^2\delta}{\lambda}\frac{\partial v}{\partial t}+C\delta v.\frac{C\delta}{a}\frac{\partial v}{\partial r}+\frac{ak}{a(r+k)}Cu\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}-\frac{(Cu)^2}{a(r+k)})=-\frac{C\lambda\mu_0}{a^3}\frac{\partial P}{\partial r}+$$
$$\frac{1}{a(r+k)}\frac{1}{a}\frac{\partial}{\partial r}\left\{a(r+k)\frac{\mu_0 C}{a}S_{rr}\right\}+\frac{ak}{a(r+k)}\frac{1}{\lambda}\frac{\partial}{\partial x}\frac{\mu_0 C}{a}S_{xr}-\frac{1}{a(r+k)}.\frac{\mu_0 C}{a}S_{xx}$$

$$\rho\left(\frac{C^2}{\lambda}\frac{a}{\lambda}\frac{\partial v}{\partial t}+\frac{C^2}{a}\frac{a^2}{\lambda^2}v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\frac{C^2}{\lambda}\frac{a}{\lambda}u\frac{\partial v}{\partial x}-\frac{C^2}{a(r+k)}u^2\right)=-\frac{C\lambda\mu_0}{a^3}\frac{\partial P}{\partial r}+$$
$$\frac{\mu_0 C}{a^2(r+k)}\frac{\partial}{\partial r}\{(r+k)S_{rr}\}+\frac{k}{(r+k)}\frac{\mu_0 C}{\lambda a}\frac{\partial}{\partial x}S_{xr}-\frac{\mu_0 C}{a^2(r+k)}.S_{xx}$$

$$\rho\left(\frac{C^2}{\lambda}\frac{a}{a}\frac{a}{\lambda}\frac{\partial v}{\partial t}+\frac{C^2}{a}\frac{a^2}{\lambda^2}v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\frac{C^2}{\lambda}\frac{a}{a}\frac{a}{\lambda}u\frac{\partial v}{\partial x}-\frac{C^2}{a(r+k)}u^2\right)=-\frac{C\lambda\mu_0}{a^3}\frac{\partial P}{\partial r}+$$
$$\frac{\mu_0 C}{a^2(r+k)}\frac{\partial}{\partial r}\{(r+k)S_{rr}\}+\frac{k}{(r+k)}\frac{\mu_0 C}{\lambda a}\frac{\partial}{\partial x}S_{xr}-\frac{\mu_0 C}{a^2(r+k)}.S_{xx}$$

$$\rho\frac{C^2}{a}\left(\delta^2\frac{\partial v}{\partial t}+\delta^2 v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\delta^2 u\frac{\partial v}{\partial x}-\frac{u^2}{(r+k)}\right)=-\frac{C\lambda\mu_0}{a^3}\frac{\partial P}{\partial r}+\frac{\mu_0 C}{a^2(r+k)}$$
$$\frac{\partial}{\partial r}\{(r+k)S_{rr}\}+\frac{k}{(r+k)}\frac{\mu_0 C}{\lambda a}\frac{\partial}{\partial x}S_{xr}-\frac{\mu_0 C}{a^2(r+k)}.S_{xx} \qquad ....(6-24)$$

Now, multiplying both sides of (6-24) by $\left(\frac{a^3}{C\lambda\mu}\right)$ we get:

$$\rho\frac{\cancel{C^2}}{\cancel{a}}\frac{\cancel{a^3}}{\cancel{C}\lambda\mu}\left(\delta^2\frac{\partial v}{\partial t}+\delta^2 v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\delta^2 u\frac{\partial v}{\partial x}-\frac{u^2}{(r+k)}\right)=-\frac{\partial P}{\partial r}+\frac{\cancel{\mu_0 C}}{a^2(r+k)}.\frac{a^3}{\cancel{C\mu_0}\lambda}$$
$$\frac{\partial}{\partial r}\{(r+k)S_{rr}\}+\frac{k}{(r+k)}\frac{\cancel{\mu_0 C}}{\lambda\cancel{a}}\frac{\cancel{a^3}}{\cancel{C\mu_0}\lambda}\frac{\partial}{\partial x}S_{xr}-\frac{\cancel{\mu_0 C}}{\cancel{a^2}(r+k)}.\frac{\cancel{a^3}}{\cancel{C\mu_0}\lambda}.S_{xx}$$

$$\frac{\rho Ca}{\mu_0}\frac{a}{\lambda}\left(\delta^2\frac{\partial v}{\partial t}+\delta^2 v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\delta^2 u\frac{\partial v}{\partial x}-\frac{u^2}{(r+k)}\right)=-\frac{\partial P}{\partial r}+\delta\frac{\partial}{\partial r}\{(r+k)S_{rr}\}+$$
$$\frac{k}{(r+k)}\delta^2\frac{\partial}{\partial x}S_{xr}-\frac{\delta}{(r+k)}.S_{xx}$$

Which is can be written as:

$$\mathrm{Re}\,\delta\left(\delta^2\frac{\partial v}{\partial t}+\delta^2 v\frac{\partial v}{\partial r}+\frac{k}{(r+k)}\delta^2 u\frac{\partial v}{\partial x}-\frac{u^2}{(r+k)}\right)=-\frac{\partial P}{\partial r}+\frac{\delta}{(r+k)}\frac{\partial}{\partial r}\{(r+k)S_{rr}\}$$
$$+\frac{k}{(r+k)}\delta^2\frac{\partial}{\partial x}S_{xr}-\frac{\delta}{(r+k)}.S_{xx} \qquad ......(6-25)$$

From eq.(6-18) we have:

$$\rho(\frac{\partial \bar{U}}{\partial \bar{t}}+\bar{V}\,\frac{\partial \bar{U}}{\partial \bar{r}}+\frac{R}{\bar{r}+R}\bar{U}\,\frac{\partial \bar{U}}{\partial \bar{X}}+\frac{\bar{U}\bar{V}}{\bar{r}+R})=-\frac{R}{\bar{r}+R}\frac{\partial \bar{P}}{\partial \bar{X}}+\frac{R}{\bar{r}+R}\frac{\partial}{\partial \bar{X}}\bar{S}_{\bar{X}\bar{X}}+\frac{1}{(\bar{r}+R)^2}$$
$$\frac{\partial}{\partial \bar{r}}\left\{(\bar{r}+R)^2\bar{S}_{\bar{X}\bar{r}}\right\}-\sigma(\frac{R}{\bar{r}+R})^2 B_0^{\,2}\bar{U}.$$

$$\rho(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+C\,\delta v\,\frac{C}{a}\frac{\partial u}{\partial r}+\frac{ak}{a(r+k)}Cu\,\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{Cu.C\,\delta v}{a(r+k)})=-\frac{ak}{a(r+k)}\frac{C\,\mu_0}{a^2}\frac{\partial P}{\partial x}+\frac{ak}{a(r+k)}$$
$$\frac{1}{\lambda}\frac{\partial}{\partial x}\frac{\mu_0 C}{a}S_{xx}+\frac{1}{(a(r+k))^2}.\frac{1}{a}\frac{\partial}{\partial r}\left\{(a(r+k))^2\frac{\mu_0 C}{a}S_{xr}\right\}-\sigma(\frac{ak}{a(r+k)})^2 B_0^{\,2}Cu.$$

$$\rho(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\cancel{a}}\frac{\cancel{a}}{\lambda}v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}\frac{C^2}{\lambda}u\,\frac{\partial u}{\partial x}+\frac{C^2}{a}\frac{a}{\lambda}\frac{u\,v}{(r+k)})=-\frac{k}{(r+k)}\frac{C\,\mu_0}{a^2}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}$$
$$\frac{1}{\lambda}\frac{\mu_0 C}{a}\frac{\partial}{\partial x}S_{xx}+\frac{1}{a^2(r+k)^2}.\frac{1}{a}\frac{\mu_0 C}{a}a^2\frac{\partial}{\partial r}\left\{(r+k)^2S_{xr}\right\}-\sigma(\frac{k}{(r+k)})^2 B_0^{\,2}Cu.$$

$$\rho(\frac{C^2}{\lambda}\frac{\partial u}{\partial t}+\frac{C^2}{\lambda}v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}\frac{C^2}{\lambda}u\,\frac{\partial u}{\partial x}+\frac{C^2}{\lambda}\frac{u\,v}{(r+k)})=-\frac{k}{(r+k)}\frac{C\,\mu_0}{a^2}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}$$
$$\frac{1}{\lambda}\frac{\mu_0 C}{a}\frac{\partial}{\partial x}S_{xx}+\frac{\mu_0 C}{a^2(r+k)^2}.\frac{\partial}{\partial r}\left\{(r+k)^2S_{xr}\right\}-\sigma(\frac{k}{(r+k)})^2 B_0^{\,2}Cu.$$

$$\rho\frac{C^2}{\lambda}(\frac{\partial u}{\partial t}+v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial u}{\partial x}+\frac{u\,v}{(r+k)})=-\frac{k}{(r+k)}\frac{C\,\mu_0}{a^2}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}$$
$$\frac{1}{\lambda}\frac{\mu_0 C}{a}\frac{\partial}{\partial x}S_{xx}+\frac{\mu_0 C}{a^2(r+k)^2}.\frac{\partial}{\partial r}\left\{(r+k)^2S_{xr}\right\}-\sigma(\frac{k}{(r+k)})^2 B_0^{\,2}Cu. \qquad ....(6-26)$$

Now, multiplying both sides of eq.(6-26) $(\frac{a^2}{C\,\mu_0})$ we get:

$$\rho\frac{\cancel{C^2}}{\lambda}\frac{a^2}{\cancel{C}\,\mu_0}(\frac{\partial u}{\partial t}+v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial u}{\partial x}+\frac{u\,v}{(r+k)})=-\frac{k}{(r+k)}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}\frac{1}{\lambda}\frac{\cancel{\mu_0 C}}{a}$$
$$\frac{a^2}{\cancel{C\mu_0}}\frac{\partial}{\partial x}S_{xx}+\frac{\cancel{\mu_0 C}}{\cancel{a^2}(r+k)^2}.\frac{\cancel{a^2}}{\cancel{C\mu_0}}\frac{\partial}{\partial r}\left\{(r+k)^2S_{xr}\right\}-\sigma(\frac{k}{(r+k)})^2 B_0^{\,2}\cancel{C}u.\frac{a^2}{\cancel{C}\,\mu_0}.$$

$$\frac{\rho C a}{\mu_0}\frac{a}{\lambda}(\frac{\partial u}{\partial t}+v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial u}{\partial x}+\frac{u\,v}{(r+k)})=-\frac{k}{(r+k)}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}\frac{a}{\lambda}\frac{\partial}{\partial x}S_{xx}+$$
$$\frac{1}{(r+k)^2}.\frac{\partial}{\partial r}\left\{(r+k)^2S_{xr}\right\}-(\frac{k}{(r+k)})^2\frac{\sigma B_0^{\,2}a^2}{\mu_0}u.$$

Which can be written as:

$$\mathrm{Re}\,\delta\left(\frac{\partial u}{\partial t}+v\,\frac{\partial u}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial u}{\partial x}+\frac{u\,v}{(r+k)}\right)=-\frac{k}{(r+k)}\frac{\partial P}{\partial x}+\frac{k}{(r+k)}\delta\,\frac{\partial}{\partial x}S_{xx}+\frac{1}{(r+k)^{2}}.$$

$$\frac{\partial}{\partial r}\left\{(r+k)^{2}S_{xr}\right\}-\left(\frac{k}{(r+k)}\right)^{2}M^{2}u. \qquad .....(6-27)$$

From eq. (6-19) we have:

$$\rho C_{\rho}\left(\frac{\partial}{\partial \bar{t}}+\bar{V}\,\frac{\partial}{\partial \bar{r}}+\frac{R}{\bar{r}+R}\bar{U}\,\frac{\partial}{\partial \bar{X}}\right)T=k_{1}\left[\frac{\partial^{2}T}{\partial \bar{r}^{2}}+\frac{1}{\bar{r}+R}\frac{\partial T}{\partial \bar{r}}+\left(\frac{R}{\bar{r}+R}\right)^{2}\frac{\partial^{2}T}{\partial \bar{X}^{2}}\right]+$$

$$(S_{\bar{r}\bar{r}}-S_{\bar{X}\bar{X}})\frac{\partial \bar{V}}{\partial \bar{r}}+S_{\bar{X}\bar{r}}\left(\frac{\partial \bar{U}}{\partial \bar{r}}+\frac{R}{\bar{r}+R}\frac{\partial \bar{V}}{\partial \bar{X}}-\frac{\bar{U}}{\bar{r}+R}\right)$$

$$\rho C_{\rho}\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+C\,\delta v\,\frac{1}{a}\frac{\partial T}{\partial r}+\frac{ak}{a(r+k)}Cu\,\frac{1}{\lambda}\frac{\partial T}{\partial x}\right)=k_{1}\left[\frac{1}{a^{2}}\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{a(r+k)}\frac{1}{a}\frac{\partial T}{\partial r}+\left(\frac{ak}{a(r+k)}\right)^{2}\right.$$

$$\left.\frac{1}{\lambda^{2}}\frac{\partial^{2}T}{\partial x^{2}}\right]+\left(\frac{\mu_{0}C}{a}S_{rr}-\frac{\mu_{0}C}{a}S_{xx}\right)\frac{C\,\delta}{a}\frac{\partial v}{\partial r}+\frac{\mu_{0}C}{a}S_{rx}\left(\frac{C}{a}\frac{\partial u}{\partial r}+\frac{ak}{a(r+k)}\frac{C\,\delta}{\lambda}\frac{\partial v}{\partial x}-\frac{Cu}{a(r+k)}\right)$$

$$\rho C_{\rho}\left(\frac{C}{\lambda}\frac{\partial T}{\partial t}+\frac{C}{\cancel{a}}v\,\frac{\cancel{a}}{\lambda}\frac{\partial T}{\partial r}+\frac{k}{(r+k)}\frac{C}{\lambda}u\,\frac{\partial T}{\partial x}\right)=k_{1}\left[\frac{1}{a^{2}}\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{a^{2}(r+k)}\frac{\partial T}{\partial r}+\left(\frac{k}{(r+k)}\right)^{2}\right.$$

$$\left.\frac{1}{\lambda^{2}}\frac{\partial^{2}T}{\partial x^{2}}\right]+\frac{\mu_{0}C}{a}(S_{rr}-S_{xx})\frac{C}{\cancel{a}}\frac{\cancel{a}}{\lambda}\frac{\partial v}{\partial r}+\frac{\mu_{0}C}{a}S_{rx}\left(\frac{C}{a}\frac{\partial u}{\partial r}+\frac{k}{(r+k)}\frac{C}{\lambda}\frac{a}{\lambda}\frac{\partial v}{\partial x}-\frac{Cu}{a(r+k)}\right)$$

$$\rho C_{\rho}\frac{C}{\lambda}\left[\frac{\partial T}{\partial t}+v\,\frac{\partial T}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial T}{\partial x}\right)=k_{1}\left[\frac{1}{a^{2}}\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{a^{2}(r+k)}\frac{\partial T}{\partial r}+\left(\frac{k}{(r+k)}\right)^{2}\frac{1}{\lambda^{2}}\frac{a^{2}}{a^{2}}\right.$$

$$\left.\frac{\partial^{2}T}{\partial x^{2}}\right]+\frac{\mu_{0}C}{a}\frac{C}{\lambda}(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_{0}C}{a}S_{rx}\left(\frac{C}{a}\frac{\partial u}{\partial r}+\frac{k}{(r+k)}\frac{C}{\lambda}\frac{a}{\lambda}\frac{a}{a}\frac{\partial v}{\partial x}-\frac{C}{a}\frac{u}{(r+k)}\right).$$

$$\rho C_{\rho}\frac{C}{\lambda}\left[\frac{\partial T}{\partial t}+v\,\frac{\partial T}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial T}{\partial x}\right)=\frac{k_{1}}{a^{2}}\left[\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{(r+k)}\frac{\partial T}{\partial r}+\left(\frac{k}{(r+k)}\right)^{2}\delta^{2}\frac{\partial^{2}T}{\partial x^{2}}\right]+$$

$$\frac{\mu_{0}C}{a}\frac{C}{\lambda}(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_{0}C}{a}\frac{C}{a}S_{rx}\left(\frac{\partial u}{\partial r}+\delta^{2}\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}\right) \qquad .....(6-28)$$

Now, multiplying both sides of (6-28) by $(\frac{a^{2}}{k_{1}})$ we get :

$$\frac{a^{2}}{k_{1}}.\rho C_{\rho}\frac{C}{\lambda}\left[\frac{\partial T}{\partial t}+v\,\frac{\partial T}{\partial r}+\frac{k}{(r+k)}u\,\frac{\partial T}{\partial x}\right)=\left[\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{(r+k)}\frac{\partial T}{\partial r}+\left(\frac{k}{(r+k)}\right)^{2}\delta^{2}\frac{\partial^{2}T}{\partial x^{2}}\right]+$$

$$\frac{\mu_0 C}{a}\frac{C}{\lambda}\frac{a^2}{k_1}(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu C}{a}\frac{C}{a}\frac{a^2}{k_1}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)})$$

$$\rho C_\rho\frac{C}{\lambda}\frac{a^2}{k_1}\frac{\mu_0}{\mu_0}[\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}+\frac{k}{(r+k)}u\frac{\partial T}{\partial x})=[\frac{\partial^2 T}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial T}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2 T}{\partial x^2}]+$$

$$\frac{\mu_0 C^2}{\lambda}\frac{a}{k_1}(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_0 C^2}{k_1}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}) \qquad ......(6-29)$$

$$\text{Now, since}\quad \theta=\frac{T-T_0}{T_0}\Rightarrow T-T_0=\theta T_0\Rightarrow T=\theta T_0+T_0\Rightarrow\partial T=T_0\partial\theta \qquad ......(6-30)$$

Thus, we can write eq.(6-29) by the following form:

$$\rho C_\rho\frac{C}{\lambda}\frac{a^2}{k_1}\frac{\mu_0}{\mu_0}[T_0\frac{\partial\theta}{\partial t}+vT_0\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}uT_0\frac{\partial\theta}{\partial x})=[T_0\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}T_0\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2$$

$$\delta^2 T_0\frac{\partial^2\theta}{\partial x^2}]+\frac{\mu_0 C^2}{k_1}\delta(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_0 C^2}{k_1}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}).$$

$$\rho C_\rho\frac{C}{\lambda}\frac{a^2}{k_1}\frac{\mu_0}{\mu_0}T_0[\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\theta}{\partial x})=T_0[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]$$

$$+\frac{\mu_0 C^2}{k_1}\delta(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_0 C^2}{k_1}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}) \qquad ....(6-31)$$

Multiplying both sides of eq.(6-31) by $(\frac{1}{T_0})$ we obtain:

$$\rho C_\rho\frac{C}{\lambda}\frac{a^2}{k_1}\frac{\mu_0}{\mu_0}[\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\theta}{\partial x})=[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]+$$

$$\frac{\mu_0 C^2}{k_1}\delta\frac{1}{T_0}(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_0 C^2}{k_1}\frac{1}{T_0}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}).$$

$$\frac{\rho Ca}{\mu_0}\frac{\mu_0 C_\rho}{k_1}\frac{a}{\lambda}[\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\theta}{\partial x})=[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]+$$

$$\frac{\mu_0 C^2}{k_1}\frac{1}{T_0}\frac{C_\rho}{C_\rho}\delta(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{\mu_0 C^2}{k_1}\frac{1}{T_0}\frac{C_\rho}{C_\rho}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}).$$

$$\frac{\rho Ca}{\mu_0}\frac{\mu_0 C_\rho}{k_1}\frac{a}{\lambda}[\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\theta}{\partial x})=[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]+$$

$$\frac{C^2}{C_\rho T_0}\frac{\mu_0 C_\rho}{k_1}\delta(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\frac{C^2}{C_\rho T_0}\frac{\mu_0 C_\rho}{k_1}S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}). \quad ....(6-32)$$

Which is can be written as:

$$\operatorname{Re}\Pr\delta[\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\theta}{\partial x})=[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{r+k})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]+$$
$$\operatorname{Ec}\Pr\delta(S_{rr}-S_{xx})\frac{\partial v}{\partial r}+\operatorname{Ec}\Pr S_{rx}(\frac{\partial u}{\partial r}+\delta^2\frac{k}{(r+k)}\frac{\partial v}{\partial x}-\frac{u}{(r+k)}). \quad ....(6-33)$$

From eq.(6-20) we have:

$$[\frac{\partial}{\partial\bar{t}}+\bar{V}\frac{\partial}{\partial\bar{r}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}]\bar{C}=D[\frac{\partial^2\bar{C}}{\partial\bar{r}^2}+\frac{1}{\bar{r}+R}\frac{\partial\bar{C}}{\partial\bar{r}}+(\frac{R}{\bar{r}+R})^2\frac{\partial^2\bar{C}}{\partial\bar{X}^2}]+\frac{DK_T}{T_m}[\frac{\partial^2 T}{\partial\bar{r}^2}$$
$$+\frac{1}{\bar{r}+R}\frac{\partial T}{\partial\bar{r}}+(\frac{R}{\bar{r}+R})^2\frac{\partial^2 T}{\partial\bar{X}^2}]$$

$$[\frac{C}{\lambda}\frac{\partial\bar{C}}{\partial t}+C\delta v\frac{1}{a}\frac{\partial\bar{C}}{\partial r}+\frac{ak}{a(r+k)}Cu\frac{1}{\lambda}\frac{\partial\bar{C}}{\partial x}]=D[\frac{1}{a^2}\frac{\partial^2\bar{C}}{\partial r^2}+\frac{1}{a(r+k)}\frac{1}{a}\frac{\partial\bar{C}}{\partial r}+(\frac{\cancel{a}k}{\cancel{a}(r+k)})^2$$
$$\frac{1}{\lambda^2}\frac{\partial^2\bar{C}}{\partial x^2}]+\frac{DK_T}{T_m}[\frac{1}{a^2}\frac{\partial^2 T}{\partial r^2}+\frac{1}{a(r+k)}\frac{1}{a}\frac{\partial T}{\partial r}+(\frac{\cancel{a}k}{\cancel{a}(r+k)})^2\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}]$$

$$[\frac{\bar{C}}{\lambda}\frac{\partial\bar{C}}{\partial t}+\frac{\bar{C}}{a}\frac{a}{\lambda}v\frac{\partial\bar{C}}{\partial r}+\frac{k}{(r+k)}\frac{\bar{C}}{\lambda}u\frac{\partial\bar{C}}{\partial x}]=D[\frac{1}{a^2}\frac{\partial^2\bar{C}}{\partial r^2}+\frac{1}{a^2(r+k)}\frac{\partial\bar{C}}{\partial r}+(\frac{k}{(r+k)})^2\frac{1}{\lambda^2}$$
$$\frac{\partial^2\bar{C}}{\partial x^2}]+\frac{DK_T}{T_m}[\frac{1}{a^2}\frac{\partial^2 T}{\partial r^2}+\frac{1}{a^2(r+k)}\frac{\partial T}{\partial r}+(\frac{k}{(r+k)})^2\frac{1}{\lambda^2}\frac{\partial^2 T}{\partial x^2}]$$

$$\frac{\bar{C}}{\lambda}[\frac{\partial\bar{C}}{\partial t}+v\frac{\partial\bar{C}}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\bar{C}}{\partial x}]=D[\frac{1}{a^2}\frac{\partial^2\bar{C}}{\partial r^2}+\frac{1}{a^2(r+k)}\frac{\partial\bar{C}}{\partial r}+(\frac{k}{(r+k)})^2\frac{1}{\lambda^2}\frac{a^2}{a^2}\frac{\partial^2\bar{C}}{\partial x^2}]+$$
$$\frac{DK_T}{T_m}[\frac{1}{a^2}\frac{\partial^2 T}{\partial r^2}+\frac{1}{a^2(r+k)}\frac{\partial T}{\partial r}+(\frac{k}{(r+k)})^2\frac{1}{\lambda^2}\frac{a^2}{a^2}\frac{\partial^2 T}{\partial x^2}]$$

$$\frac{\bar{C}}{\lambda}[\frac{\partial\bar{C}}{\partial t}+v\frac{\partial\bar{C}}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\bar{C}}{\partial x}]=\frac{D}{a^2}[\frac{\partial^2\bar{C}}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\bar{C}}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\bar{C}}{\partial x^2}]+$$
$$\frac{DK_T}{T_m}\frac{1}{a^2}[\frac{\partial^2 T}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial T}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2 T}{\partial x^2}] \quad ....(6-34)$$

Multiplying both sides of (6-34) by $(\frac{a^2}{D})$ we get:

$$\frac{\overline{C}}{\lambda}\frac{a^2}{D}[\frac{\partial \overline{C}}{\partial t}+v\frac{\partial \overline{C}}{\partial r}+\frac{k}{(r+k)}u\frac{\partial \overline{C}}{\partial x}]=[\frac{\partial^2 \overline{C}}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial \overline{C}}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2 \overline{C}}{\partial x^2}]+\frac{DK_T}{T_m}\frac{1}{a^2}$$

$$\frac{a^2}{D}[\frac{\partial^2 T}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial T}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2 T}{\partial x^2}]. \quad ......(6-35)$$

Since $\theta=\frac{T-T_0}{T_0}$ and $\varphi=\frac{\overline{C}-\overline{C}_0}{\overline{C}_0}$ thus $\partial T=T_0\partial\theta$ and $\partial\overline{C}=\overline{C}_0\partial\varphi$

So, we can write (6-35) by the following form:

$$\frac{\overline{C}}{\lambda}\frac{a^2}{D}[\overline{C}_0\frac{\partial\varphi}{\partial t}+v\overline{C}_0\frac{\partial\varphi}{\partial r}+\frac{k}{(r+k)}u\overline{C}_0\frac{\partial\varphi}{\partial x}]=[\overline{C}_0\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\overline{C}_0\frac{\partial\varphi}{\partial r}+(\frac{k}{(r+k)})^2\delta^2$$

$$\overline{C}_0\frac{\partial^2\varphi}{\partial x^2}]+\frac{DK_T}{T_m}\frac{1}{D}[T_0\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}T_0\frac{\partial\theta}{\partial r}+(\frac{k}{(r+k)})^2\delta^2 T_0\frac{\partial^2\theta}{\partial x^2}].$$

$$\frac{\overline{C}}{\lambda}\frac{a^2}{D}\overline{C}_0\frac{\rho}{\rho}[\frac{\partial\varphi}{\partial t}+v\frac{\partial\varphi}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\varphi}{\partial x}]=\overline{C}_0[\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\varphi}{\partial x^2}]$$

$$+\frac{DK_T}{T_m}\frac{1}{D}T_0[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]. \quad ....(6-36)$$

Multiplying both sides of (6-36) by $(\frac{1}{\overline{\overline{C}}_0})$ we get :

$$\frac{\rho}{\rho}\frac{\mu_0}{\mu_0}\frac{a^2}{D}\frac{\overline{C}}{\lambda}[\frac{\partial\varphi}{\partial t}+v\frac{\partial\varphi}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\varphi}{\partial x}]=[\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\varphi}{\partial x^2}]+$$

$$\frac{DK_T}{T_m}\frac{1}{D}\frac{T_0}{\overline{\overline{C}}_0}\frac{\rho}{\rho}\frac{\mu_0}{\mu_0}[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\theta}{\partial x^2}]$$

$$\frac{\rho\overline{C}a}{\mu_0}\frac{a}{\lambda}\frac{\mu_0}{\rho D}[\frac{\partial\varphi}{\partial t}+v\frac{\partial\varphi}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\varphi}{\partial x}]=[\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\varphi}{\partial x^2}]+$$

$$\frac{\rho DK_T T_0}{\mu_0 T_m\overline{C}_0}\frac{\mu_0}{\rho D}[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+(\frac{k}{(r+k)})^2\delta^2\frac{\partial^2\theta}{\partial x^2}].$$

Which can be written as:

$$\mathrm{Re}\,\delta Sc[\frac{\partial\varphi}{\partial t}+v\frac{\partial\varphi}{\partial r}+\frac{k}{(r+k)}u\frac{\partial\varphi}{\partial x}]=[\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi}{\partial r}+\delta^2(\frac{k}{(r+k)})^2\frac{\partial^2\varphi}{\partial x^2}]+$$

$$Sr\,Sc[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+\delta^2(\frac{k}{(r+k)})^2\frac{\partial^2\theta}{\partial x^2}]. \quad ....(6-37)$$

From eq.(6-13) we have:

$$S_{\bar r\bar r} = \frac{2\bar\mu(\bar T)}{1+\lambda_1}(1+\lambda_2[\frac{\partial}{\partial \bar t}+(\frac{R}{\bar r+R}\bar U\frac{\partial}{\partial \bar X}+\bar V\frac{\partial}{\partial \bar r})]\frac{\partial \bar V}{\partial \bar r}$$

$$\frac{\mu_0 C}{a} S_{rr} = \frac{2\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+(\frac{ak}{a(r+k)})Cu\frac{1}{\lambda}\frac{\partial}{\partial x}+C\delta v\frac{1}{a}\frac{\partial}{\partial r})]\frac{C\delta}{a}\frac{\partial v}{\partial r}$$

$$\frac{\mu_0 C}{a} S_{rr} = \frac{2\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+(\frac{k}{(r+k)})\frac{C}{\lambda}u\frac{\partial}{\partial x}+\frac{C}{\cancel{a}}\frac{\cancel{a}}{\lambda}v\frac{\partial}{\partial r})]\frac{C}{\cancel{a}}\frac{\cancel{a}}{\lambda}\frac{\partial v}{\partial r}$$

$$\frac{\mu_0 C}{a} S_{rr} = \frac{2\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+(\frac{k}{(r+k)})\frac{C}{\lambda}u\frac{\partial}{\partial x}+\frac{C}{\lambda}v\frac{\partial}{\partial r})]\frac{C}{\lambda}\frac{\partial v}{\partial r}$$

$$\frac{\mu_0 C}{a} S_{rr} = \frac{2\mu_0\mu(\theta)}{1+\lambda_1}\frac{C}{\lambda}(1+\lambda_2\frac{C}{\lambda}[\frac{\partial}{\partial t}+(\frac{k}{(r+k)})u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r})]\frac{\partial v}{\partial r} \qquad ....(6-38)$$

Multiplying both sides of (6-38) by $\frac{a}{C\mu_0}$ we get:

$$S_{rr} = \frac{2\cancel{\mu_0}}{1+\lambda_1}\frac{\cancel{C}}{\lambda}\frac{a}{\cancel{C\mu_0}}\mu(\theta)(1+\lambda_2\frac{C}{\lambda}\frac{a}{a}[\frac{\partial}{\partial t}+(\frac{k}{(r+k)})u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r})]\frac{\partial v}{\partial r}$$

$$S_{rr} = \frac{2\delta}{1+\lambda_1}\mu(\theta)(1+\frac{\lambda_2 C\delta}{a}[\frac{\partial}{\partial t}+(\frac{k}{(r+k)})u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r})]\frac{\partial v}{\partial r} \qquad ....(6-39)$$

From eq.(6-14) we have:

$$\bar S_{\bar r\bar X} = \frac{\bar\mu(\bar T)}{1+\lambda_1}(1+\lambda_2[\frac{\partial}{\partial\bar t}+\frac{R}{\bar r+R}\bar U\frac{\partial}{\partial\bar X}+\bar V\frac{\partial}{\partial\bar r}])(\frac{\partial\bar U}{\partial\bar r}+\frac{R}{\bar r+R}\frac{\partial\bar V}{\partial\bar X}-\frac{\bar U}{\bar r+R})$$

$$\frac{\mu_0 C}{a}S_{rx} = \frac{\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+\frac{\cancel{a}k}{\cancel{a}(r+k)}Cu\frac{1}{\lambda}\frac{\partial}{\partial x}+C\delta v\frac{1}{a}\frac{\partial}{\partial r}])(\frac{C}{a}\frac{\partial u}{\partial r}+$$

$$\frac{\cancel{a}k}{\cancel{a}(r+k)}\frac{C\delta}{\lambda}\frac{\partial v}{\partial x}-\frac{C}{a(r+k)}u)$$

$$\frac{\mu_0 C}{a}S_{rx} = \frac{\mu_0}{1+\lambda_1}\mu(\theta)(1+\lambda_2\frac{C}{\lambda}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{C}{a}\frac{\partial u}{\partial r}+\frac{k}{(r+k)}$$

$$\frac{C}{\lambda}\frac{a}{\lambda}\frac{\partial v}{\partial x}-\frac{C}{a}\frac{1}{(r+k)}u).$$

$$\frac{\mu_0 C}{a}S_{rx} = \frac{\mu_0}{1+\lambda_1}\mu(\theta)(1+\lambda_2\frac{C}{\lambda}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{C}{a}\frac{\partial u}{\partial r}+\frac{k}{(r+k)}$$

$$\frac{C}{\lambda}\frac{a}{\lambda}\frac{a}{a}\frac{\partial v}{\partial x}-\frac{C}{a}\frac{1}{(r+k)}u)$$

$$\frac{\mu_0 C}{a}S_{rx}=\frac{\mu_0}{1+\lambda_1}\mu(\theta)\frac{C}{a}(1+\lambda_2\frac{C}{\lambda}\frac{a}{a}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{\partial u}{\partial r}+\frac{k}{(r+k)}$$

$$\delta^2\frac{\partial v}{\partial x}-\frac{u}{(r+k)})\qquad ......(6\text{-}40)$$

Multiplying both sides of (6-40) by $(\frac{a}{\mu_0 C})$ we have:

$$S_{rx}=\frac{\cancel{\mu_0}}{1+\lambda_1}\frac{\cancel{C}}{\cancel{a}}\frac{\cancel{a}}{\cancel{\mu_0}\cancel{C}}\mu(\theta)(1+\frac{\lambda_2 C\delta}{a}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{\partial u}{\partial r}+\frac{k}{(r+k)}$$

$$\delta^2\frac{\partial v}{\partial x}-\frac{u}{(r+k)}).$$

$$S_{rx}=\frac{\mu(\theta)}{1+\lambda_1}(1+\frac{\lambda_2 C\delta}{a}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{\partial u}{\partial r}+\frac{k}{(r+k)}\delta^2\frac{\partial v}{\partial x}-$$

$$\frac{u}{(r+k)})\qquad ......(6-41)$$

From eq.(6-15) we have:

$$\bar{S}_{\bar{X}\bar{X}}=\frac{2\bar{\mu}(T)}{1+\lambda_1}(1+\lambda_2[\frac{\partial}{\partial\bar{t}}+\frac{R}{\bar{r}+R}\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{r}}])(\frac{R}{\bar{r}+R}\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\bar{V}}{\bar{r}+R}),$$

$$\frac{\mu_0 C}{a}S_{xx}=\frac{2\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+\frac{\cancel{a}k}{\cancel{a}(r+k)}Cu\frac{1}{\lambda}\frac{\partial}{\partial x}+C\delta v\frac{1}{a}\frac{\partial}{\partial r}])$$

$$(\frac{\cancel{a}k}{\cancel{a}(r+k)}\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{C\delta v}{a(r+k)}),$$

$$\frac{\mu_0 C}{a}S_{xx}=\frac{2\mu_0\mu(\theta)}{1+\lambda_1}(1+\lambda_2[\frac{C}{\lambda}\frac{\partial}{\partial t}+\frac{k}{(r+k)}\frac{C}{\lambda}u\frac{\partial}{\partial x}+\frac{C}{a}\frac{a}{\lambda}v\frac{\partial}{\partial r}])$$

$$(\frac{k}{(r+k)}\frac{C}{\lambda}\frac{\partial u}{\partial x}+\frac{C}{a}\frac{a}{\lambda}\frac{v}{(r+k)}),$$

$$\frac{\mu_0 C}{a}S_{xx}=\frac{2\mu_0}{1+\lambda_1}(1+\lambda_2\frac{C}{\lambda}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])\frac{C}{\lambda}(\frac{k}{(r+k)}$$

$$\frac{\partial u}{\partial x}+\frac{v}{(r+k)}),\qquad .\ ....(6\text{-}42)$$

Multiplying both sides of (6-42) by $(\frac{a}{\mu_0 C})$ we get:

$$S_{xx}=\frac{2\cancel{\mu_0}}{1+\lambda_1}\frac{C}{\lambda}\frac{a}{\cancel{\mu_0 C}}(1+\lambda_2\frac{C}{\lambda}\frac{a}{a}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{k}{(r+k)}\frac{\partial u}{\partial x}+\frac{v}{(r+k)}),$$

$$S_{xx}=\frac{2\delta}{1+\lambda_1}\mu(\theta)(1+\frac{\lambda_2 C\delta}{a}[\frac{\partial}{\partial t}+\frac{k}{(r+k)}u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}])(\frac{k}{(r+k)}\frac{\partial u}{\partial x}+\frac{v}{(r+k)}) \quad ...(6-43)$$

The general solution of the governing equations (6-25)-(6-43) in the general case seems to be difficult and not easy, therefore we shall can fine the analysis under the assumption of small wave length $(\delta << 1)$ and low Reynolds number approximation, thus we can write the above equations in the form of stream function:

$$\frac{\partial P}{\partial r}=0 \quad ....(6-44)$$

$$\frac{\partial P}{\partial x}=\frac{1}{k(r+k)}.\frac{\partial}{\partial r}\left\{(r+k)^2 S_{xr}\right\}-\frac{k}{r+k}.M^2\frac{\partial \psi}{\partial r} \quad ....(6-45)$$

$$S_{rr}=0,\ S_{xx}=0, S_{rx}=\frac{\mu(\theta)}{1+\lambda_1}(-\frac{\partial^2\psi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial \psi}{\partial r}) \quad .....(6-46)$$

$$0=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}+BrS_{rx}(-\frac{\partial^2\psi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\psi}{\partial r}) \quad ......(6-47)$$

$$0=\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi}{\partial r}+SrSc(\frac{\partial^2\theta}{\partial r^2}-\frac{1}{(r+k)}\frac{\partial\theta}{\partial r}) \quad ......(6-48)$$

6-6 Rate of Volume Flow and Boundary Conditions:

The relation between volume flow rate and time average flow rate is [78]:

$$F(x,t)=Q+2(h(x,t)-1) \quad .....(6-49)$$

The corresponding dimensionless boundary conditions are given by:

$$\psi=\pm\frac{F}{2}, \text{ at } r=\mp h=\mp(1+\varphi\cos 2\pi(x-t))$$

$$\frac{\partial\psi}{\partial r}=0, \text{ at } r=\mp h$$

$$\theta=0, \sigma=0, r=\mp h \quad ......(6-50)$$

The coefficient of heat transfer at the upper wall is given by:

$$Z = h_x (\theta_r)_{r=h} \qquad .....(6\text{-}51)$$

6-7 Reynolds Model of Viscosity

The Reynolds model of viscosity is used to describe the variable of viscosity with temperature. The Reynolds model of viscosity is defined as: [81]

$$\mu(\theta) = e^{-\alpha\theta} \qquad .....(6\text{-}52)$$

Using the McLaurin series expansion the above expression can be written as:

$$\mu(\theta) = 1 - \alpha\theta, \quad \text{for } \alpha << 1 \qquad ....(6\text{-}53)$$

If $\alpha = 0$ Thus the constant viscosity will be achieve.

Now, Compensating equation (6-53)) into equation (6-46),(3) we have :

$$S_{rx} = \frac{1-\alpha\theta}{1+\lambda_1}\left(-\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r+k}\frac{\partial\psi}{\partial r}\right) \qquad ....(6\text{-}54)$$

Substitute eq. (6-54) into eq. (6-45) and eq. (6-47) we have:

$$\frac{\partial p}{\partial x} = \frac{1}{K(r+k)}.\frac{\partial}{\partial r}\{(r+k)^2\frac{(1-\alpha\theta)}{1+\lambda_1}.(-\frac{\partial^2\psi}{\partial r^2} + \frac{1}{(r+k)}\frac{\partial\psi}{\partial r})\} + \frac{k}{(r+k)}k^2 M^2 \frac{\partial\psi}{\partial r} \qquad ....(6\text{-}55)$$

$$0 = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{(r+k)}.\frac{\partial\theta}{\partial r} + Br\frac{(1-\alpha\theta)}{1+\lambda_1}.(-\frac{\partial^2\psi}{\partial r^2} + \frac{1}{(r+k)}\frac{\partial\psi}{\partial r})^2 \qquad ....(6\text{-}56)$$

6-8 Perturbation Analysis

Equation (6-44) shows that P depends on x only. equation (6-55) is nonlinear and it is not easy to get a closed form solution. However for vanishing α, the boundary value problem is agreeable to an easy analytical solution. In this case the equation can be solved. So for this analysis we suggests small α for the perturbation technique to solve the non-linear problem. Accordingly, we write:

$$\psi = \psi_0 + \alpha\psi_1 + ...$$
$$F = F_0 + \alpha F_1 + ...$$
$$p = p_0 + \alpha p_1 + ...$$
$$\varphi = \varphi_0 + \alpha\varphi_1 + ... \qquad ....(6\text{-}57)$$

Now, if we substituting Eq.(6-57) into Eq. (6-55),(6-56),(6-48) and (6-50) we see that:

$$\frac{\partial p}{\partial x}=\frac{1}{K(r+k)}.\frac{1}{1+\lambda_1}\frac{\partial}{\partial r}\{-(r+k)^2.\frac{\partial^3\psi}{\partial r^3}-(r+k)\frac{\partial^2\psi}{\partial r^2}+\frac{\partial\psi}{\partial r}+\alpha(\theta.(r+k)^2.\frac{\partial^3\psi}{\partial r^3}+2\theta$$

$$(r+k).\frac{\partial^2\psi}{\partial r^2}+(r+k)^2\frac{\partial^2\psi}{\partial r^2}.\frac{\partial\theta}{\partial r}-\theta.(r+k)\frac{\partial^2\psi}{\partial r^2}-\theta\frac{\partial\psi}{\partial r}-(r+k)\frac{\partial\psi}{\partial r}\frac{\partial\theta}{\partial r})\}+\frac{k}{(r+k)}M^2\frac{\partial\psi}{\partial r}.$$

$$\frac{\partial p_0}{\partial x}+\alpha\frac{\partial p_1}{\partial x}=\frac{1}{K(r+k)}.\frac{1}{1+\lambda_1}\{-(r+k)^2.(\frac{\partial^3\psi_0}{\partial r^3}+\alpha\frac{\partial^3\psi_1}{\partial r^3})-(r+k)(\frac{\partial^2\psi_0}{\partial r^2}+\alpha\frac{\partial^2\psi_1}{\partial r^2})+(\frac{\partial\psi_0}{\partial r}$$

$$+\alpha\frac{\partial\psi_1}{\partial r})+\alpha((\theta_0+\alpha\theta_1).(r+k)^2.(\frac{\partial^3\psi_0}{\partial r^3}+\alpha\frac{\partial^3\psi_1}{\partial r^3})+2(\theta_0+\alpha\theta_1)(r+k).(\frac{\partial^2\psi_0}{\partial r^2}+\alpha\frac{\partial^2\psi_1}{\partial r^2})+$$

$$(r+k)^2(\frac{\partial^2\psi_0}{\partial r^2}+\alpha\frac{\partial^2\psi_1}{\partial r^2}).(\frac{\partial\theta_0}{\partial r}+\alpha\frac{\partial\theta_1}{\partial r})-(\theta_0+\alpha\theta_1).(r+k)(\frac{\partial^2\psi_0}{\partial r^2}+\alpha\frac{\partial^2\psi_1}{\partial r^2})-(\theta_0+\alpha\theta_1)(\frac{\partial\psi_0}{\partial r}$$

$$+\alpha\frac{\partial\psi_1}{\partial r})-(r+k)(\frac{\partial\psi_0}{\partial r}+\alpha\frac{\partial\psi_1}{\partial r})(\frac{\partial\theta_0}{\partial r}+\alpha\frac{\partial\theta_1}{\partial r}))\}+\frac{k}{(r+k)}M^2(\frac{\partial\psi_0}{\partial r}+\alpha\frac{\partial\psi_1}{\partial r})$$

Thus we have after some simplifications

$$\frac{\partial p_0}{\partial x}+\alpha\frac{\partial p_1}{\partial x}=\frac{1}{K(r+k)}.\frac{1}{1+\lambda_1}\{-(r+k)^2.(\frac{\partial^3\psi_0}{\partial r^3}+\alpha\frac{\partial^3\psi_1}{\partial r^3})-(r+k)(\frac{\partial^2\psi_0}{\partial r^2}+\alpha\frac{\partial^2\psi_1}{\partial r^2})+$$

$$(\frac{\partial\psi_0}{\partial r}+\alpha\frac{\partial\psi_1}{\partial r})+\alpha(r+k)^2\theta_0.\frac{\partial^3\psi_0}{\partial r^3}+\alpha^2(r+k)^2\theta_0\frac{\partial^3\psi_1}{\partial r^3}+\alpha^2(r+k)^2\theta_1\frac{\partial^3\psi_0}{\partial r^3}+\alpha^3$$

$$(r+k)^2\theta_1\frac{\partial^3\psi_1}{\partial r^3}+\alpha(r+k).\theta_0\frac{\partial^2\psi_0}{\partial r^2}+\alpha^2(r+k)\theta_0\frac{\partial^2\psi_1}{\partial r^2}+\alpha^2(r+k)\theta_1.\frac{\partial^2\psi_0}{\partial r^2}+\alpha^3(r+k)$$

$$\theta_1\frac{\partial^2\psi_1}{\partial r^2})+\alpha(r+k)^2\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial\theta_0}{\partial r}+\alpha^2(r+k)^2\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial\theta_1}{\partial r}+\alpha^2(r+k)^2\frac{\partial^2\psi_1}{\partial r^2}\frac{\partial\theta_0}{\partial r}+\alpha^3(r+k)^2$$

$$\frac{\partial^2\psi_1}{\partial r^2}\frac{\partial\theta_1}{\partial r})-\alpha\theta_0\frac{\partial\psi_0}{\partial r}-\alpha^2\theta_0\frac{\partial\psi_1}{\partial r}-\alpha^2\theta_1\frac{\partial\psi_0}{\partial r}-\alpha^3\theta_1\frac{\partial\psi_1}{\partial r}-\alpha(r+k)\frac{\partial\psi_0}{\partial r}.\frac{\partial\theta_0}{\partial r}-\alpha^2(r+k)$$

$$\frac{\partial\psi_0}{\partial r}\frac{\partial\theta_1}{\partial r}-\alpha^2(r+k)\frac{\partial\psi_1}{\partial r}\frac{\partial\theta_0}{\partial r}-\alpha^3(r+k)\frac{\partial\psi_1}{\partial r}\frac{\partial\theta_1}{\partial r}\}+\frac{k}{(r+k)}M^2(\frac{\partial\psi_0}{\partial r}+\alpha\frac{\partial\psi_1}{\partial r})\qquad .....(6-58)$$

Also we have about the equation of temperature:

$$0=\frac{\partial^2}{\partial r^2}(\theta_0+\alpha\theta_1)+\frac{1}{(r+k)}\frac{\partial}{\partial r}(\theta_0+\alpha\theta_1)+\frac{Br}{1+\lambda_1}(1-\alpha(\theta_0+\alpha\theta_1))(\frac{\partial^2}{\partial r^2}(\psi_0+\alpha\psi_1))^2$$

$$-\frac{2}{(r+k)}.\frac{\partial^2}{\partial r^2}(\psi_0+\alpha\psi_1).\frac{\partial}{\partial r}(\psi_0+\alpha\psi_1)+\frac{1}{(r+k)^2}(\frac{\partial}{\partial r}(\psi_0+\alpha\psi_1))^2)\qquad ......(6-59)$$

and the equation of concentration can be written by:

$$0=\frac{\partial^2}{\partial r^2}(\varphi_0+\alpha\varphi_1)+\frac{1}{(r+k)}\frac{\partial}{\partial r}(\varphi_0+\alpha\varphi_1)+SrSc(\frac{\partial^2}{\partial r^2}(\theta_0+\alpha\theta_1)+\frac{1}{(r+k)}\frac{\partial}{\partial r}(\theta_0+\alpha\theta_1)) \quad ......(6-60)$$

So, if we collecting the coefficient of powers of α, then we can get the zeroth and first order equation with it's boundary equations:

6-8-1 Zeros- order system $(\alpha^{(0)})$

$$\frac{\partial p_0}{\partial x}=\frac{1}{k}\frac{1}{1+\lambda_1}\{-(r+k)\frac{\partial^3\psi_0}{\partial r^3}-\frac{\partial^2\psi_0}{\partial r^2}+\frac{1}{(r+k)})\frac{\partial\psi_0}{\partial r}\}+\frac{k}{r+k}.\mathrm{M}^2\frac{\partial\psi_0}{\partial r} \quad ....(6-61)$$

Differentiable eq.(6-61) with respect to (r) we have :

$$0=\frac{1}{1+\lambda_1}((r+k).\frac{\partial^4\psi_0}{\partial r^4}+2\frac{\partial^3\psi_0}{\partial r^3}-\frac{1}{(r+k)}\frac{\partial^2\psi_0}{\partial r^2}+\frac{1}{(r+k)^2}\frac{\partial\psi_0}{\partial r}-\mathrm{k}^2\mathrm{M}^2(\frac{1}{(r+k)}.$$
$$\frac{\partial^2\psi_0}{\partial r^2}-\frac{1}{(r+k)^2}\frac{\partial\psi_0}{\partial r}) \quad ....(6-62)$$

$$0=\frac{\partial^2\theta_0}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta_0}{\partial r}+\frac{Br}{1+\lambda_1}\{(\frac{\partial^2\psi_0}{\partial r^2})^2-\frac{2}{(r+k)}.\frac{\partial^2\psi_0}{\partial r^2}.\frac{\partial\psi_0}{\partial r}+\frac{1}{(r+k)^2}(\frac{\partial\psi_0}{\partial r})^2\} \quad ...(6-63)$$

$$0=\frac{\partial^2\varphi_0}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi_0}{\partial r}+SrSc(\frac{\partial^2\theta_0}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta_0}{\partial r}) \quad .....(6-64)$$

Along the corresponding boundary conditions:

$$\psi_0=\frac{-F_0}{2},\frac{\partial\psi_0}{\partial r}=0,\ \theta_0=0,\ \varphi_0=0,\ \ at\left(r=+h\right)$$
$$\psi_0=\frac{F_0}{2},\frac{\partial\psi_0}{\partial r}=0,\ \theta_0=0,\ \varphi_0=0,\ \ at\left(r=-h\right) \quad ....(6-65)$$

6-8-2 First order system $(\alpha^{(1)})$

$$\frac{\partial p_1}{\partial x}=\frac{1}{k}\frac{1}{1+\lambda_1}\{-(r+k)\frac{\partial^3\psi_1}{\partial r^3}-\frac{\partial^2\psi_1}{\partial r^2}+\frac{1}{r+k}\frac{\partial\psi_1}{\partial r}+(r+k)\theta_0\frac{\partial^3\psi_0}{\partial r^3}+\theta_0\frac{\partial^2\psi_0}{\partial r^2}+$$
$$(r+k)\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial\theta_0}{\partial r}-\frac{1}{r+k}\theta_0\frac{\partial\psi_0}{\partial r}-\frac{\partial\psi_0}{\partial r}\frac{\partial\theta_0}{\partial r}\}+\frac{k}{r+k}.\mathrm{M}^2\frac{\partial\psi_1}{\partial r} \quad ....(6-66)$$

Differentiable eq.(6-66) with respect to (r) we have:

$$0=\frac{1}{1+\lambda_1}\{(r+k)\frac{\partial^4\psi_1}{\partial r^4}+2\frac{\partial^3\psi_1}{\partial r^3}-\frac{1}{r+k}\frac{\partial^2\psi_1}{\partial r^2}+\frac{1}{(r+k)^2}\frac{\partial\psi_1}{\partial r}-(r+k)\theta_0\frac{\partial^4\psi_0}{\partial r^4}-$$
$$2(r+k)\frac{\partial^3\psi_0}{\partial r^3}\frac{\partial\theta_0}{\partial r}-2\theta_0\frac{\partial^3\psi_0}{\partial r^3}-(r+k)\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial^2\theta_0}{\partial r^2}-\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial\theta_0}{\partial r}+\frac{1}{r+k}\theta_0\frac{\partial^2\psi_0}{\partial r^2}+$$
$$\frac{1}{r+k}\frac{\partial\psi_0}{\partial r}\frac{\partial\theta_0}{\partial r}-\frac{1}{(r+k)^2}(\theta_0\frac{\partial\psi_0}{\partial r})+\frac{\partial\psi_0}{\partial r}\frac{\partial^2\theta_0}{\partial r^2}-\mathrm{k}^2.\mathrm{M}^2\{\frac{1}{r+k}.\frac{\partial^2\psi_1}{\partial r^2}-\frac{1}{(r+k)^2}\frac{\partial\psi_1}{\partial r}\}$$
$$\text{......(6-67)}$$

$$0=\frac{\partial^2\theta_1}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta_1}{\partial r}+\frac{Br}{1+\lambda_1}\{2\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial^2\psi_1}{\partial r^2}-\frac{2}{(r+k)}.(\frac{\partial^2\psi_0}{\partial r^2}.\frac{\partial\psi_1}{\partial r}+\frac{\partial^2\psi_1}{\partial r^2}.\frac{\partial\psi_0}{\partial r})+$$
$$\frac{1}{(r+k)^2}.2\frac{\partial\psi_0}{\partial r}\frac{\partial\psi_1}{\partial r}-\theta_0(\frac{\partial^2\psi_0}{\partial r^2})^2+\frac{2}{(r+k)}\theta_0\frac{\partial^2\psi_0}{\partial r^2}\frac{\partial\psi_0}{\partial r}-\frac{1}{(r+k)^2}\theta_0(\frac{\partial\psi_0}{\partial r})^2\}$$
$$\text{.....(6-68)}$$

$$0=\frac{\partial^2\varphi_1}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\varphi_1}{\partial r}+SrSc(\frac{\partial^2\theta_1}{\partial r^2}+\frac{1}{(r+k)}\frac{\partial\theta_1}{\partial r}) \qquad \text{.....(6-69)}$$

The corresponding boundary condition are given by :

$$\psi_1=\frac{-F_0}{2},\frac{\partial\psi_1}{\partial r}=0,\ \theta_1=0,\ \varphi_1=0,\ \ at\left(r=+h\right)$$
$$\psi_1=\frac{F_1}{2},\frac{\partial\psi_1}{\partial r}=0,\ \theta_1=0,\ \varphi_1=0,\ \ at\left(r=-h\right) \qquad \text{....(6-70)}$$

6-9 Solution of The Problem

6-9-1 Solution of the zero's order $(\alpha^{(0)})$

The solution of Eq.(6-62), (6-63) and (6-64) subset to the associates boundary conditions (6-65) are found to be the form:

$$\psi_0=a_4+a_3kr+\frac{a_3}{2}r^2-\frac{a_2}{-1+n_1}(k+r)^{1-n_1}+\frac{a_1(k+r)^{1+n_1}}{1+n_1} \qquad \text{....(6-71)}$$

$$\theta_0=\frac{-a_2{}^2(1+n_1)^2(k+r)^{-2n_1}\alpha_1}{4n_1{}^2(1+\lambda_1)}-\frac{a_1{}^2(-1+n_1)^2(k+r)^{2n_1}\alpha_1}{4n_1{}^2(1+\lambda_1)}+$$
$$C_2+C_1\log(k+r)+\frac{a_1a_2(-1+n_1)^2\alpha_1\log(k+r)^2}{1+\lambda_1} \qquad \text{....(6-72)}$$

$$\varphi_0 = \frac{a_2{}^2(1+n_1)^2(k+r)^{-2n_1}\alpha_1\alpha_2\alpha_3}{4n_1{}^2(1+\lambda_1)} + \frac{a_1{}^2(-1+n_1)^2(k+r)^{2n_1}\alpha_1\alpha_2\alpha_3}{4n_1{}^2(1+\lambda_1)} +$$

$$C_6 + C_5\log(k+r) - \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(k+r)^2}{1+\lambda_1} \qquad \text{........}(6\text{-}73)$$

where ($(a_i, \mathrm{i}=1,2,3,4), (\mathrm{C}_j, \mathrm{j}=1,2), (\mathrm{C}_k, \mathrm{k}=5,6)$ are constants and can be written as:

$$a_1 = \frac{-f_0(k(-(-h+k)^{n_1}+(h+k)^{n_1})+h((-h+k)^{n_1}+(h+k)^{n_1})(-1+n_1)(1+n_1)}{\begin{array}{l}(2(k^2((-h+k)^{n_1}-(h+k)^{n_1})^2n_1+h^2((-h+k)^{n_1}+(h+k)^{n_1})^2n_1\\ +hk((-h+k)^{2n_1}-(h+k)^{2n_1})(1+n_1{}^2)));\end{array}}$$

$$a_2 = \frac{\begin{array}{l}(f_0(-h+k)^{n_1}(h+k)^{n_1})(k((-h+k)^{n_1}-(h+k)^{n_1})+h((-h+k)^{n_1}-(h+k)^{n_1})\\ (-1+n_1)(1+n_1)\end{array}}{\begin{array}{l}(2(k^2((-h+k)^{n_1}-(h+k)^{n_1})^2n_1+h^2((-h+k)^{n_1}+(h+k)^{n_1})^2n_1\\ +hk((-h+k)^{2n_1}-(h+k)^{2n_1})(1+n_1{}^2)));\end{array}}$$

$$a_3 = \frac{-(f_0((-h+k)^{2n_1}-(h+k)^{2n_1})(-1+n_1)(1+n_1)}{\begin{array}{l}(2(k^2((-h+k)^{n_1}-(h+k)^{n_1})^2n_1+h^2((-h+k)^{n_1}+(h+k)^{n_1})^2n_1\\ +hk((-h+k)^{2n_1}-(h+k)^{2n_1})(1+n_1{}^2)));\end{array}}$$

$$a_4 = \frac{\begin{array}{l}(f_0(2k^2((-h+k)^{2n_1}-(h+k)^{2n_1})+4hk((-h+k)^{2n_1}+\\ (h+k)^{2n_1})n_1+h^2((-h+k)^{2n_1}-(h+k)^{2n_1})(1+n_1{}^2)\end{array}}{\begin{array}{l}(4(k^2((-h+k)^{n_1}-(h+k)^{n_1})^2n_1+h^2((-h+k)^{n_1}+(h+k)^{n_1})^2n_1\\ +hk((-h+k)^{2n_1}-(h+k)^{2n_1})(1+n_1{}^2)));\end{array}} \qquad \text{.......}(6\text{-}74)$$

$$c_1 = \frac{\begin{array}{l}-(\alpha_1(-\dfrac{a_1^2(-h+k)^{2n_1})(-1+n_1{}^2)}{n_1{}^2}+\dfrac{a_1^2(h+k)^{2n_1})(-1+n_1{}^2)}{n_1{}^2}-\dfrac{a_2^2(-h+k)^{-2n_1})(1+n_1{}^2)}{n_1{}^2}\\ \dfrac{a_2^2(h+k)^{-2n_1})(1+n_1{}^2)}{n_1{}^2}+4a_1a_2(-1+n_1{}^2)\log(-h+k)^2-4a_1a_2(-1+n_1{}^2)\log(h+k)^2\end{array}}{(4(1+\lambda_1)(\log(-h+k)-\log(h+k)))}$$

$$c_2 = \frac{1}{4n_1{}^2(1+\lambda_1)(-\log(-h+k)+\log(h+k))}(-h+k)^{-2n_1}(h+k)^{-2n_1}\alpha_1((h+k)^{2n_1}$$
$$(a_1^2(-h+k)^{4n_1})(-1+n_1{}^2)+a_2^2(1+n_1{}^2))\log(h+k)-4a_1a_2(-h+k)^{2n_1}(h+k)^{2n_1}n_1{}^2$$
$$(-1+n_1{}^2)\log(-h+k)^2\log(h+k)-(-h+k)^{2n_1}\log(-h+k)(a_1^2(h+k)^{4n_1}(-1+n_1{}^2)^2$$
$$+a_2^2(1+n_1{}^2)-4a_1a_2(h+k)^{2n_1}n_1{}^2(-1+n_1{}^2)\log(h+k)^2)); \qquad \text{.......}(6\text{-}75)$$

$$c_5 = \frac{\begin{array}{l}(\frac{a_1^2(-h+k)^{2n_1}(-1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \frac{a_1^2(h+k)^{2n_1}(-1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} + \\ \frac{a_2^2(-h+k)^{-2n_1}(1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \frac{a_2^2(h+k)^{-2n_1}(1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \\ \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(-h+k)^2}{(1+\lambda_1)} + \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(h+k)^2}{(1+\lambda_1)}\end{array}}{\log(-h+k)-\log(h+k)}$$

$$c_6 = \frac{\begin{array}{l}(-\frac{a_1^2(h+k)^{2n_1}(-1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \frac{a_2^2(h+k)^{-2n_1}(1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} + \\ \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(h+k)^2}{(1+\lambda_1)} + \log(h+k)(\frac{a_1^2(-h+k)^{2n_1}(-1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \\ \frac{a_1^2(h+k)^{2n_1}(-1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} + \frac{a_2^2(-h+k)^{-2n_1}(1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \\ \frac{a_2^2(h+k)^{-2n_1}(1+n_1)^2\alpha_1\alpha_2\alpha_3}{4n_1^{\ 2}(1+\lambda_1)} - \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(-h+k)^2}{(1+\lambda_1)} + \\ \frac{a_1a_2(-1+n_1)^2\alpha_1\alpha_2\alpha_3\log(h+k)^2}{(1+\lambda_1)}\end{array}}{\log(-h+k)-\log(h+k)}$$

$$\text{.......(6-76)}$$

<u>6- 9-2 Solution of the first order</u> $(\alpha^{(1)})$

The solutions of eq.(6-67), (6-68) and (6-69) subset to the associates boundary conditions (6-70) are found to be at the form:

$$\psi_1 = \alpha(\mathrm{b}_4 + \frac{1}{96n_1^4(1+\lambda_1)}(\mathrm{k}+\mathrm{r})(3\mathrm{a}_2^3(1+n_1)^3(\mathrm{k}+\mathrm{r})^{-3n_1}$$
$$\alpha_1 - 3\mathrm{a}_1^3(-1+\mathrm{n}_1)^3(\mathrm{k}+\mathrm{r})^{3n_1}\alpha_1 + 48n_1^4(\mathrm{k}+\mathrm{r})(1+\lambda_1)\,\mathrm{b}_2 +$$

$$\frac{1}{\begin{array}{l}(1+\mathrm{n}_1).2(\mathrm{k}+\mathrm{r})^{n_1}(-3\mathrm{a}_1^2 a_2(3-10n_1^2+7n_1^4)\alpha_1+2a_1n_1(c_1+3c_1n_1^2\\ 2c_2n_1(-1+5\mathrm{n}_1{}^2))(1+\lambda_1)-16n_1^4(1+\lambda_1)b_3)+6a_1n_1(-1+\mathrm{n}_1{}^2)(3\mathrm{a}_1a_2\\ (-1+\mathrm{n}_1{}^2)\alpha_1-2c_1n_1(1+\lambda_1)+4c_2n_1^2(1+\lambda_1))\log(k+r)+12a_1n_1{}^2\\ (-1+\mathrm{n}_1{}^2)(-a_1a_2(-1+\mathrm{n}_1{}^2)\alpha_1+c_1n_1(1+\lambda_1))\log(k+r)^2+8a_1{}^2a_2n_1^3\\ (-1+\mathrm{n}_1{}^2)\alpha_1\log(k+r)^3)\end{array}}$$

Where $(b_i, \mathrm{i}=1,2,3,4), (\mathrm{c}_j, j=3,4), (\mathrm{c}_k, k=7,8)$ are constants can be obtained by using "MATHEMATICA" software.

6-9-3 Solution of heat transfer coefficient z(x)

The solution of eq.(6-51) can be found by the form:

$$\begin{array}{l} z(x)=\dfrac{-1}{96(k+r)}\pi\phi(192\mathrm{c}_1+\dfrac{96a_2^2(1+n_1)^2(k+r)^{-2n_1}\alpha_1}{n_1(1+\lambda_1)}-\dfrac{96a_1^2(-1+n_1)^2(k+r)^{2n_1}\alpha_1}{n_1(1+\lambda_1)}\\ +\dfrac{384a_1a_2(-1+n_1)^2\alpha_1 Log(k+r)}{(1+\lambda_1)}+\alpha(192\mathrm{c}_3-\dfrac{3a_2^4(1+n_1)^4(-1+5n_1{}^2)(k+r)^{-4n_1}\alpha_1{}^2}{n_1{}^5(1+\lambda_1)^2}+\\ \dfrac{3a_1^4(-1+n_1)^4(-1+5n_1{}^2)(k+r)^{4n_1}\alpha_1{}^2}{n_1{}^5(1+\lambda_1)^2}+\dfrac{96a_1a_2c_1(-1+n_1{}^4)\alpha_1 Log(k+r)^2}{n_1{}^2(1+\lambda_1)}-\\ +\dfrac{1}{\begin{array}{l}(-1+\mathrm{n}_1).2(\mathrm{k}+\mathrm{r})^{-n_1}(3\mathrm{a}_2^2a_1(3-10n_1^2+7n_1^4)\alpha_1+2a_2n_1(c_1+3c_1n_1^2\\ 2c_2n_1(1-5\mathrm{n}_1{}^2))(1+\lambda_1)-16n_1^4(1+\lambda_1)b_1)+6a_2n_1(-1+\mathrm{n}_1{}^2)(3\mathrm{a}_1a_2\\ (-1+\mathrm{n}_1{}^2)\alpha_1+2c_1n_1(1+\lambda_1)+4c_2n_1^2(1+\lambda_1))\log(k+r)+12a_1n_1{}^2\\ (-1+\mathrm{n}_1{}^2)(a_1a_2(-1+\mathrm{n}_1{}^2)\alpha_1+c_1n_1(1+\lambda_1))\log(k+r)^2+8a_2{}^2a_1n_1^3\\ (-1+\mathrm{n}_1{}^2)\alpha_1\log(k+r)^3)\end{array}}\end{array}$$

$$.......(6-77)$$

$$\theta_1=\alpha(c_4+\frac{1}{192}((\frac{3a_2^4(1+n_1)^4(-1+5n_1^{\ 2})(k+r)^{-4n_1}\alpha_1^{\ 2}}{4n_1^6(1+\lambda_1)^2}+$$

$$\frac{3a_1^4(-1+n_1)^4(-1+5n_1^{\ 2})(k+r)^{4n_1}\alpha_1^{\ 2}}{4n_1^6(1+\lambda_1)^2}+192c_3\log(k+r)-$$

$$\frac{(24(-1+n_1^{\ 2})\alpha_1(a_1^2a_2^{\ 2}(3-8n_1^{\ 2}+5n_1^{\ 4})\alpha_1-8a_2b_3n_1^4(1+\lambda_1)-4a_1n_1^2(2b_1n_1^2+a_2(c_2-3c_2n_1^2))(1+\lambda_1)\log(k+r)^2}{n_1^4(1+\lambda_1)^2}$$

$$+\frac{16a_1^2a_2^{\ 2}(-1+n_1^{\ 2})^2(1+n_1^{\ 2})\alpha_1^{\ 2}\log(k+r)^4}{n_1^2(1+\lambda_1)^2}-\ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots.(6-78)$$

$$\varphi=\alpha(c_8+\frac{1}{768n_1^6(1+\lambda_1)^2}(k+r)^{-4n_1}(-3\alpha_1\alpha_2\alpha_3(a_2^4(1+n_1)^4(-1+5n_1^{\ 2})\alpha_1$$

$$+12a_1a_2^3(1+n_1)^3\ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots..(6-79)$$

$$\frac{(48(-1+n_1)^2\alpha_1(a^2{}_1a^2{}_2(3-8n_1^{\ 2}+5n_1^{\ 4})\alpha_1-8a_2b_3n_1^{\ 4}(1+\lambda_1)-4a_1n_1^{\ 2}(2b_1n_1^{\ 2}+a_2(c_2-3c_2n_1^{\ 2}))(1+\lambda_1))\log(k+r))}{n_1^{\ 5}(1+\lambda_1)^2}$$

$$+\frac{64a_1^2a_2^2(1+n_1^{\ 2})(-1+n_1^{\ 2})^2\alpha_1^{\ 2}\log(k+r)^3}{n_1^{\ 2}(1+\lambda_1)^2} \qquad \ldots..(6-80)$$

6-10 Results and Discussion

In this section, the numerical and computation results are discussed for the problem of an incompressible viscous non-Newtonian Jeffrey fluid with variable viscosity of temperature in the curved channel with the effects of radial magnetic field and heat/ mass transfer through the graphical illustrations of some important results. (MATHEMATICA) program is used to find out numerical and illustrations.

6-10-1 Velocity distribution

Influence of different parameters on the velocity distribution have been illustrated in figures (6-2)-(6-8). These figures are scratched at the fixed value of (x=0.2, t=0.05). from figure (6-2)(a) displays the effect of Hartmann number parameter (M) on velocity u, it is noticed that the velocity increase at upper wall on region of $r\in[0.5,1]$ and decrease at lower wall on region of $r\in[-1,0]$ Figure (6-3)(a), illustrates the effect of the parameter ϕ on velocity, we see that velocity u increase on upper and lower walls of channel with an increase of ϕ. From figure (6-4)(a), it observed that there is similar behavior of Jeffrey parameter λ_1 of parameter M on velocity u. Figure (6-5)(a), show that velocity distribution increase at upper part of channel and decrease at lower part of channel with an increase of cultivator parameter k. Figure (6-6)(a) illustrates that velocity distribution u enhances at the central region and walls of the channel with an increase of Q. From figures (6-2)(b),(6-3)(b),(6-4)(b),(6-5)(b) and (6-6)(b) of effects of M, ϕ, λ_1, k and Q respectively, observed that the velocity profiles are not symmetric in curved channel (for small values of cultivator parameter k) and it is symmetric in the straight channel (for large values of k). the effect of Brinkman number (Br) on velocity is seen in figure (6-7)(a),(b), it is noticed that velocity u increase at core and upper wall of channel but the fluid is reflected will be reduced at the point (0.4813,0.8085) with an increase of (Br), and it's graph model parameter (α) is illustrated and displayed in figure (6-8)(a),(b), it is showed that it's behavior is similar to the manner of (Br) on velocity but it's graph can be seen in the symmetric profile in the straight channel.

<u>6-10-2 Trapping phenomenon</u>

The effects of various parameters like $M, \phi, \lambda_1, k, Q, \alpha_1$ and α on trapping can be see through figures (6-9)-(6-15). Figure (6-9) show that the numbered and size of trapping bolus decrease with an increase of value of M in the upper and lower part of channel. The figures (6-10),(6-11),(6-12),(6-13) illustrates the effects of ϕ, Q, α_1 and α on circulating bolus, it is seen that there is rise up in number and size of bolus with an increase of these parameters. Opposite behavior is showed for the effects of λ_1 and k and their influence are displayed in figures (6-14) and (6-15) respectively.

<u>6-10-3 Temperature characteristics</u>

The expressions for temperature are given by eq.(6-72) and (6-78) for zeros and first order solution. The effects of various parameters on temperature for fixed values of (x=0.2, t=0.05) are shown, the results are presented in fig (6-16)-(6-21). From figure (6-16)(a),(b), it can found that temperature profile decrease at the center line and walls but the fluid will conversed it's flow at the upper wall of channel which make it's temperature may be increase with an increase of

Hartmann parameter (M). The effects of parameter ϕ on temperature distribution is plotted in fig.(6-17)(a),(b), it is noticed that temperature increase at the central line and the walls of channel. Figure (6-18) (a),(b), showed the influence of parameter λ_1, it is observed that an increase in λ_1 leads to decrease in temperature profile at the central line of channel, that is temperature profile θ is smaller for non-Newtonian fluid ($\lambda_1 \neq 0$) when compared with viscous fluid ($\lambda_1 = 0$). The effect of parameter k is noticed in figure (6-19)(a),(b), it is observed that an increase in k leads to decrease in temperature profile at the central line of channel with an increase of k. Figure (6-20)(a),(b), illustrate the influence of Brinkman number (Br) on temperature which is showed that an increase in (Br) results rise up on temperature, it is due to the fact that (Br) incorporates viscous dissipation effects which extends the fluid temperature. Figure (6-21)(a),(b), displayed the influence of temperature (Q) on temperature, which is noticed that the temperature increasing with an increase of Q at the central line of channel. In all graphs of temperature distribution of effects of all parameters mentioned above that the profiles of temperature are not symmetric in curved channel and it is symmetric in straight channel.

6-10-4 Mass transfer distribution

The expression for concentration are given by eq.(6-73) and (6-79) for the zeros and first order solution. The effects of various parameters on concentration for fixed values of (x=0.2, t=0.05) are shown, the results are presented in fig (6-22)-(6-29). The profile of concentration is reverse of profile of temperature and the parameters behaved opposite manner on concentration than a temperature distribution. The effects of parameters $M, \phi, \lambda_1, k, Br, Sr, Sc$ and Q on concentration are considered. The impact of M, λ_1, k are plotted in figures(6-22)(a),(b)-(6-23)(a),(b),(6-24). It is noticed that an increase in these last parameters lead to an increase on magnitude of concentration. Opposite behavior is obtained for the parameters φ, Br, Sr, Sc and Q which is illustrated into figures (6-25)(a),(b)-(6-29)(a),(b). In fact the reason behind the reducing of concentration when we increase the values of (Sc.) is due that the mass diffusion decrease which show decrease in concentration. We observed that all graphs of concentration distribution are not symmetric in curved channel and it has symmetry characteristic in straight channel. It is warth mentioning that the negative values of concentration profiles for some values of parameters agree with natural process when nutrients diffuse out of blood to the neighboring tissues.

6-10-5 Heat transfer coefficient:

Figs. (6-30)-(6-35) are drawn to examine the impact of Hartmann number (M), inclusion parameter (ϕ), Jeffrey parameter (λ_1), curvature parameter(k), Brinkman number (Br) and flow rate volume parameter (Q) on heat transfer coefficient (Z). it is observed that due to contraction and expansion of peristaltic channel walls. The behavior of heat transfer coefficient Z is oscillatory. Figure (6-30) shows that absolute value of Z increase with an increase in M. However greater impact is noticed near $0 < x < 0.4$ and $-0.4 < x < 0$. Similar behavior is noticed for the impact of ϕ, Br and Q which is displayed in figures (6-31), (6-32) and (6-33) respectively. The effect of λ_1 and k on heat transfer coefficient Z is illustrated in figures (6-34) and (6-35) respectively, it is observed that the decreasing response of absolute heat transfer coefficient Z with an increase of above parameters. More clear results are noticed in the range where $0 < x < 0.4$ and $-0.4 < x < 0$.

6-10-6 pressure gradient distribution

Effects of various parameters on the pressure gradient versus x have been illustrated in figures (6-36)-(6-41). These figures are scratched at the fixed values of (r=0.2,t=0.05). from figure (6-36) displays the effect of parameter (M) on pressure gradient, it is noticed that an increase in M leads to reduce in pressure gradient. Figure (6-37) illustrates the effect of the parameter ϕ on pressure gradient, it is observed that pressure gradient increase on the center of channel at the region of $\left(-0.2 < x < 0.2\right)$ and reduce at the edges of walls at the regions of $\left(-0.4 < x < -0.2\right)$ and $\left(0.2 < x < 0.4\right)$. The impact of parameters λ_1 and (k) are plotted in figures (6-38) and (6-39) respectively which is noticed that an increase in these parameters lead to rise in pressure gradient. Similar behavior is obtained for the effects of parameter (Br) and Reynolds model (α) and Brinkman number. Their behavior is plotted in figure (6-40) and (6-41).

6-11 Concluding Remarks

The present study deals with the combined effects of radial magnetic field and heat/mass transfer on the peristaltic transport of viscous incompressible Jeffrey fluid with variable viscosity dependent on temperature in curved channel. We obtained the analytical solution of the problem under long wave length and low Reynolds number and using perturbation method for Reynolds parameter of viscosity. The results are analyzed for different values of parameters namely Hartmann number (M), amplitude ratio ϕ, Jeffrey parameter λ_1, culvature parameter (k), Reynolds perturbation parameter α, volume flow rate (Q), Brinkman number(Br). Soret number (Sr), Schmidt number (Sc). Thus through our work we observe the following notations:

1. The influence of Hartmann number (M), Jeffrey parameter (λ_1),calvature parameter (k), Reynolds parameter of viscosity (α), Brinkman number (Br) on axial velocity is oscillator.
2. The axial velocity is rise up and enhance with an increase of ϕ, Q
3. The profiles of axial velocity are parabolic and symmetric for large values of culvature parameter (k) (straight channel) and non-symmetric in the curved channel.
4. The size and number of trapping bolus increase with an increase of $\phi, \mathrm{Q}, \mathrm{Br}$ and α and they are decrease with an increase of λ_1, M and k.
5. The effect of inclusion parameter or amplitudes ratio ϕ on pressure gradient is vacillating.
6. The pressure gradient increase with an increase of $\lambda_1, \mathrm{k}, \mathrm{Br}$ and has opposite manner with an increase of M .
7. The temperature distribution increase with an increase of ϕ, Br and Q and decrease with an increase of M, λ_1 *and* k .
8. The concentration distribution decrease with an increase of Sr and Sc. Opposite behavior for concentration distribution is noted when compared with temperature.
9. The profiles of temperature and concentration are symmetric and parabolic for large values of k (straight channel) and non-symmetric for curved channel for small values of k.
10. The action of heat transfer coefficient is wobbling, that is on region of $0 < x < 0.4$ we see that z(x) is increasing function of M, Br, Q, ϕ.

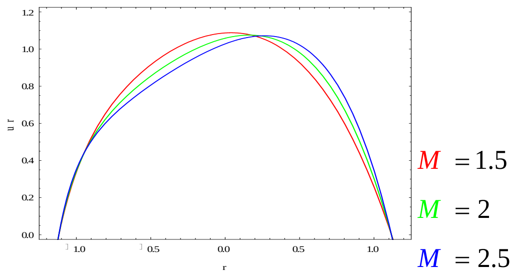


Fig.(6-2-a) Effect of (M) on velocity u

$\phi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 2), (curved channel)

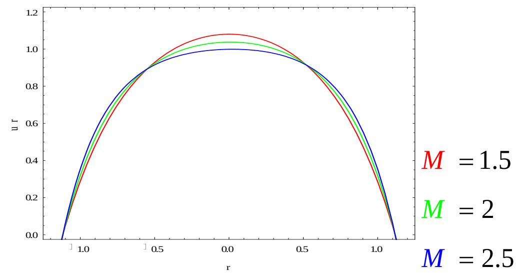


Fig.(6-2-b) Effect of (M) on velocity u

$\phi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 50), (straight channel)

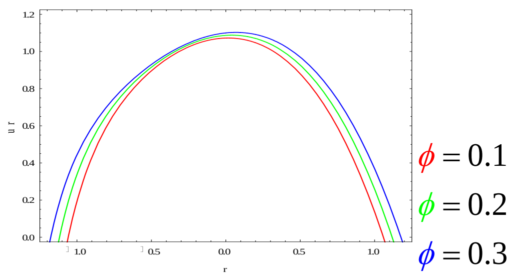


Fig.(6-3-a) Effect of (ϕ) on velocity u

$t = 0.05, \lambda_1 = 1, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 2), (curved channel)

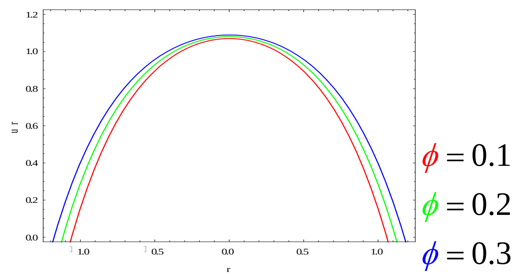


Fig.(6-3-b) Effect of (ϕ) on velocity u

$t = 0.05, \lambda_1 = 1, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 50), (straight channel)

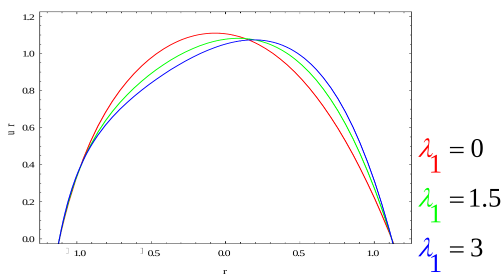


Fig.(6-4-a) Effect of (λ_1) on velocity u

$t = 0.05, \phi = 0.2, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 2), (curved channel)

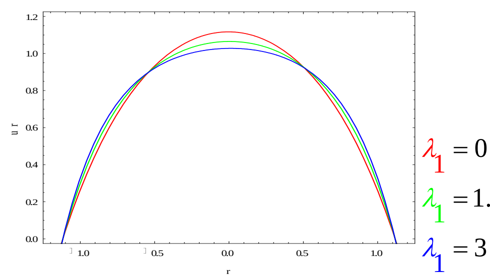


Fig.(6-4-b) Effect of (λ_1) on velocity u

$t = 0.05, \phi = 0.2, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 50), (Straight channel)

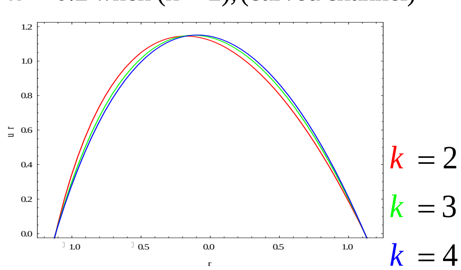


Fig.(6-5-a) Effect of (k) on velocity u

$t = 0.05, \phi = 0.2, M = 0.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01,$

$Q = 1.5 x = 0.2$ (curved channel)

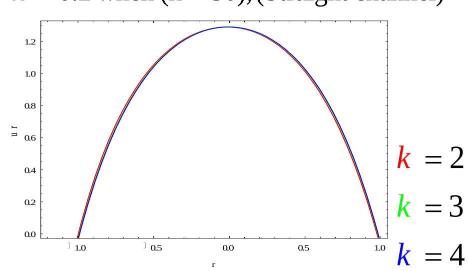


Fig.(6-5-b) Effect of (k) on velocity u

$t = 0.05, \phi = 0.2, M = 0.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01,$

$Q = 1.5, x = 0.2$ (straight channel)

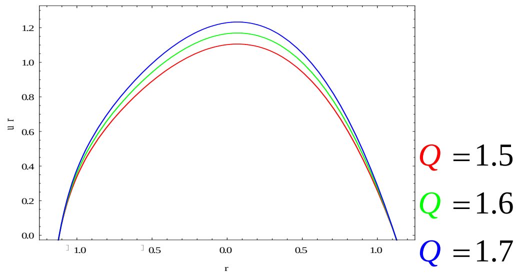


Fig.(6-6-a) Effect of (Q) on velocity u

$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01,$
$x = 0.2, (k = 1.5)$ (curved channel)

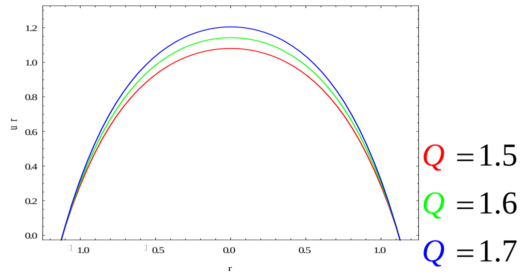


Fig.(6-6-b) Effect of (Q) on velocity u

$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 1.5, \alpha = 0.1, \alpha_1 = 0.01,$
$x = 0.2, (k = 50)$ (straight channel)

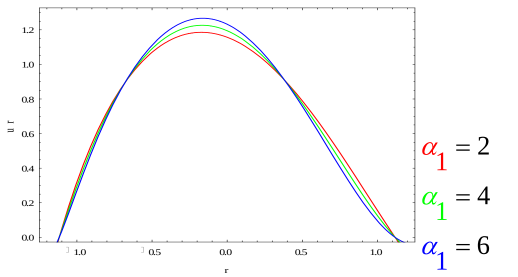


Fig.(6-7-a) Effect of (α_1) on velocity u

$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 0.5, \alpha = 0.6, Q = 1.5$
$x = 0.2, (k = 2)$ (curved channel)

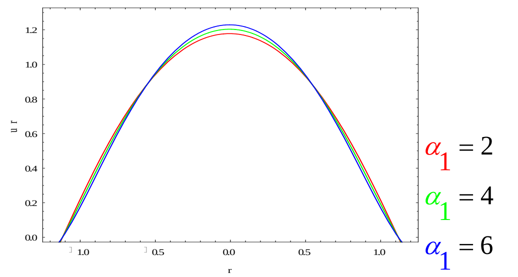


Fig.(6-7-b) Effect of (α_1) on velocity u

$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 0.5, \alpha = 0.6, Q = 1.5$
$x = 0.2, (k = 50)$ straight channel)

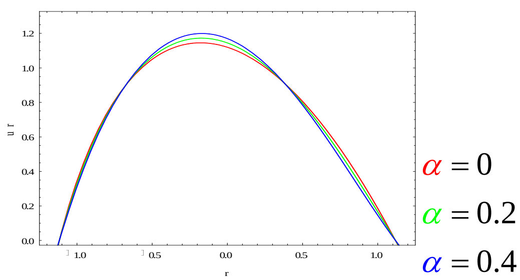


Fig.(6-8-a) Effect of (α) on velocity u

$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 0.5,, \alpha_1 = 4, Q = 1.5$
$x = 0.2, (k = 2)$ (curved channel)

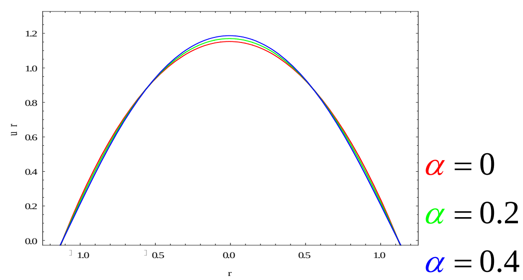
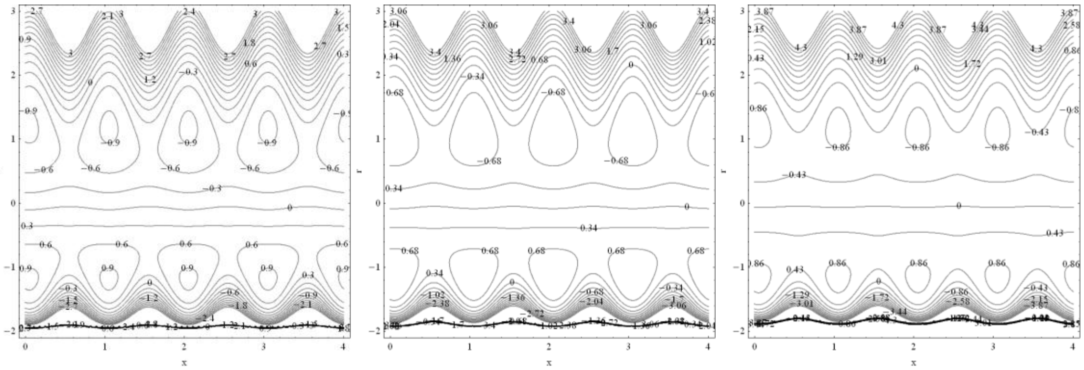


Fig.(6-8-b) Effect of (α) on velocity u

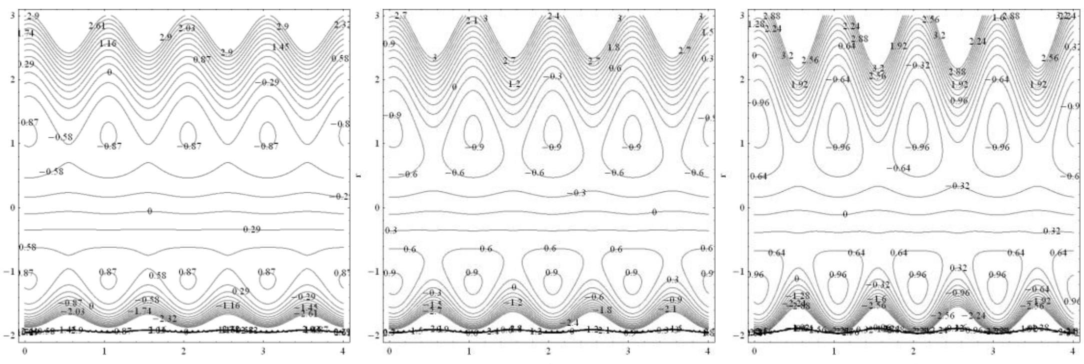
$t = 0.05, \phi = 0.2, \lambda_1 = 1, M = 0.5,, \alpha_1 = 4, Q = 1.5$
$x = 0.2, (k = 50)$ (Straight channel)



Fig(6-9) Stream lines for

$t = 0.05, \phi = 0.2, \alpha = 0.1, \alpha_1 = 0.01, \lambda_1 = 1, k = 2, Q = 1.5$

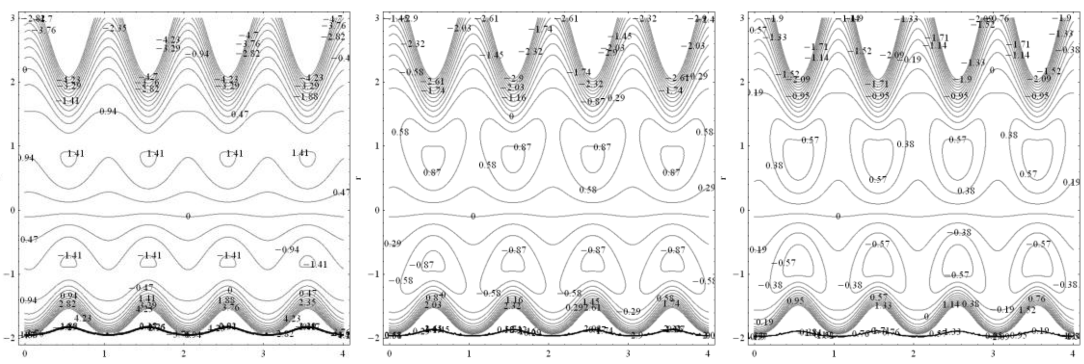
(a)M = 0.5, (b) M = 0.7, (c) M = 0.9



Fig(6-10) Stream lines for

$t = 0.05, M = 0.5, \alpha = 0.1, \alpha_1 = 0.01, \lambda_1 = 1, k = 2, Q = 1.5$

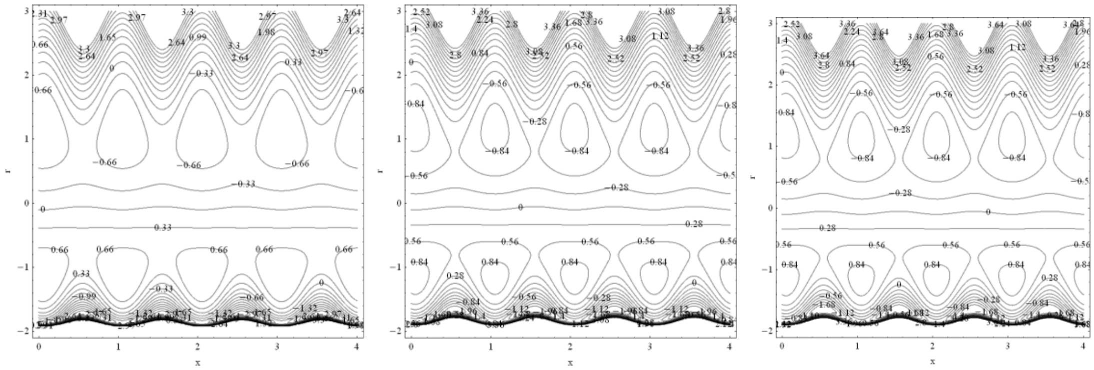
(a)$\phi = 0.15$ (b)$\phi = 0.2$, (c) $\phi = 0.3$



Fig(6-11) Stream lines for

$t = 0.05, M = 0.5, \alpha = 0.1, \alpha_1 = 0.01, \lambda_1 = 1, k = 2, Q = 1.5$

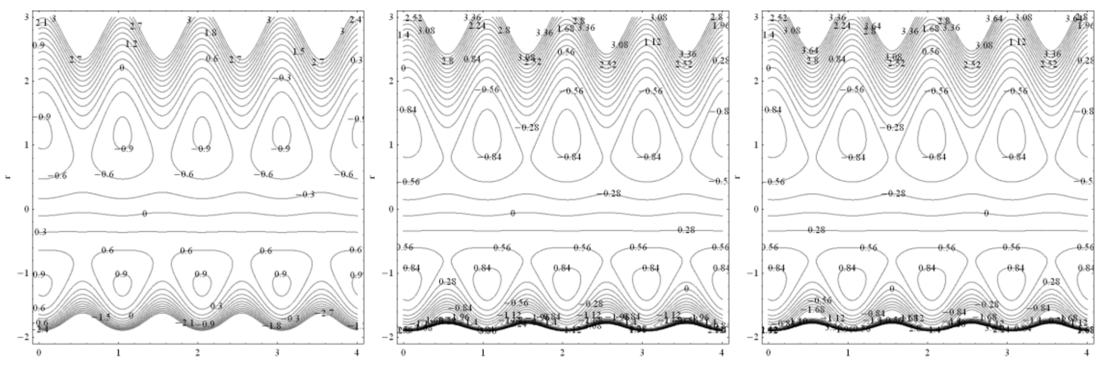
(a)$Q = -2.5$ (b)$Q = -1.5$, (c)$Q = -1$



Fig(6-12) Stream lines for

$t = 0.05, M = 0.5, \phi = 0.2, \alpha = 0.1, \lambda_1 = 1, k = 2, Q = 1.5$

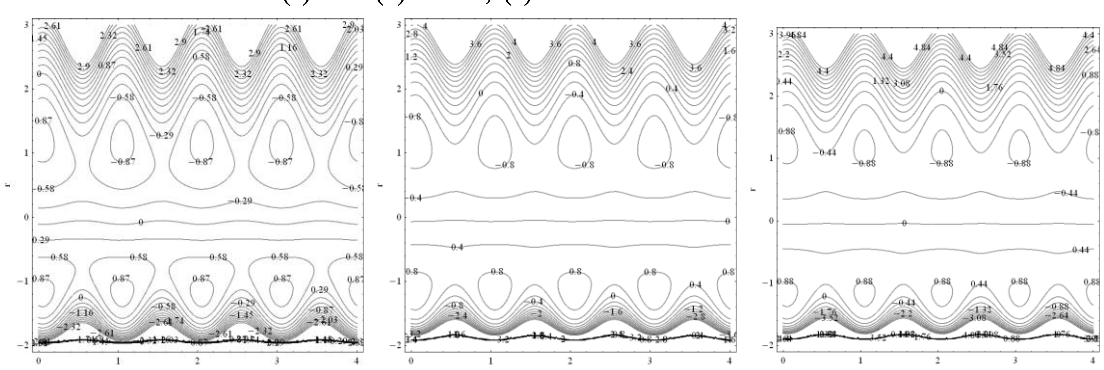
(a)$\alpha_1 = 0.1$ (b)$\alpha_1 = 0.2$, (c) $\alpha_1 = 0.22$



Fig(6-13) Stream lines for

$t = 0.05, M = 0.5, \phi = 0.2, \alpha_1 = 0.1, \lambda_1 = 1, k = 2, Q = 1.5$

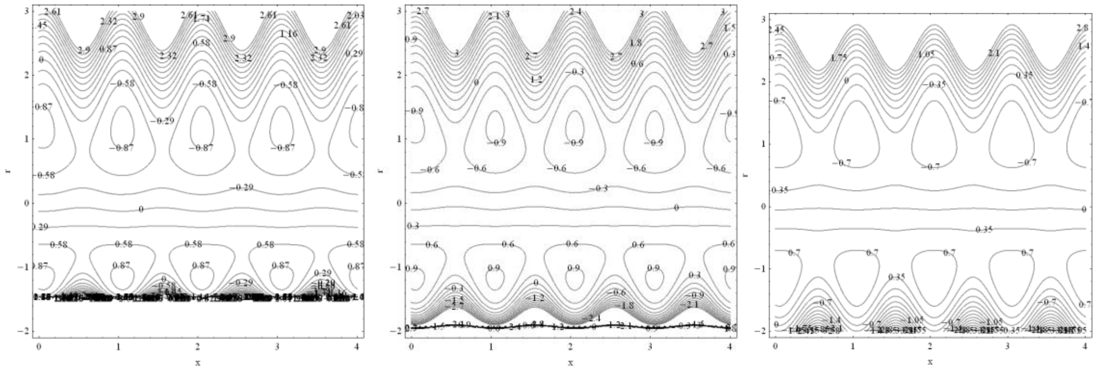
(a)$\alpha = 0$ (b)$\alpha = 0.2$, (c)$\alpha = 0.22$



Fig(6-14) Stream lines for

$t = 0.05, M = 0.5, \phi = 0.2, \alpha_1 = 0.01, \alpha = 0.1, k = 2, Q = 1.5$

(a)$\lambda_1 = 0$ (b)$\lambda_1 = 5$, (c)$\lambda_1 = 7$



Fig(6-15) Stream lines for

$t = 0.05, M = 0.5, \phi = 0.2, \alpha_1 = 0.01, \alpha = 0.1, Q = 1.5$

(a)$k = 1.5$, (b)$k = 2$, (c)$k = 4$,

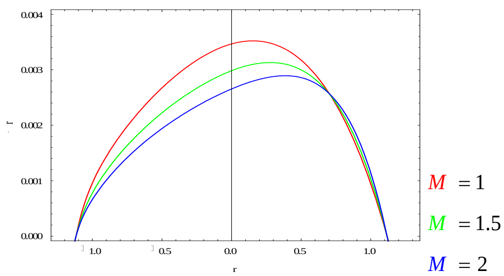


Fig.(6-16-a) Effect of (M) on temperature

$\phi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.001, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 1.5), (curved channel)

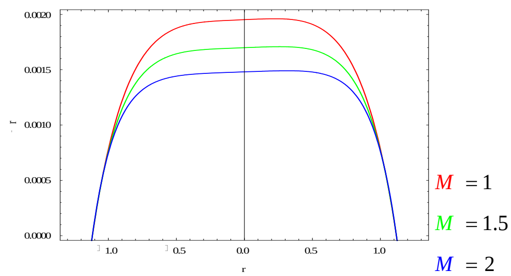


Fig.(6-16-b) Effect of (M) on temperature

$\phi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.001, \alpha_1 = 0.01, Q = 1.5$

$x = 0.2$ when (k = 50), (straight channel)

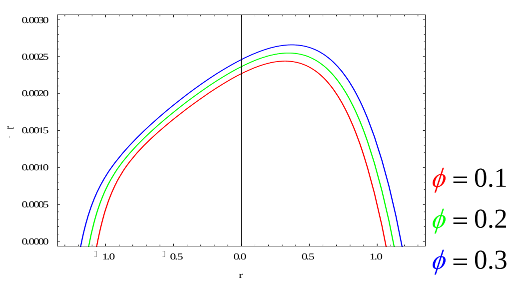


Fig.(6-17-a) Effect of (ϕ) on temperature

$M = 1.5, t = 0.05, \lambda_1 = 1, \alpha = 0.001, \alpha_1 = 0.01,$

$Q = 1.5, x = 0.2$ when (k = 1.5), (curved channel)

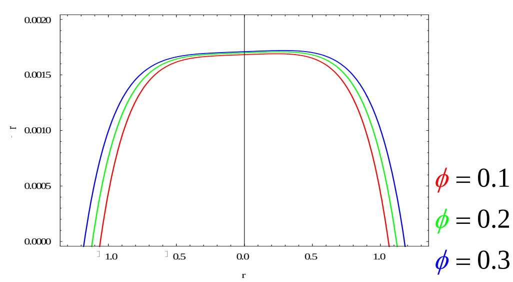


Fig.(6-17-b) Effect of (ϕ) on temperature

$M = 1.5, t = 0.05, \lambda_1 = 1, \alpha = 0.001, \alpha_1 = 0.01,$

$Q = 1.5, x = 0.2$ when (k = 50), (straight channel)

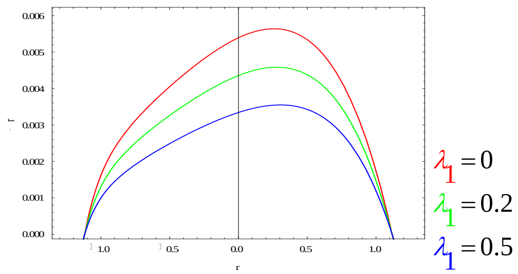


Fig.(6-18-a) Effect of (λ_1) on temperature
$M = 1.5, t = 0.05, \phi = 0.2, \alpha = 0.001, \alpha_1 = 0.01,$
$Q = 1.5, x = 0.2$ when (k = 2), (curved channel)

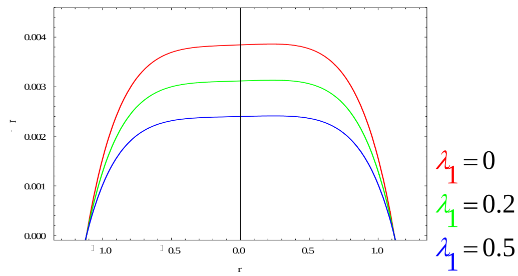


Fig.(6-18-b) Effect of (λ_1) on temperature
$M = 1.5, t = 0.05, \phi = 0.2, \alpha = 0.001, \alpha_1 = 0.01,$
$Q = 1.5, x = 0.2$ when (k = 50), (straight channel)

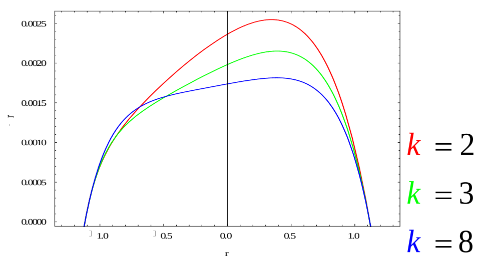


Fig.(6-19) Effect of (k) on temperature
$M = 1.5, t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha = 0.001,$
$\alpha_1 = 0.01, Q = 1.5, x = 0.2$, (curved channel)

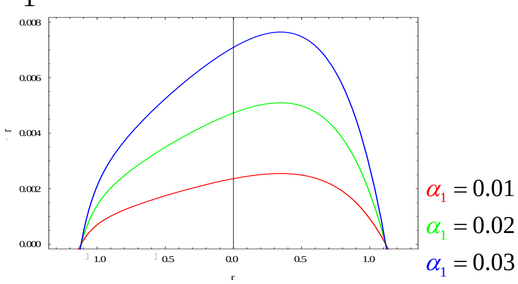


Fig.(6-20-a) Effect of (Br) on temperature
$M = 1.5, t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha = 0.001,$
$Q = 1.5, x = 0.2$ when (k = 2), (curved channel)

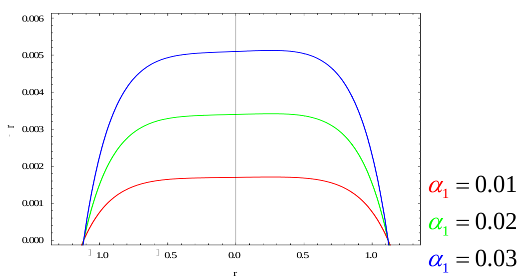


Fig.(6-20-b) Effect of (Br) on temperature
$M = 1.5, t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha = 0.001,$
$Q = 1.5, x = 0.2$ when (k = 50), (straight channel)

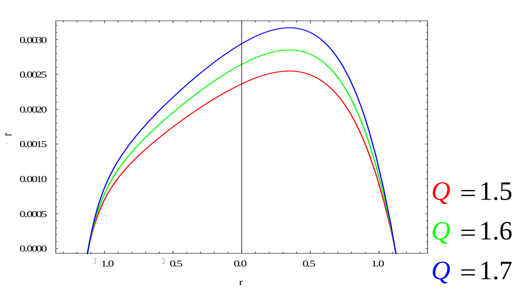


Fig.(6-21-a) Effect of (Q) on temperature
$M = 1.5, t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha = 0.001,$
$\alpha_1 = 0.01, x = 0.2$ when (k = 2), (curved channel)

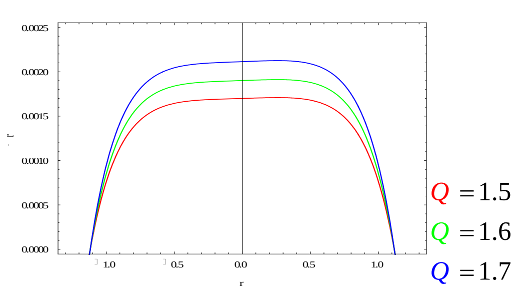


Fig.(6-21-b) Effect of (Q) on temperature
$M = 1.5, t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha = 0.001,$
$\alpha_1 = 0.01, x = 0.2$ when (k = 50), (straight channel)

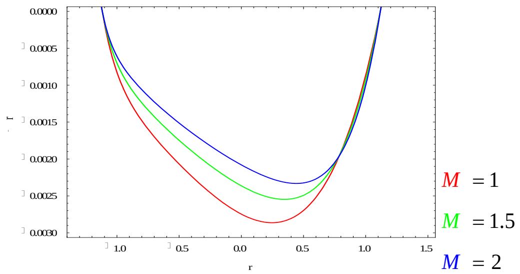


Fig.(6-22-a) Effect of (M) on mass transfer

$t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 2), (curved channel)

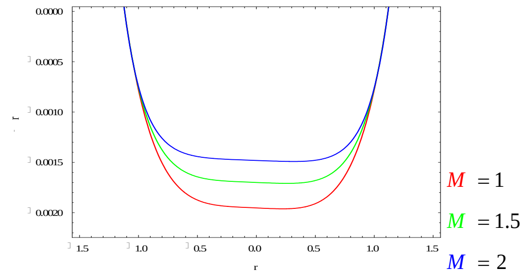


Fig.(6-22-b) Effect of (M) on mass transfer

$t = 0.05, \lambda_1 = 1, \phi = 0.2, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 50), (straight channel)

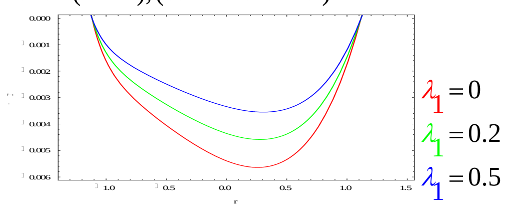


Fig.(6-23-a) Effect of (λ_1) on mass transfer

$t = 0.05, M = 1.5, \phi = 0.2, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 2), (curved channel)

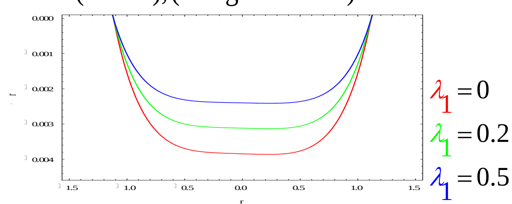


Fig.(6-23-b) Effect of (λ_1) on mass transfer

$t = 0.05, M = 1.5, \phi = 0.2, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 50), (straight channel)

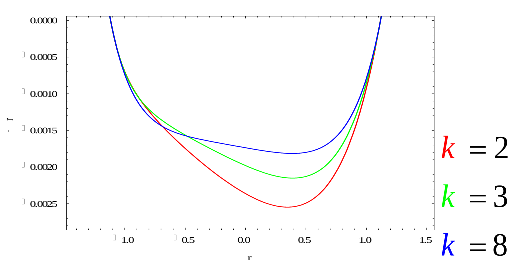


Fig.(6-24) Effect of (k) on mass transfer

$t = 0.05, M = 1.5, \phi = 0.2, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

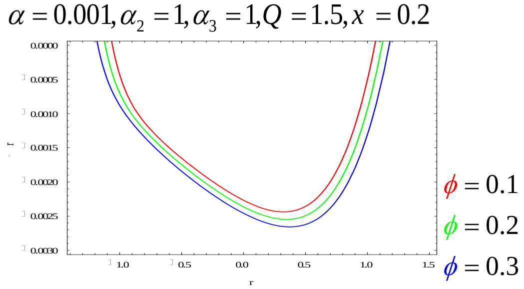


Fig.(6-25-a) Effect of (ϕ) on mass transfer

$t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 2), (curved channel)

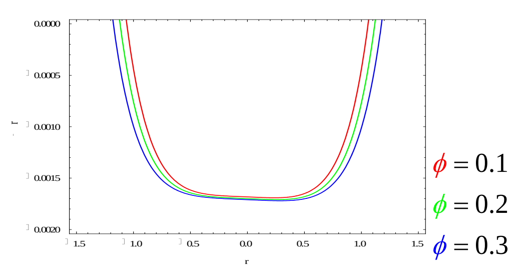


Fig.(6-25-b) Effect of (ϕ) on mass transfer

$t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

when (k = 50), (straight channel)

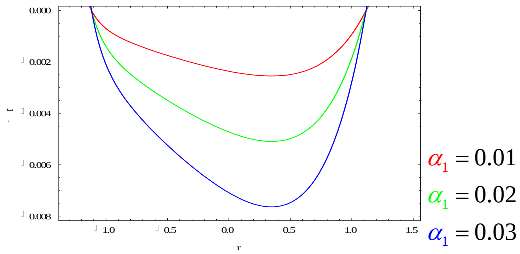


Fig.(6-26-a) Effect of (Br) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1,$
$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$
when (k = 2), (curved channel)

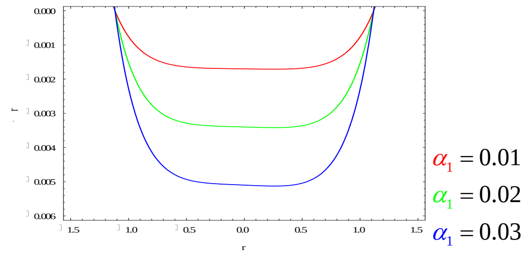


Fig.(6-26-b) Effect of (Br) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1,$
$\alpha = 0.001, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$
when (k = 50), (straight channel)

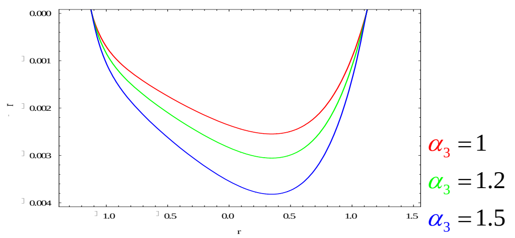


Fig.(6-27-a) Effect of (Sc) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$
$\alpha = 0.001, \alpha_2 = 1, Q = 1.5, x = 0.2$
when (k = 2), (curved channel)

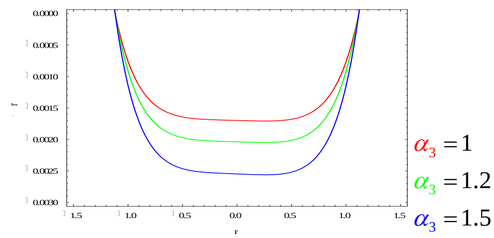


Fig.(6-27-b) Effect of (Sc) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$
$\alpha = 0.001, \alpha_2 = 1, Q = 1.5, x = 0.2$
when (k = 50), (straight channel)

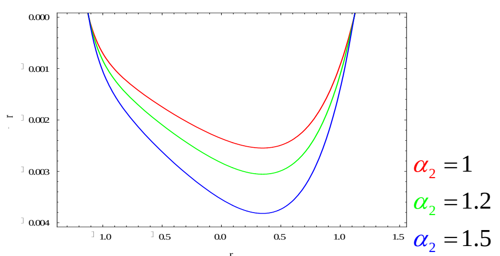


Fig.(6-28-a) Effect of (Sr) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$
$\alpha = 0.001, \alpha_3 = 1, Q = 1.5, x = 0.2$
when (k = 2), (curved channel)

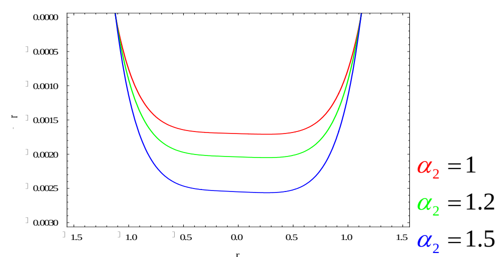


Fig.(6-28-a) Effect of (Sr) on mass transfer
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$
$\alpha = 0.001, \alpha_3 = 1, Q = 1.5, x = 0.2$
when (k = 50), (straight channel)

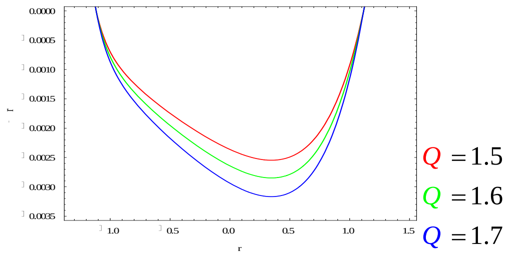


Fig.(6-29-a) Effect of (Q) on mass transfer

$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_3 = 1, \alpha_2 = 1, x = 0.2$

when (k = 2), (curved channel)

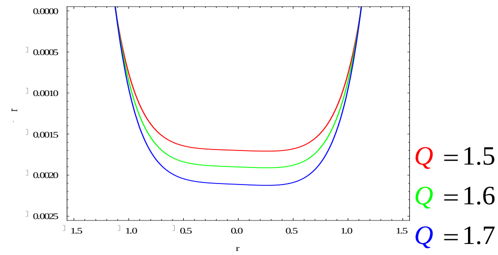


Fig.(6-29-b) Effect of (Q) on mass transfer

$\phi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01,$

$\alpha = 0.001, \alpha_3 = 1, \alpha_2 = 1, x = 0.2$

when (k = 50), (straight channel)

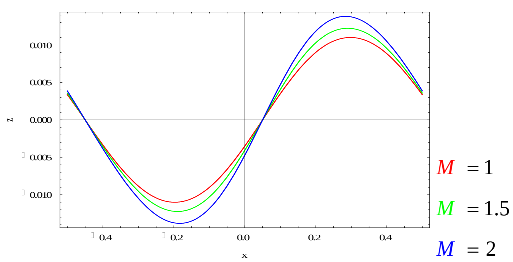


Fig.(6-30) Effect of (M) on heat transfer

$\phi = 0.2, t = 0.05, \lambda_1 = 1, k = 1.5, \alpha_1 = 0.01,$

$\alpha = 0.001, Q = 1.5$

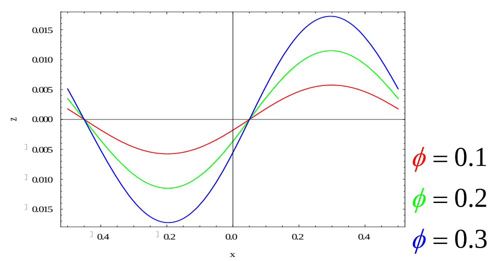


Fig.(6-31) Effect of (ϕ) on heat transfer

$M = 1.5, t = 0.05, \lambda_1 = 1, k = 2, \alpha_1 = 0.01,$

$\alpha = 0.001, Q = 1.5$

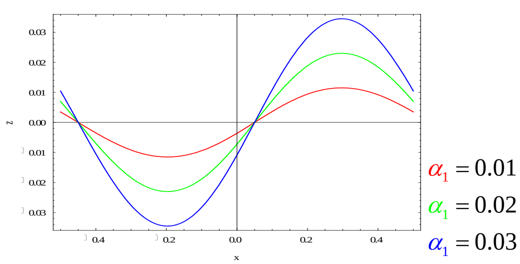


Fig.(6-32) Effect of (Br) on heat transfer

$M = 1.5, \phi = 0.2, t = 0.05, \lambda_1 = 1, k = 2,$

$\alpha = 0.001, Q = 1.5$

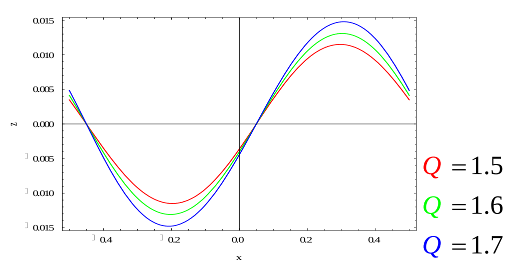


Fig.(6-33) Effect of (Q) on heat transfer

$M = 1.5, \phi = 0.2, t = 0.05, \lambda_1 = 1, k = 2$

$, \alpha = 0.001, \alpha_1 = 0.01$

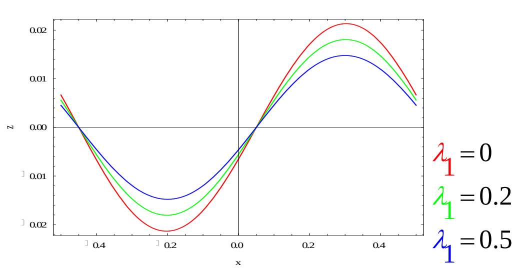


Fig.(6-34) Effect of (λ_1) on heat transfer

$M = 1.5, \phi = 0.2, t = 0.05, Q = 1.5, k = 2,$

$\alpha = 0.001, \alpha_1 = 0.01$

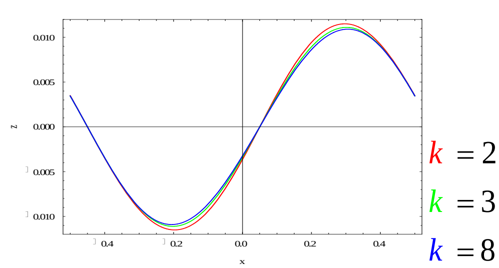


Fig.(6-35) Effect of (k) on heat transfer

$M = 1.5, \phi = 0.2, t = 0.05, Q = 1.5,$

$\alpha = 0.001, \alpha_1 = 0.01, \lambda_1 = 1$

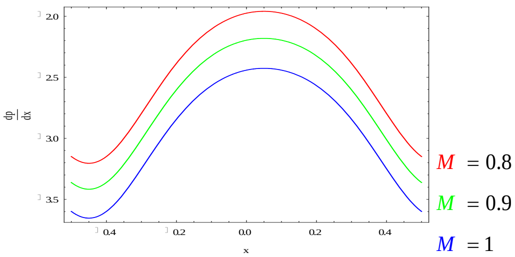


Fig.(6-36) Effect of (M) on pressure gradient.
$\phi = 0.2, t = 0.05, \lambda_1 = 1, k = 2, \alpha_1 = 0.01,$
$\alpha = 0.001, Q = 2, r = 0.1$

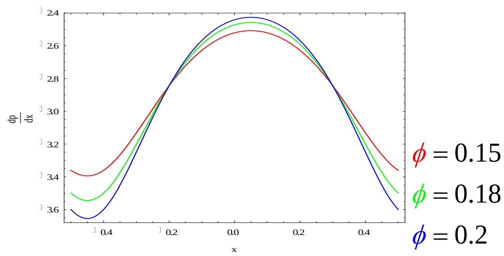


Fig.(6-37) Effect of (ϕ) on pressure gradient.
$t = 0.05, M = 1, \lambda_1 = 1, k = 2, \alpha_1 = 0.01,$
$\alpha = 0.001, Q = 2, r = 0.5$

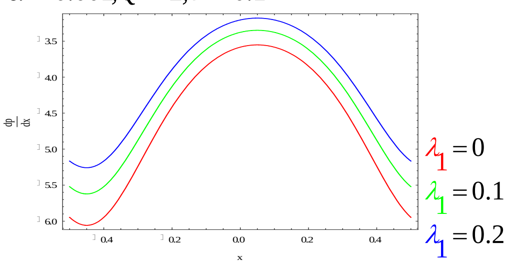


Fig.(6-38) Effect of (λ_1) on pressure gradient.
$\phi = 0.2, t = 0.05, M = 1, k = 2, \alpha_1 = 0.01,$
$\alpha = 0.001, Q = 2, r = 0.5$

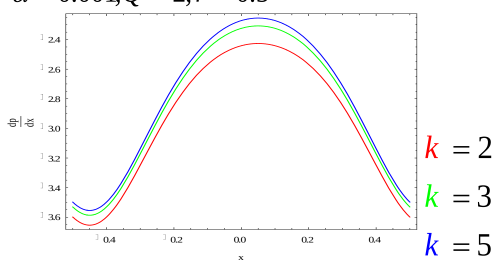


Fig.(6-39) Effect of (k) on pressure gradient.
$\phi = 0.2, t = 0.05, M = 1, \lambda_1 = 1, \alpha_1 = 0.01,$
$\alpha = 0.001, Q = 2, r = 0.5$

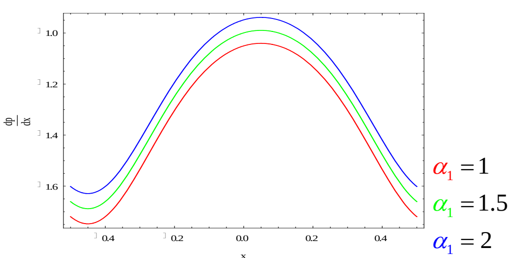


Fig.(6-40) Effect of (Br) on pressure gradient.
$\phi = 0.2, t = 0.05, M = 0.5, \lambda_1 = 1, k = 2,$
$\alpha = 0.6, Q = 1.5, r = 0.5$

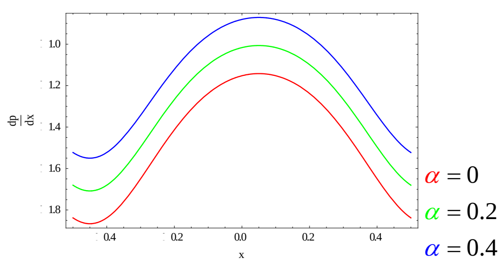


Fig.(6-41) Effect of (α) on pressure gradient.
$\phi = 0.2, t = 0.05, M = 0.5, \lambda_1 = 1, k = 2,$
$\alpha_1 = 4, Q = 1.5, r = 0.2$

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Recommendation for future work

In the light of the study and results obtained in this thesis, the following future work suggestions are given:

- A study of peristaltic transport of Walter's –B fluid under the effect of inclined magnetic field through a porous medium in an inclined tapered asymmetric channel by using the properties of the wall.
- A study of peristaltic transport of MHD flow of blood and heat\mass transfer in a tapered asymmetric channel through porous medium by using the effect of variable viscosity with temperature and properties of the wall.
- A study of peristaltic transport of Williamson fluid with variable viscosity of space under the effect of hall magnetic field in a tapered channel.
- A study of peristaltic transport of Jeffrey fluid under the effect of variable viscosity with space of radial direction.

المستخلص

في هذه الاطروحة ناقشنا التدفق التمعجي لموائع لزجة لا نيوتنية والتي تحمل اسم (وولتر-ب ، مائع دموي، ويليامسون، جيفري) تحت تأثير لزوجة ثابتة ومتغيرة، وشروط الانزلاق وعدم الانزلاق على سرعة المائع، الحقل الهيدرومغناطيسية، الجدران المرنة (القابلة للتمدد والتقلص)، الوسط المسامي، وانتقال الحرارة عبر قنوات مختلفة ثنائية البعد كالقنوات المستقيمة، المائلة، المستدقة، المنحنية.

الحلول بالنسبة للموديلات السابقة من الموائع قد أُعتبرت وحُللت تحت تأثير طول موجي طويل وعدد تقريبي صغير لراينولدز

معادلات الحركة، معادلات درجة الحرارة (الطاقة)، معادلة التركيزقد تم اشتقاقها. هذه المعادلات التفاضلية تم حلها تحليليا بطرق الاضطراب المنتظم. السمات البارزة لصفات التدفق، قوة الاحتكاك، دالة الجريان، جميعها حللت خلال دراسة تأثير الاعداد عديمة الابعاد التي تسيطر على المعادلات التي تحكم الجريان.

خمسة مسائل قد نوقشت خلال عملنا والتي يمكن ان تُتبع بما يلي:

المسألة الاولى، تتعلق بالتدفق التمعجي لمائع وولتر-ب الغير قابل للأنضغاط خلال وسط مسامي تحت تأثير الحقل المغناطيسي المنتظم في قناة مستدقة غير متناظرة ومستقيمة. لقد وجدنا بأن سرعة المائع تزداد بالقرب من مركز القناة تحت تأثير معامل الذاكرة القصير.

المسألة الثانية، تتعلق بالتدفق التمعجي لمائع دموي متغير اللزوجة خلال وسط مسامي في قناة مستدقة غير متناظرة ومستقيمة تحت تأثير الحقل المغناطيسي المنتظم وانتقال الحرارة. لقد وجدنا بأن معامل راينولدز من موديل اللزوجة له سلوك متزعزع على سرعة المائع ووجدنا حرارة المائع تتعزز تحت زيادة مُعامل مصدرالتشتت الحراري.

المسألة الثالثة، تتعلق بالتدفق التمعجي لمائع ويليامسون خلال وسط مسامي في قناة مستدقة غير متناظرة ومائلة تحت تاثير الحقل المغناطيسي المائل وانتقال الحرارة. لقد وجدنا ان سرعة المائع تزداد تحت تأثير زاوية ميل الحقل المغناطيسي ، وكذلك وجدنا ان حرارة المائع تزداد تحت تأثيرعدد برينكمان.

المسألة الرابعة، تتعلق بالتدفق التمعجي لمائع جيفري خلال وسط مسامي في قناة مستدقة متناظرة ومائلة تحت تاثير الحقل المغناطيسي المائل وانتقال الحرارة وبمساعدة خواص الجدار. لقد وُجد ان سرعة المائع وحرارته تزداد تحت تأثير خواص الجدار وزاوية ميل القناة.

المسألة الأخيرة، تتعلق بالتدفق التمعجي لمائع جيفري متغير اللزوجة بالنسبة للحرارة في قناة منحنية ومتناظرة تحت تاثير الحقل المغناطيسي الشعاعي وانتقال الحرارة والكتلة. لقد وُجد بأن معامل الاحداثي المنحني له سلوك متذبذب على سرعة المائع كذلك وُجدت ان منحنيات السرعة غير متناظرة في حالة القناة المنحنية وتكون متناظرة في حالة القناة المستقيمة ذات القيم الكبرى من المعامل المنحني. ووُجد ان تركيز المائع يقل عند زيادة عدد سيكماتد.

من الجدير بالذكر ان الحقل المغناطيسي والوسط المسامي يسبب اعاقة للجريان للأنواع السابقة من الموائع في كل المسائل اعلاه.

هذه الدراسة قد تمت بأستخدام برنامج ماثماتيكا لرسم المخططات والحصول على النتائج العددية.

جمهورية العراق

وزارة التعليم العالي والبحث العلمي

جامعة بغداد/ كلية التربية للعلوم الصرفة /ابن الهيثم

قسم الرياضيات

تأثير معلمات مختلفة على النقل التمعجي لبعض انواع من الموائع في قنوات مستدقة ومنحنية

أطروحة

مقدمة إلى مجلس كلية التربية للعلوم الصرفة /ابن الهيثم / جامعة بغداد

وهي جزء من متطلبات نيل شهادة الدكتوراه في علوم فلسفة الرياضيات

من قبل

تمارا شهاب احمد

بإشراف

أ. د. أحمد مولود عبد الهادي

1439 ه   **2018 م**