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Determining Effect of Rainwater in Contaminated Soil in Baghdad City Using Mathematical Methods

A Thesis

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بِسْمِ اللَّـهِ الرَّحْمَـٰنِ الرَّحِيمِ

{ وَعِندَهُ مَفَاتِحُ الْغَيْبِ لاَ يَعْلَمُهَا إِلاَّ هُوَ وَيَعْلَمُ مَا فِي الْبَرِّ وَالْبَحْرِ وَمَا تَسْقُطُ مِن وَرَقَةٍ إِلاَّ يَعْلَمُهَا وَلاَ حَبَّةٍ فِي ظُلُمَاتِ الأَرْضِ وَلاَ رَطْبٍ وَلاَ يَابِسٍ إِلاَّ فِي كِتَابٍ مُّبِينٍ }

صدق اللَّهُ العظيم

سورة الأنعام(59)

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Researcher

List of Symbols

Symbol	Definition
XRF	X-Ray Fluorescence
ADM	Adomian decomposition method
PDE	Partial differential equation
PDEs	Partial differential equations
C	Solute concentration ($\frac{mg}{L}$)
V_x	Darcy's flux ($\frac{cm}{hr}$)
D_L	Hydrodynamic dispersion coefficient ($\frac{\text{cm}^2}{\text{hr}}$)
C_a	Concentration of adsorbed chemical
f	Irreversible reaction decay rate
C_0	Initial concentration
$\frac{dc}{dx} = C_x$	Concentration for depth x ($\frac{\text{mg}}{\text{L}}$)
V	The average pore – water velocity, ($\frac{\text{cm}}{\text{hr}}$)
k(A)	condition number

Author's Publications

Journal Papers

[1] Tawfiq, L. N. M., Hasan, M. A., (2017), "Mathematical Model for Estimation the Effect of Rainwaters in Contaminated Soil and its Application in Baghdad", Journal of Education College, Vol.1, pp:315-320.

[2] Tawfiq, L. N. M., Hasan, M. A., (2017), "Estimation the Effect of Rain waters in Contaminated Soil by Using Mathematical Method and its Application in Baghdad", Iraqi Journal of Agriculture, Accepted.

[3] Tawfiq, L. N. M., Hasan, M. A., (2017), "Estimate the Effect of Rainwaters in Contaminated Soil by Using Simulink Technique", Journal of Physics: Conference Series (JPCS).

[4] Tawfiq, L. N. M., Hasan, M. A., (2017), "Design Simulink Model to Estimate the Dispersion Parameter with Application in Iraq", Global Journal of Engineering Science And Researches, Vol. 4, No. 10.

[5] Tawfiq, L. N. M., Hasan, M. A., (2017), "Solution Partial differential equations using Simulink", Australian Journal of Basic and Applied Sciences, Accepted.

Conference Papers

[1] Tawfiq, L. N. M., Hasan, M. A., (2017), "Mathematical Model for Estimation the Effect of Rainwaters in Contaminated Soil and its Application in Baghdad", 23th Scientific Conference, College of Education, University of Al-Mustansiryah in 26-27/4/2017.

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[2] Tawfiq, L. N. M., Hasan, M. A., (2017), "Estimation the Effect of Rain waters in Contaminated Soil by Using Mathematical Method and its Application in Baghdad", 10th Scientific Conference for Agricultural Research, Ministry of Agriculture, December 2017.

[3] Tawfiq, L. N. M., Hasan, M. A., (2017), "Estimate the Effect of Rainwaters in Contaminated Soil by Using Simulink Technique", First International Conference for Science, College of Education for pure science – Ibn Al Haitham \ University of Baghdad, in13-14/ 12/ 2017.

Abstract

The aim of this thesis is to design a Simulink model which estimates the effect of rainwaters in the contaminated soil by heavy metal based on estimate the concentration of Copper, Lead, Zinc, Cadmium, Cobalt, Nickel, Crum and Iron. So, this model would be a method to determine the contamination levels of these metals with the potential for this contamination sources with their impact and the effect of rainwater on it.

New approach to estimate dispersion parameter based on the Simulink environment is proposed.

The aim of using Simulink environment is to solve differential equations individually depending on the given data in parallel with analytical mathematics trends.

In general, mathematical models of the spread heavy metals in soil and the effect of rainwater are modeled and solved to predict the behavior of the system under different conditions.

The main code that utilized and presented is MATLAB/ode45 to enable solving model equation and experience the response of the engineering systems for different applied conditions.

Finally, we compare the performance of the suggested model with the traditional methods, by estimating the concentration of heavy metals before and after the rain, and then apply them in Baghdad soils after classifying it to different zones, such as: residential, industrial, commercial, agricultural and main roads. Then, compare the results of suggested methods with the results obtained by laboratory inspecting using X-Ray Fluorescence to determine the rate of accuracy. The results of this work show that the suggested model can be successfully applied to the rapid and accuracy estimation of concentration of heavy metals.

Contents

Introduction

Mathematical Models are simplified representations of some real world entity can be in equations or computer code are intended to mimic essential features while leaving out inessentials, that is, models describe our beliefs about how the world functions [7]. The nonlinear models of real-life problems are still difficult to solve either numerically or analytically.

Recently, there has been much attention devoted to the researches for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models, especially contamination model. Usually differential equations can be used for solving different applications of different fields. In general, students experience applied differential equations through the engineering applications such as Simulink environment.

Simulink model is a block diagram environment for multi task simulation and model based design. It supports system design, simulation, code generation, and continuous testing [17]. Simulink environment can be used to describe and solve different applications of various fields especially contamination model.

A contamination can be defined generally as the presence of any material in the environment in quantities that results directly or indirectly, alone or reacting with other materials, harmful effects on health of organisms. Soil contamination can be defined as the damage and change that affects soil specifications and the nature, chemical and vital properties such that it will effect negatively in a direct or indirect way at the organisms living on its surface: humans, animals, and plants [21].

Several researchers have already investigated the mobility of heavy metal in the soil amended with sewage sludge and concluded that only relatively small amount of metal were available for transport in the soil water immediately after sludge application [16]. Giordano and Mortvedt [11] show that under excessive leaching condition, movement of heavy metal in soil is somewhat greater from inorganic than from complexes sources found in sewage sludge. Other works study the effect of rainwaters in soil depending on laboratory testing for many samples of soil before and after the rain such [1-3] [12] [15] [19] [22] [27] and the references therein.

Dispersion parameter is the foundation of researching the pollution regular pattern and predicting the water quality. It is indispensable for the study of pesticides, heavy metals in farmland, chemical fertilizers, movement of water and groundwater.

The hydrodynamic dispersion parameter is named of convection dispersion. It is a comprehensive reflection parameter of soil, and it is depended on the effect of moisture content and the velocity of pore water [5]. There are many researches about hydrodynamic dispersion to determine the hydro dispersive.

Several methods based on a series of laboratory dispersion tests from the tracer experiments such [4] [8] [18] [19] [20][32] and the references therein.

In this thesis a new approach is used for calculating the dispersion parameter by designing suitable Simulink model. Then design a model equation based on Simulink environment that can estimate the effect of rainwaters in the contaminated soil.

The organization of this thesis is as follows: Chapter one, contains two sections, section one represents mathematical concepts, consisting some of the definitions, hypotheses, axioms and theorems that are needed. Section two, consists an introduction to using Simulink that are needed throughout the thesis. Chapter two describes, develops and designs Simulink model for spread of contamination through soils which can be used to determine the rate of contamination and the effect of rainwater on it by estimating the concentration of heavy metals. Chapter three, illustrates how the suggested method is well-suited for describe model, by discussion the global error. Also, suggests a new modification of the error estimation which helps to reduce the computational time of classic estimation of error. It also helps to perform well for a given data and existed samples. Chapter four introduces the practical applications of the suggested method to determine the effect of rainwater in contaminated soil by estimate the concentration of heavy metals such: Copper (Cu), Lead (Pb), Cadmium (Cd), Cobalt (Co), Zinc (Zn), Iron (Fe), Nickel (Ni) and Crum (Cr) for different zone in Baghdad. Finally, chapter five contains the conclusions and future works. All algorithms in this thesis have been implemented in MATLAB version 7.12.

CHAPTER ONE

PRELIMINARIES

Chapter One

Preliminaries

1.1. Introduction

This chapter offers a brief description to some background ideas for terms or topics that is used and needed throughout the thesis. In several cases this material could also be familiar; but a restricted discussion is provided here in an endeavor to form the thesis self-contained.

This chapter consist two sections. Section one consists of some mathematical concepts which will be needed in the thesis.

In section two, a general idea about Simulink environment have been introduced, especially building Simulink model that is needed in the formulation of the model equation.

1.1.Mathematical Concepts

This section consists of the definitions, hypotheses, axioms and theorems that are needed throughout the thesis.

1.1.1. Partial Differential Equations

Partial differential equations (PDEs) are well known that most of the phenomena arise in science and engineering fields can be described by PDEs. It is an equation that involving the unknown function (the dependent variable), and its partial derivatives, the dependent variable must depend on more than one independent variable. So, it is a useful tool for describing natural phenomena of science and engineering models. A partial differential equation is called linear if the power of the dependent variable and each partial derivative contained in the equation is one. However, if any of these conditions is not satisfied, the equation is called nonlinear. [30]

There are many methods to find the solutions of PDEs like numerical methods, approximate methods and analytic methods. It is important to note that several methods, such as the method of finite element and the variational principle, are among the methods that are used to solve the nonlinear PDEs. Moreover, nonlinear PDEs are not easy to solve especially if the questions of uniqueness and stability of solutions are to be discussed. It is necessary to point out that some methods that used for the linear PDEs cannot be applied to nonlinear PDEs. For this reason, numerical solutions are usually established for nonlinear PDE. The initial conditions are important to start solved numerically. That is, initial values of the dependent variable at the independent variable should be given. A particular solution is frequently required that will satisfy prescribed conditions. Given a PDE that controls the mathematical behavior of a physical phenomenon in a bounded domain. The boundary data is called boundary conditions. There are three types of boundary conditions defined as follows:

i. Dirichlet boundary conditions: The function is known on the boundary of the bounded domain.

ii. Neumann boundary conditions: The derivative of the function at the boundary of the domain is known.

iii. Mixed (Robin) boundary conditions: A linear combination of the dependent variable and its derivative is known on the boundary [17].

Note a PDE with initial condition given in the one independent variable and boundary condition given in the others is said to be PDE with *initial-boundary condition* or *initial-boundary* PDE [31].

The topic in this thesis is the field of PDEs. Most real-world physical systems, including gas dynamics, elasticity, relativity, thermodynamics, fluid mechanics ecology, neurology, and much more, are modeled by nonlinear PDEs. The applicability (model our problem using PDE) of this thesis is to estimate the effect of rainwater in contaminated soil by heavy metals.

1.1.2. Existence and Uniqueness of Solutions

The existence and uniqueness theory plays an important role in analyzing approximate methods for solving PDE′s, therefore, the main theorems are introduced here while its proof is given in [35].

Theorem 1.1 (Existence)

Let U, V be Hilbert spaces and L: U → V be a bounded linear operator.

Then R(L) = V if and only if R(L) is closed and if $R(L)^{\perp} = \{0\}$ [35].

Theorem 1.2 (Existence and uniqueness)

Let U, V be Hilbert spaces and L: U → V be a closed linear operator. Suppose that there exists a constant C > 0 such that:

$\|Lv\|_V \geq C\|v\|_U$, for all v ϵ U (coercivity estimate).

If $R(L)^{\perp} = \{0\}$, then the operator equation Lu = f has a unique solution.

Note The theorem guarantees that the general PDE will always have (existence of) exactly one (uniqueness) solution [37].

1.1.3. Condition Number[9]

The **condition number** of the matrix A is defined as the number $\|A\|\,\|A^{-1}\|$ and is denoted by ƙ(A).

To calculate the condition number of an invertible square matrix, we need to know what the norm of a matrix means. How is the norm of a matrix defined?

Just like the determinant, the norm of a matrix is a simple unique scalar number. However, the norm is always positive and is defined for all matrices – square or rectangular, and invertible or noninvertible square matrices. One of the popular definitions of a norm is the row sum norm (also called the uniform-matrix norm). For a m×n matrix *A*, the row sum norm of *A* is defined as:

$$||A||_{\infty} = \max_{1\leq i\leq m} \sum_{j=1}^{n} |a_{ij}|$$

That is, find the sum of the absolute value of the elements of each row for the matrix A. The maximum out of the m such values is the row sum norm of the matrix A.

There are many methods can be used to solve differential equations some of these methods are analytic others are approximated and numerical. And these method varies in accuracy, efficient, performance, easy and rapidly implementation. In this thesis we use Adomian Decomposition Method.

1.1.4. Adomian Decomposition Method [14]

Adomian Decomposition Method (ADM) is one of the decomposition methods provide the solution in an infinite series form; where the exact solution exists and the obtained series may converge to a closed form solution. If the exact solution does not exist, then the truncated series may be used for numerical purposes. ADM is a creative and effective method for exactly solving functional equations of various kinds. It is important to know that a large amount of research work has been devoted to the application of ADM to a wide class of linear or nonlinear, ordinary or partial differential equations. The idea of this technique is demonstrating the unknown perform u(x, y) of any equation in expanded series as a add of infinite variety of terms outlined by the decomposition series which may be written as:

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y), \qquad (1.1)$$

where $u_n(x, y)$, $n \geq 0$ are to be determined during a algorithmic manner.

Now, we tend to summarize ADM to resolve the linear equation by the subsequent steps:

1. Writing the PDE in operator kind by:

$$Lu + Ru = g, \qquad (1.2)$$

where L is that the lower order by-product that is assumed to be invertible, R is different linear differential operator, and g may be a supply term.

2. Applying the inverse operator L^{-1} to each side of equation (1.2) then victimization the given condition to get:

$$u = f - L^{-1}(Ru), \qquad (1.3)$$

where f is that the operate represents the terms arising from integration g and by victimization the given conditions that area unit assumed to be prescribed.

3. The elements $u_0, u_1, u_2, \dots$, square measure recurrently determined. Then add (1.1) into each side of (1.3) to get:

$$\sum_{n=0}^{\infty} u_n = f - L^{-1}\left(R\left(\sum_{n=0}^{\infty} u_n\right)\right) \qquad (1.4)$$

Rewrite equation (1.4) as follows:

$$u_0 + u_1 + u_2 + u_3 + \cdot \cdot \cdot = f - L^{-1} \left(R(u_0 + u_1 + u_2 + \cdot \cdot \cdot)\right) \qquad (1.5)$$

4. Determining the algorithmic relation as follows:

$$u_0 = f ,$$

$$u_{k+1} = - L^{-1} (R(u_k)) , k \geq 0, \qquad (1.6)$$

or equivalently:

$u_0 = f$,

$u_1 = -L^{-1}(R(u_0))$,

$u_2 = -L^{-1}(R(u_1))$,

$u_3 = -L^{-1}(R(u_2))$, (1.7)

5. Then substitute (1.7) into (1.1) to induce the answer in a very series type.

1.1.5. Interpolation

Interpolation is involves finding a polynomial that agrees exactly with some information that we have about a real-valued function f of a single real variable x. This information may be in the form of values $f(x_0), . . . , f(x_n)$ of the function f at some finite set of points $\{x_0, . . . , x_n\}$ on the real line. Interpolation is often used for estimating the values on a continual grid primarily based model [29]. Weierstrass approximation theorem guarantees that the existences of a polynomial, but this theorem is not a constructive one, i.e., it does not gift how a way to get such a polynomial, i.e., the interpolation downside may be developed in a different way viz. because the answer to the subsequent question: a way to realize an honest representative of a function that is not best-known expressly, however solely at some points of the interested domain.

Theorem 1.3 (Weierstrass Approximation Theorem) [29]

Let f(x) be a continuous real value function on [a, b], then for any $\varepsilon > 0$, there exist an integer n and polynomial $P_n(x)$ of degree n, such that:

$$| f(x) - P_n(x) | < \varepsilon, \quad \forall x \in [a, b] \qquad (1.8)$$

1.2.An Introduction to Using Simulink [10]

A Simulink model is a block diagram in the Library Browser. An empty block diagram will pop up. We can drag blocks into the diagram from the library. To connect blocks, drag a signal line from the output of a block to the input of another block. Ctrl-Click will automatically connect.

The Simulink library contains all the blocks that are available with which to build any models. The library should open by default when open Simulink, but if it is not visible for any reason, it can be brought up from any open Simulink window from the View menu see Figure (1.1).

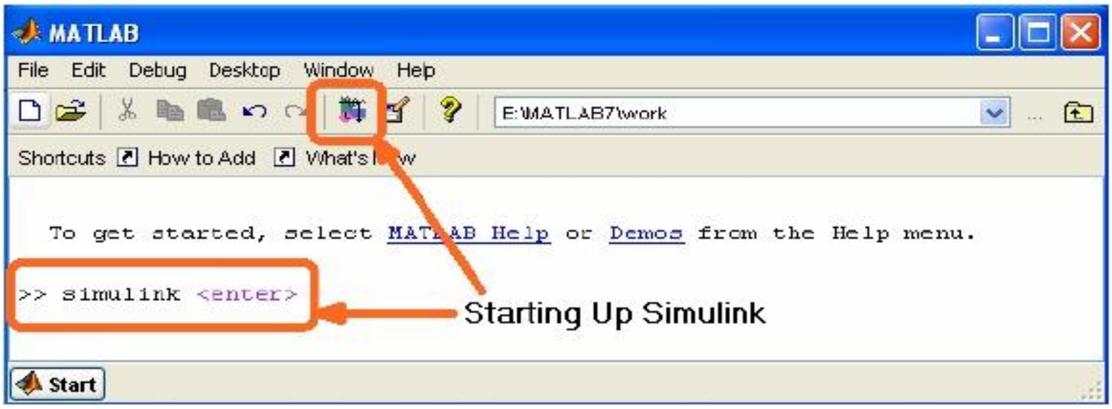


Figure 1.1: Starting Simulink

1.2.1. Adding Comments

We can add text comments anywhere in the block diagram by double clicking and typing in some text. We can change the default comments under the blocks by double clicking and editing the text.

A list of block library can seem. User will choose a block of desired characteristics from these libraries. the foremost usually use block libraries are:

Integrator, State Space, Transfer Function, Transport Delay, Add Gain, Product, Sum, Mux, Demux, Scope, XY Graph, Step, Signal Generator, Ramp, Random Number, AWGN Channel.

1.2.2. Signal Routing

We can create a branch point in a signal line by holding down the CTRL key, and clicking on the line. A summer block can be found in the "commonly used blocks" library, and in the "math" library.

To change the shape of the summer to rectangular, or to add additional inputs or change the sign, double click on the summer. Also, we can flip a block over by right clicking and looking under "Format", or by selecting it and typing CTRL-i.

1.2.3. Mathematics of Simulink Blocks

A Simulink block consists of a set of inputs, a set of states, and a set of outputs where the outputs are a function of the sample time, the inputs, and the block's states see Figure (1.2).

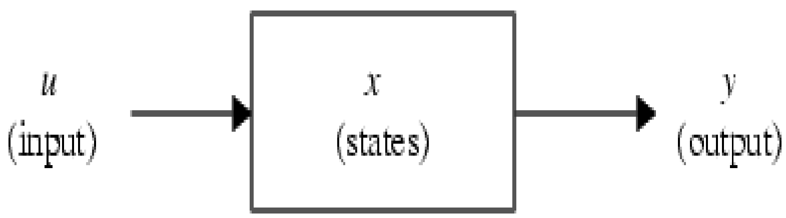


Figure 1.2: Simulink block

The following equations express the mathematical relationships between the inputs, outputs, and the states

$y = f_0(t, x, u)$ (output)

$\dot{x_c} = f_d(t, x, u)$ (derivative)

$x_{d\ (k+1)} = f_u(t, x, u)$ (update)

Where $x = x_c + x_d$

1.2.4. Specifying Parameters

Specifying Parameters is very important to run Simulink model. It is easy to do that, but we need to understand the model, familiarize with blocks intend to use. The block parameter dialog is opened by double clicking on the block (see Figure (1.3)). Parameters that may not be adjusted are grayed out. Note that all parameters of this block are currently adjustable.

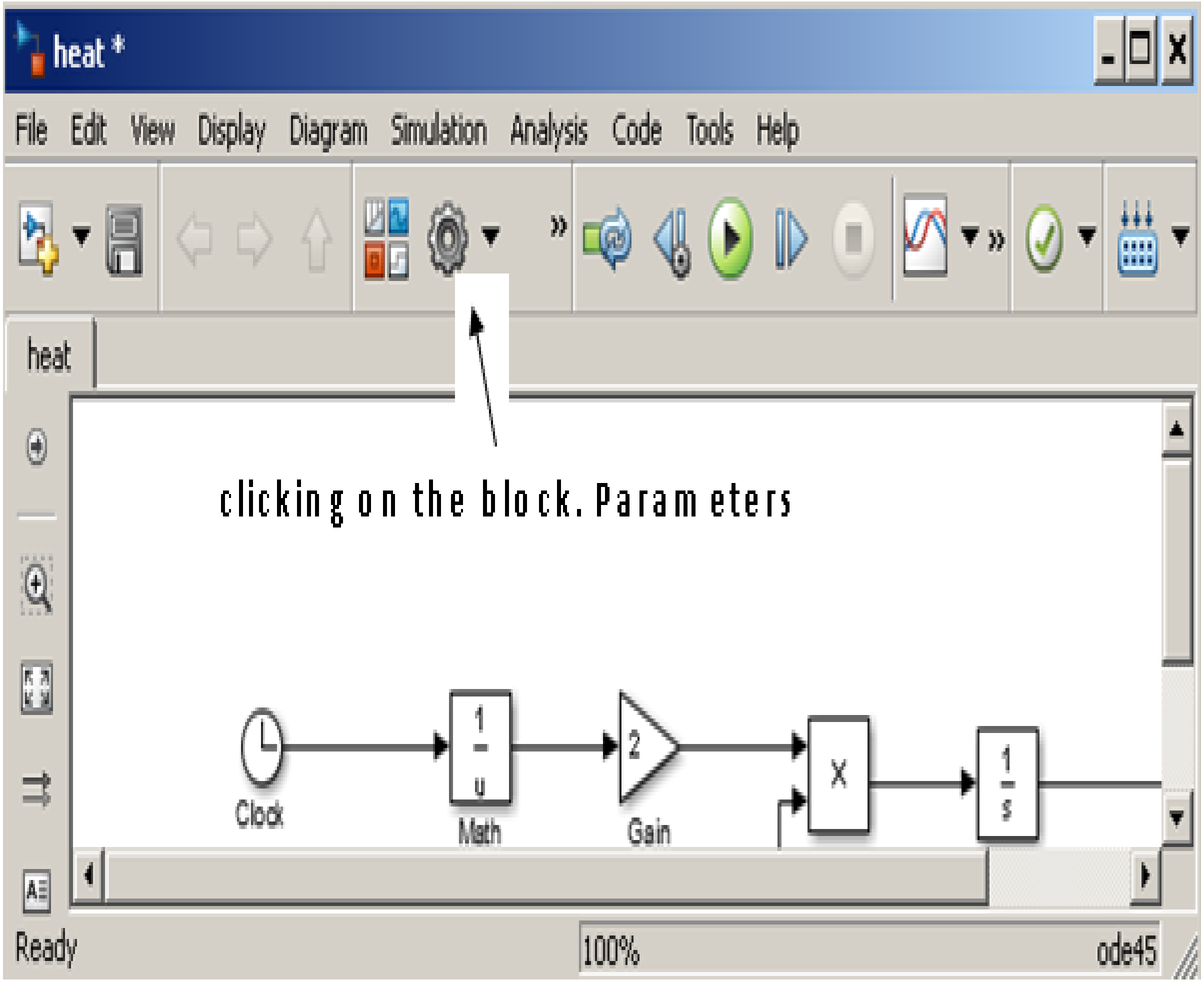


Figure 1.3: Access Model Parameters

❖ Commonly Used Blocks

These are commonly used blocks in MatLab Simulink software. Figure (1.4) is a copy of commonly used blocks window, from Matlab Simulink 2014b package. Here, some of these blocks are explained.

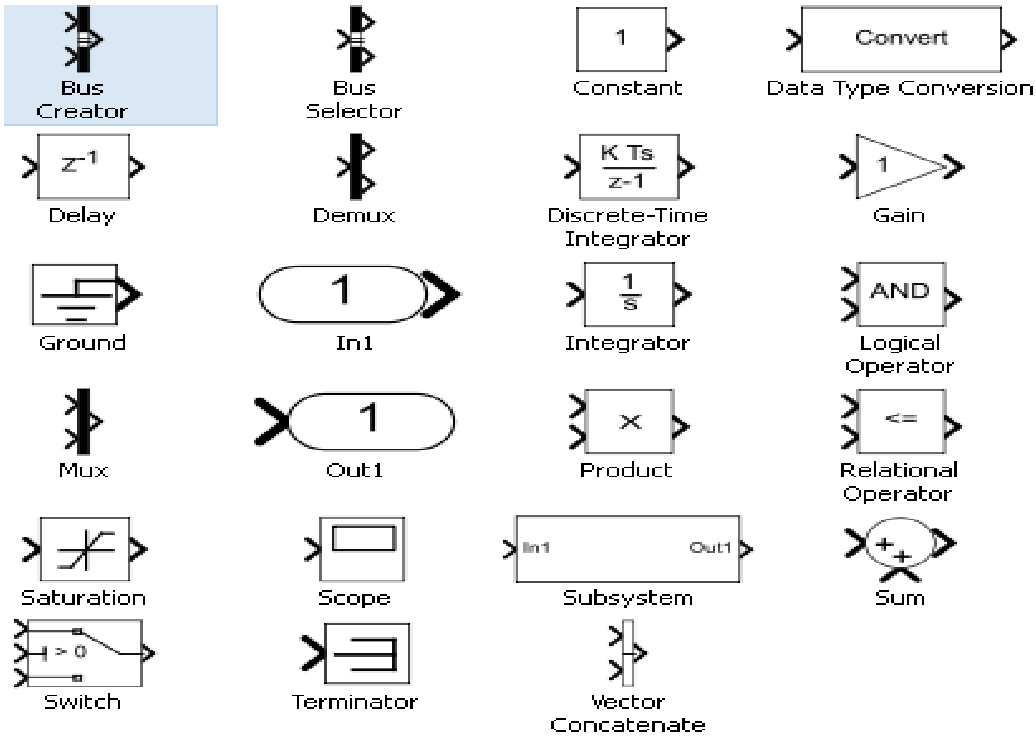


Figure 1.4: Simulink commonly used blocks.

- Summation and multiplication blocks:

The default sum block parameter, one can change the signs, number of inputs and locations from list of signs.

- Gain block is used to multiply the signal by a constant; it may be a constant number, vector, or matrix. Ground block is used to ground unconnected points.

- The integrator block integrates the input and is used with continuous time signals.

❖ Continuous

The continuous blocks are illustrated in Figure (1.5)

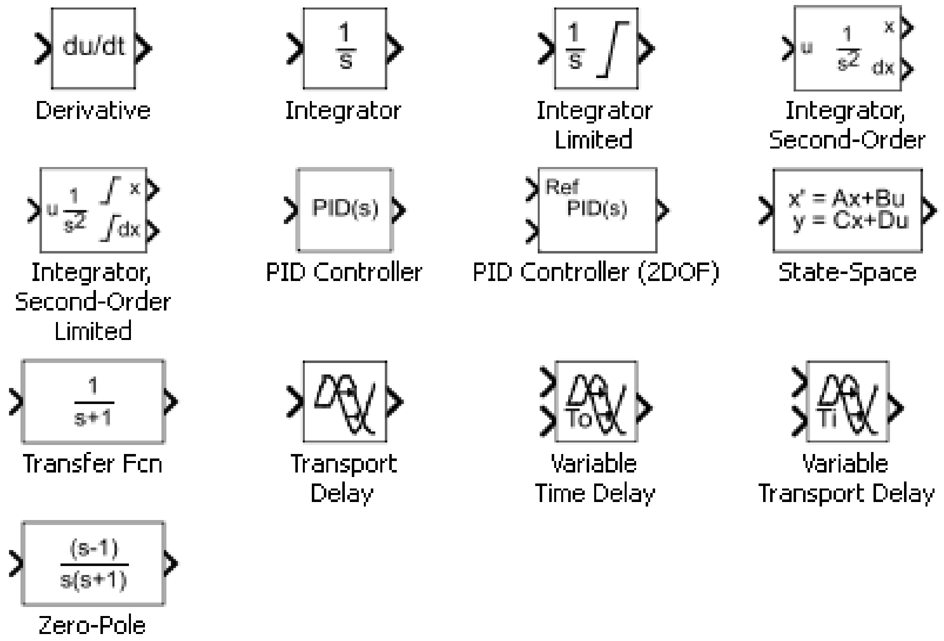


Figure 1.5: The continuous blocks

To create a new Simulink model press FILE / NEW / MODEL from the Simulink Library browser. A new model window will be opened in which we will have to transfer the required blocks and make proper connections among them to represent the equation of our interest. Let us save this file by name simbookex. The Simulink extension for model files is .mdl. To transfer the blocks (for example integrator block) from the Simulink library, click on the continuous category and then position the mouse on the integrator and drag it into the model window and release. Similarly drag all the other blocks into the model window.

❖ Discontinuous

Discontinuity is one of the most important nonlinear factors challenging both methods of nonlinear analysis and its applications. The problem becomes more complicated when the discontinuity is developing under the influence of dynamic processes in the structures.

❖ Matrix Input

Illustrated in Figure (1.6)

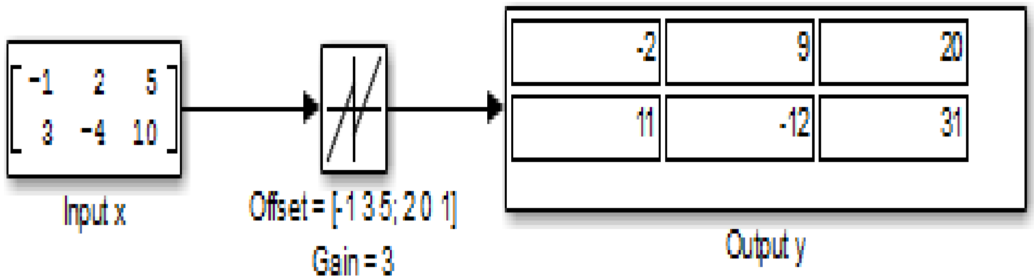


Figure 1.6: Matrix representation

❖ Saturation Block

The Saturation block is used to impose upper and lower bounds on a signal. Figure (1.7) illustrates the effect of saturation block; it limits the output between ± 1 in the lower scope.

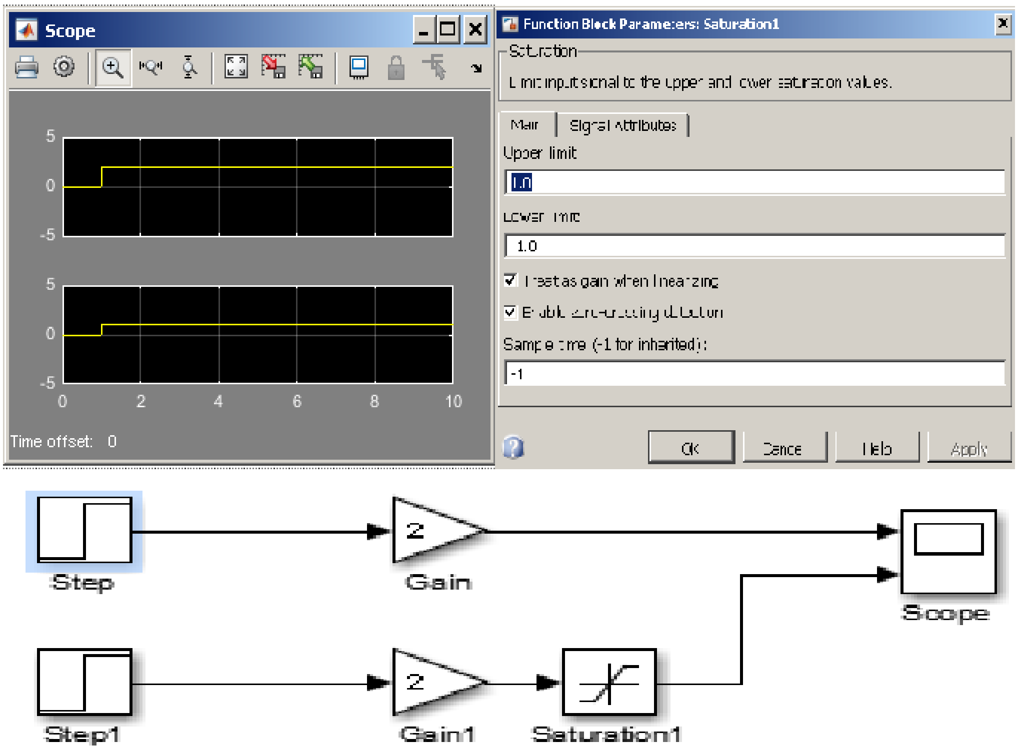


Figure 1.7 : Saturation block.

1.2.5. Logic and Bit Operations

Illustrated in the following Figure (1.8)

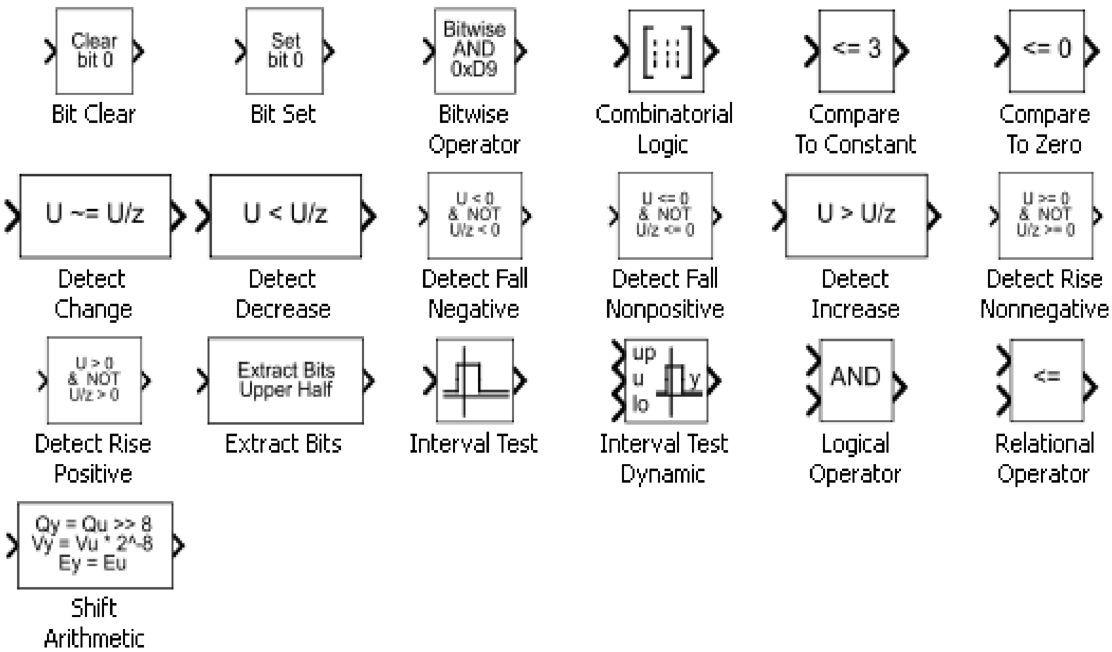


Figure 1.8: Logic and Bit Operations

1.2.6. Look up Tables

Illustrated in the following Figure (1.9)

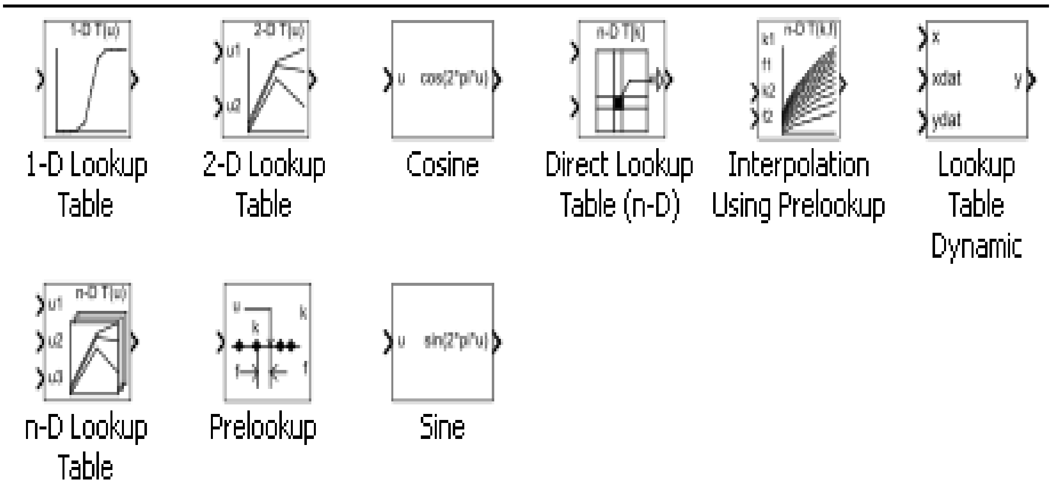


Figure 1.9: Look up Tables

1.2.7. Mathematical Operations

Illustrated in the Figure (1.10)

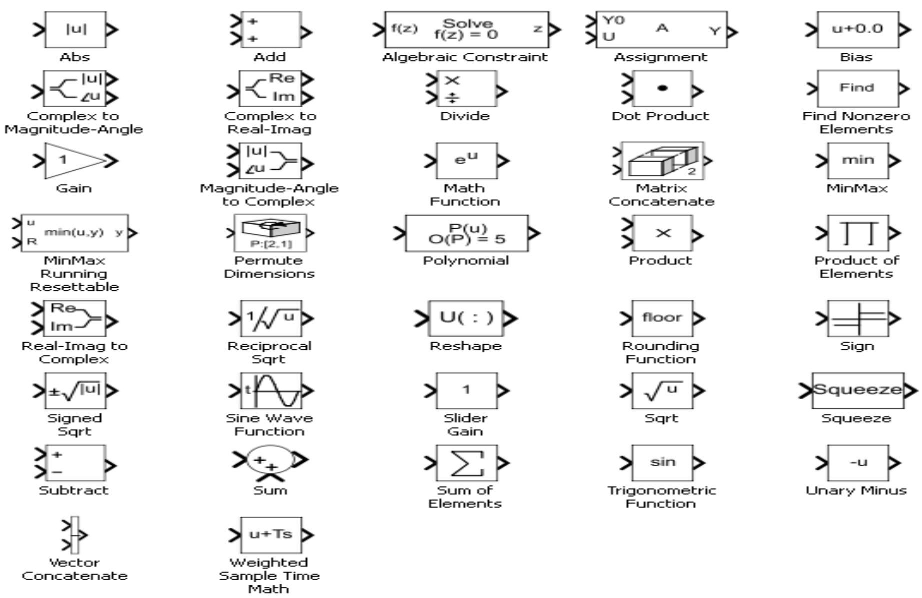


Figure 1.10: Mathematical Operations

1.2.8. Mathematical Functions

Illustrated in the Figure (1.11)

exp	Exponential	e^u	exp
log	Natural logarithm	$\ln u$	log
10^u	Power of base 10	10^u	10.^u (see power)
log10	Common (base 10) logarithm	$\log u$	log10
magnitude^2	Complex modulus	$\lvert u \rvert^2$	(abs(u)).^2 (see abs and power)
square	Power 2	u^2	u.^2 (see power)
pow	Power	u^v	power
conj	Complex conjugate	$\bar{u}$	conj
reciprocal	Reciprocal	$1/u$	1./u (see rdivide)
hypot	Square root of sum squares	$(u^2+v^2)^{0.5}$	hypot
rem	Remainder after division	—	rem
mod	Modulus after division	—	mod
transpose	Transpose	u^T	u.' (see Array vs. Matrix Operations)
hermitian	Complex conjugate transpose	u^H	u' (see Array vs. Matrix Operations)

Figure 1.11: Mathematical Function

CHAPTER TWO

DESIGN SIMULINK MODEL

Chapter Two

Design Simulink Model

2.1. Introduction

In this chapter, a mathematical model based on Simulink environment to estimate the concentration of heavy metals which can be used to determine the effect of rainwater in contamination soil will be design, which may be used to estimate the concentration of heavy metals in soil before and after the rain.

2.2. Creation the Model

Creating an operating model with Simulink is easy; the model involves four basic steps as the following:

1- Select desired blocks

2- Configure block parameters

3- Connect block inputs outputs

4- Configure simulation parameters

First we may gather all the required blocks from the Library Browser. Then modify the blocks in order that they correspond to the blocks of the required model. Lastly, connect the blocks with lines to create the entire system and set the overall simulation parameters. After this, simulate the entire system to verify that it works. We can describe the above steps in detail as follow:

A. Step one: choose desired blocks

Getting the required blocks into the model window as following steps and illustrated in Figure (2.1):

- Create a replacement model (New from the File menu). we'll get a blank model window
- Click on the Sources icon within the Library Browser. This opens the Sources window which contains the Sources block library. Sources area unit will not to generate signals.
- Drag the undulation and Clock blocks from the Sources window into the left aspect of suggested model window.
- Click on the Sinks icon within the Library Browser to open the Sinks window.
- Drag the Scope and to space blocks into the proper aspect of suggested model window.
- Click on the Signal Routing icon within the Library Browser to open the Signal Routing window.
- Drag the Mux block into the suggested model window
- Click on the mathematics Operations icon within the Library Browser to open math's Operations window.
- Drag the Gain block into suggested model window.

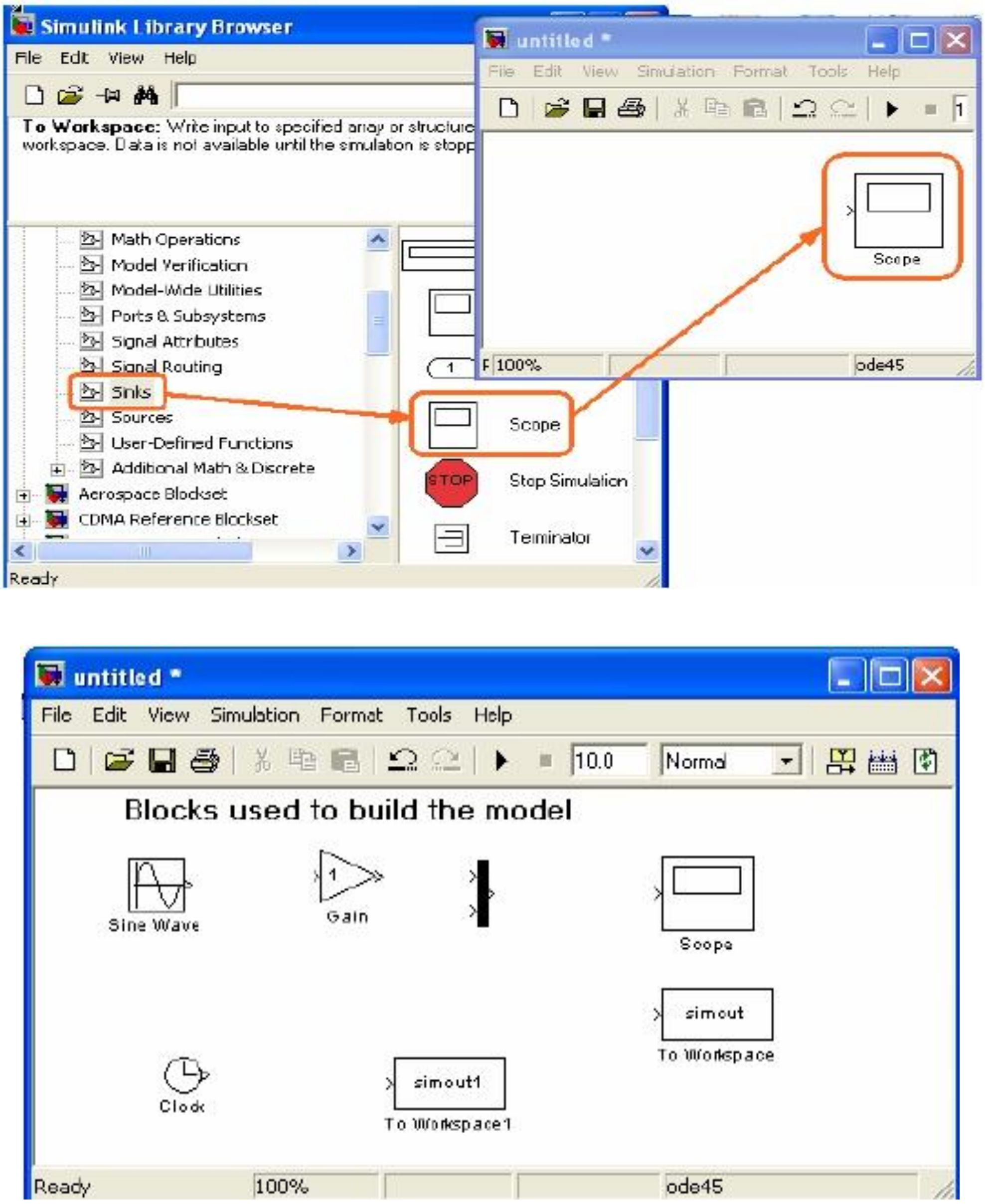


Figure 2.1: Choose desired blocks

B. Step 2: Set up block parameters:

Resizing and moving blocks as follow:

- First choose the block by clicking along with left push whereas the pointer is on the block. In every corner of the block slightly crammed sq. seems. place the pointer on one of the corner, press the left push and keep it ironed down. Move the mouse and unleash the push once the block has the required size.

- To move a block, Initially got to choose it. Then place the pointer within the block, press the left push down and keep it ironed down. Drag the block to its new position and unleash the push.

C. Step 3: Connecting block inputs and outputs:

Adding a branch line see Figure (2.2).

To connect the output of the undulation to the input of the Mux block, We will use a branch line. Drawing a line is slightly completely different therein to begin; the line must be welded to associate existing line. Position the pointer on the road that connects the trigonometric function Wave block to the Gain block. While not moving the mouse, press and hold down the CTRL key then press and hold the left push button. Drag the pointer to the input port of the Mux block and unharness the push button.

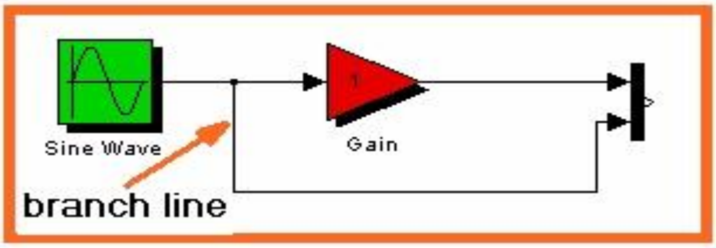


Figure 2.2: Connecting block inputs and outputs

D. Step 4: tack together simulation parameters as the following:

- **Adding Annotation:**

Annotations are adding matter data concerning the designed model. One will add associate degree annotation to any unoccupied space of suggested diagram. To make a model annotation, double click in associate degree unoccupied space. A little parallelogram seems and also the pointer changes to a vertical insertion bar. Begin writing the annotation content, to start out a brand new line, simply hit the Enter key, every line is focused among the oblong box that surrounds the annotation. To shut the annotation, click with the mouse elsewhere within the window.

In suggested diagram we will place a title at the highest and we will place the name and date at the lower right, as pictured below.

To move associate degree annotation, drag it to a brand new location. To alter the font size of associate degree annotation, choose it and with the mouse choose Format and so Font. Within the pop-up window choose the specified font size and so click OK. The font size can modification once the annotation is deselected, to try and do therefore merely click elsewhere.

- **Adding Signal Labels**

Labels are often further out lines to any annotate suggested model. To form a symbol label, double click on a line section and sort the label at the insertion purpose. Once click on another a part of the model, the label fixes its location illustrated in Figure (2.3).

To move a symbol label, purpose at it with the mouse, press and hold the left push and drag the label to a replacement location. To unselect a label, click elsewhere within the model.

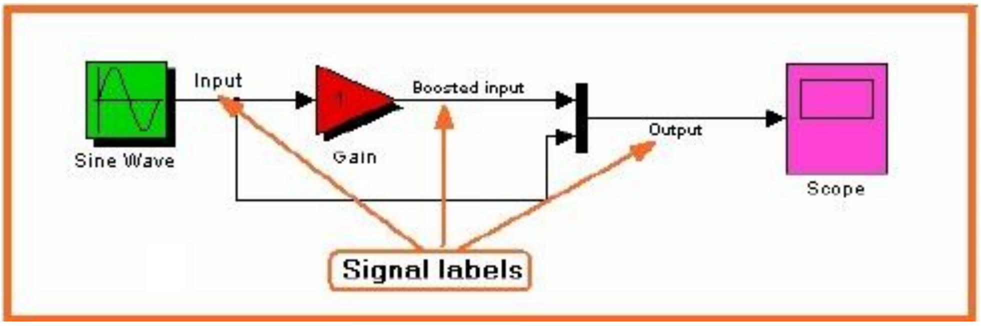


Figure 2.3: Adding Signal Labels

2.3. Run Simulation

Double-click the Scope block, Scope window can pop-up. Then click Simulation Start, we will see the simulation result's planned within the Scope. Click Auto scale (an icon of Telescope), the Scope can resize the plot and build it work the window. We will be able to additionally Zoom in/out the plot. Also, able to additionally resize the axes; by right click on the axes.

Now, we give illustrating example, to explanation integrating a sine wave, we require the following blocks: Sine wave, Integrator, Scope, Gain, Bus creator.

These blocks are arranged in the following order as shown in Figure (2.4) and the output is shown in Figure (2.5).

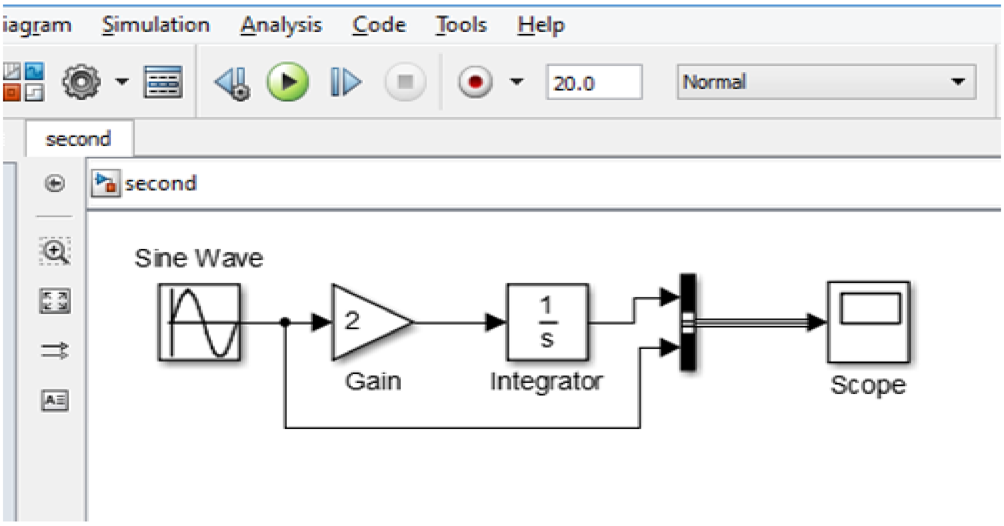


Figure 2.4: Integrating a sine wave

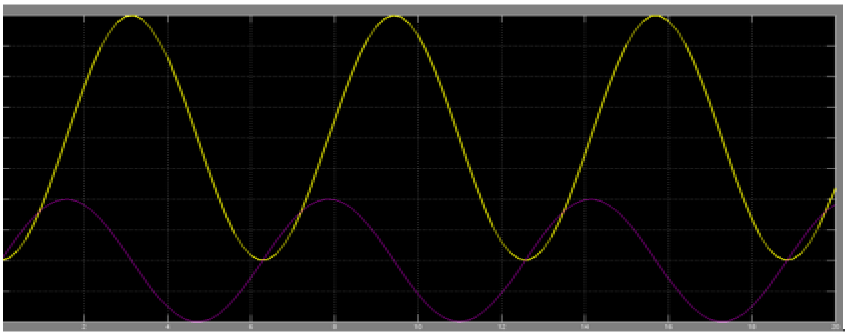


Figure 2.5: Output of the block diagram in Figure (2.4)

2.4. Scope

The scope block displays its input with relevancy simulation time. The Scope block will have multiple axis; all axes have a typical time vary. We will be able to move and size the Scope window and we will be able to modify the Scope’s parameter values throughout the simulation. When begin a simulation, Simulink doesn't open Scope windows, for illustration see Figure (2.6).

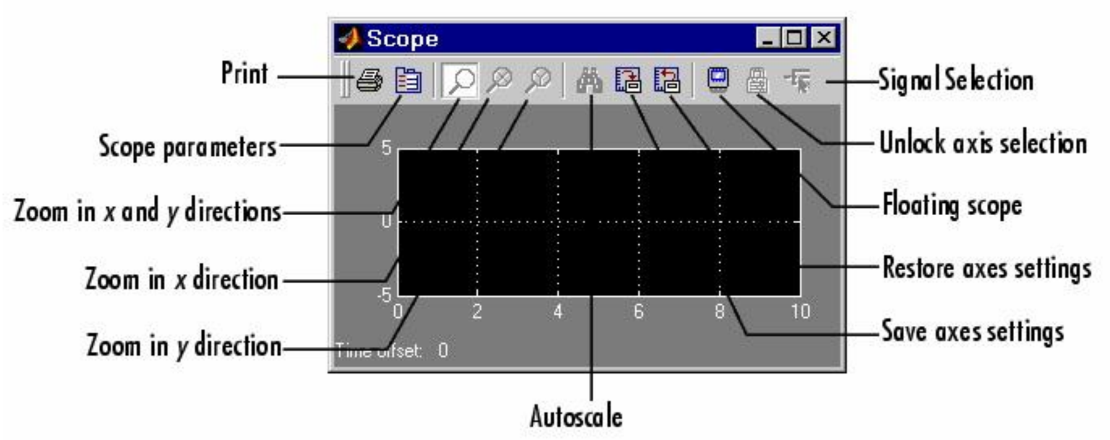


Figure 2.6: Scope block

2.4.1. Scope Parameters

We can alter axis limits, set the amount of axes, tick labels, sampling parameters, and saving choices by selecting Parameters toolbar button, as delineated below Figure (2.7).

Figure 2.7: Scope Parameters

2.5. Numbers of axes

Set the amount of y-axes during this knowledge field. With the exception of the floating Scope, there's no limit to the amount of axes the Scope block will contain. Note that the amount of axes is up to the amount of input ports.

2.6. Time range

Change the coordinate axis limits by getting into variety or car within the Time vary field, getting into range variety of seconds causes every screen to show the number of information that corresponds to it number of seconds. Enter car to line the coordinate axis to the length of the simulation Figure (2.8).

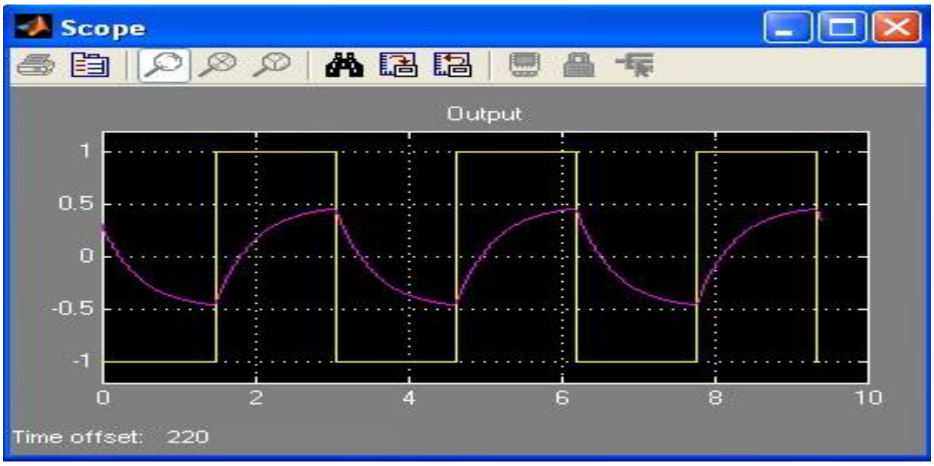


Figure 2.8: Time range

2.7. Mathematical Description of the Model

The suggested model descriptions of the contamination of soils by heavy metals which are distributed throughout the soil system remain in the soil solution as iron, organic and inorganic complexes. Some of these heavy metals are mobile for uptake by plants. This mobility and availability depends on several factors including soil texture and PH [24][27][34]. Also, in the industrial regions, where some types of factories are active, several chemical and petrochemical processes would be also active. As a result, industrial water becomes contaminated with various substances which are harmful; these are sources of environmental contamination, which added in the description of the model equation. Then, the model equation, can be written as the relation between the change of concentration C ($\frac{\text{mg}}{\text{L}}$) of heavy metals proportionally with the change of time t (day^{-1}) and the change of concentration of heavy metals proportionally with the change of space x (cm) multiplicand with the average pore-water velocity V ($\frac{\text{cm}}{\text{hr}}$)). Then adding the fluid velocity in the reactor multiplicand with Dispersion parameter D ($\frac{\text{cm}^2}{\text{hr}}$). So, the model equation can be written as:

$$\frac{\partial C}{\partial t} = D\,\frac{\partial^2 C}{\partial^2 x} - V\,\frac{\partial C}{\partial x} \qquad (2.1)$$

which is a second order linear PDE, with initial - boundary conditions. Now, how to choose initial and boundary conditions. A variety of conditions may be specified depending on the type of soil. There are different types of soil depending on properties, structure, characteristic or layers. In this thesis we classify the soil depending on characteristic and properties of soil as follows: Sand (fine and coarse), Silt, Loam, Crags and Clay [11]. If the soil is a loam land then the conditions are:

IC: $C(x, 0) = C_{x,0} = C_0\, e^{\frac{-Vx}{D}}$

BC: $C(0,t) = C_\circ$ *and* $\frac{\partial C}{\partial x}(x,t) = 0.$

When $x \longrightarrow \infty$

Where;

$C_\circ$: Initial concentration ($\frac{\text{mg}}{\text{L}}$)).

The amount of each element retained by each soil ($\frac{\text{mg}}{\text{kg}}$) was calculated from the initial concentration in solution ($\frac{\text{mg}}{\text{L}}$) and the final concentration C in solution ($\frac{\text{mg}}{\text{L}}$). Equation (2.1), which can be represented as a mathematical model for spread of contamination through soil which can be used to determine the rate of contamination. The solution of model equation gives the concentration of the heavy metals in soil for any space and time. This model can be considered as an important model to give concentrations of heavy metals without spending in traditional laboratory inspecting. Thus, the comparison between the result of model equation (2.1) before and after the rain gives the effect of the rain of contaminated soil.

2.8. Solving the Model Equation

We will use the Adomian Decomposition Method (ADM) to solve equation (2.1). First we consider the linear differential equation in an operator form by

$$L_t C = D L_{xx} C - V L_x C \qquad (2.2)$$

Where L, is the derivative symbol which assumed to be invertible.

Take the inverse operator L^{-1} to both sides of equation (2.2), and using the initial condition:

$$C(x, 0) = C_{x,0} = C_0 e^{\frac{-Vx}{D}}$$

We get:

$$C = C_0 e^{\frac{-Vx}{D}} + D L_t^{-1} L_{xx} C - V L_t^{-1} L_x C \qquad (2.3)$$

Related to Adomian method, we define the solution C by an infinite series of components given by:

$$C = \sum_{n=0}^{\infty} C_{x,n}$$

Then the equation (2.3) can be written as:

$$(\sum_{n=0}^{\infty} C_{x,n}) = C_0\, e^{\frac{-Vx}{D}} + D\, L_t^{-1}\, L_{xx}\, (\sum_{n=0}^{\infty} C_{x,n}) - V\, L_t^{-1}\, L_x\, (\sum_{n=0}^{\infty} C_{x,n}) \qquad (2.4)$$

Where $c_{x,0}$, $c_{x,1}$, $c_{x,2}$, … can be determined as far as we like.

As given in our model, the zeros component $C_{x,0} = c_0\, e^{\frac{-Vx}{D}}$, then:

$$C_{x,1} = L_t^{-1}\,(D\, L_{xx}\, c_0\, e^{\frac{-Vx}{D}} - V\, L_x\; c_0\, e^{\frac{-Vx}{D}})$$

$$= L_t^{-1}\,(\,c_0\,\frac{V^2}{D}\; e^{\frac{-Vx}{D}} + c_0\,\frac{V^2}{D}\; e^{\frac{-Vx}{D}})$$

$$= 2t\, c_0\,\frac{V^2}{D}\; e^{\frac{-Vx}{D}}$$

$$C_{x,2} = L_t^{-1}\,(D\, L_{xx}\,(2t\, c_0\,\frac{V^2}{D}\; e^{\frac{-Vx}{D}}) - V\, L_x\,(2t\, c_0\,\frac{V^2}{D}\; e^{\frac{-Vx}{D}}))$$

$$= L_t^{-1}\,(2t\, c_0\,\frac{V^4}{D^2}\; e^{\frac{-Vx}{D}} + 2t\, c_0\,\frac{V^4}{D^2}\; e^{\frac{-Vx}{D}})$$

$$= \frac{4\,t^2}{2}\, c_0\,\frac{V^4}{D^2}\; e^{\frac{-Vx}{D}}$$

$$C_{x,3} = L_t^{-1}\,(D\, L_{xx}\,(\frac{4\,t^2}{2}\; c_0\,\frac{V^4}{D^2}\; e^{\frac{-Vx}{D}}) - V\, L_x\,(\frac{4\,t^2}{2}\, c_0\,\frac{V^4}{D^2}\; e^{\frac{-Vx}{D}}))$$

$$= L_t^{-1}\,(2t^2 c_0\,\frac{V^6}{D^3}\, e^{\frac{-Vx}{D}} + 2t^2 c_0\,\frac{V^6}{D^3}\, e^{\frac{-Vx}{D}})$$

$$= 8\,\frac{t^3}{3!}\; c_0\,\frac{V^6}{D^3}\, e^{\frac{-Vx}{D}}$$

And so on for other components. Consequently, the solution in a series form is given by:

$$C = C_{x,0} + C_{x,1} + C_{x,2} + C_{x,3} \ldots\ldots$$

$$= c_0\, e^{\frac{-Vx}{D}} + 2t\, c_0\,\frac{V^2}{D}\, e^{\frac{-Vx}{D}} + \frac{4\,t^2}{2}\, c_0\,\frac{V^4}{D^2}\, e^{\frac{-Vx}{D}} + 8\frac{t^3}{3!}\; c_0\,\frac{V^6}{D^3}\; e^{\frac{-Vx}{D}} + \ldots$$

That is:

$$C = c_0\, e^{\frac{-Vx}{D}} \{\, 1 + (\frac{t}{D})(2v^2) + \frac{1}{2!}\left(\frac{t}{D}\right)^2 (2v^2)^2 + \frac{1}{3!}\left(\frac{t}{D}\right)^3 (2v^2)^3 + \ldots\}$$

i.e., C = $c_0\, e^{\frac{-Vx}{D}}\; e^{\frac{2tV^2}{D}}$

Thus

$$C = c_0 \exp\left(\frac{-Vx}{D} + \frac{2V^2}{D}t\right) = c_0 \exp\left\{\frac{V}{D}(-x + 2Vt)\right\} \qquad (2.5)$$

Thus, equation (2.5) represents the closed form solution of the model equation.

Now, we choose V = 5.14 × 10^{-6} ms^{-1} $= 44.4096 \times 10^{-2}\ \frac{m}{d}$, depending on results of [8][6].

Therefore, C = $c_0 \exp\left\{\frac{44.4096\times10^{-2}}{D}\ (-x + 2(44.4096 \times 10^{-2})t)\right\}$ (2.6)

So, C = $c_0 \exp\left\{\frac{44.4096\times10^{-2}}{D}(-x + 0.888192t)\right\}$

Then, $D = \frac{Ln\frac{C}{C_0}}{44.4096\times10^{-2}\ (-x+0.888192t)}$ (2.7)

Now, we build Simulink[23][28] model to estimate the dispersion parameter D for the equation (2.7), we start by looking at the terms in the bracket. There are two input signal lines: t and x, we also notice that both t and x are multiplied by constants, as in the case of the first- model; this can be done in Simulink by using a Gain block. The signals are then summed together as illustrated in Figure (2.9).

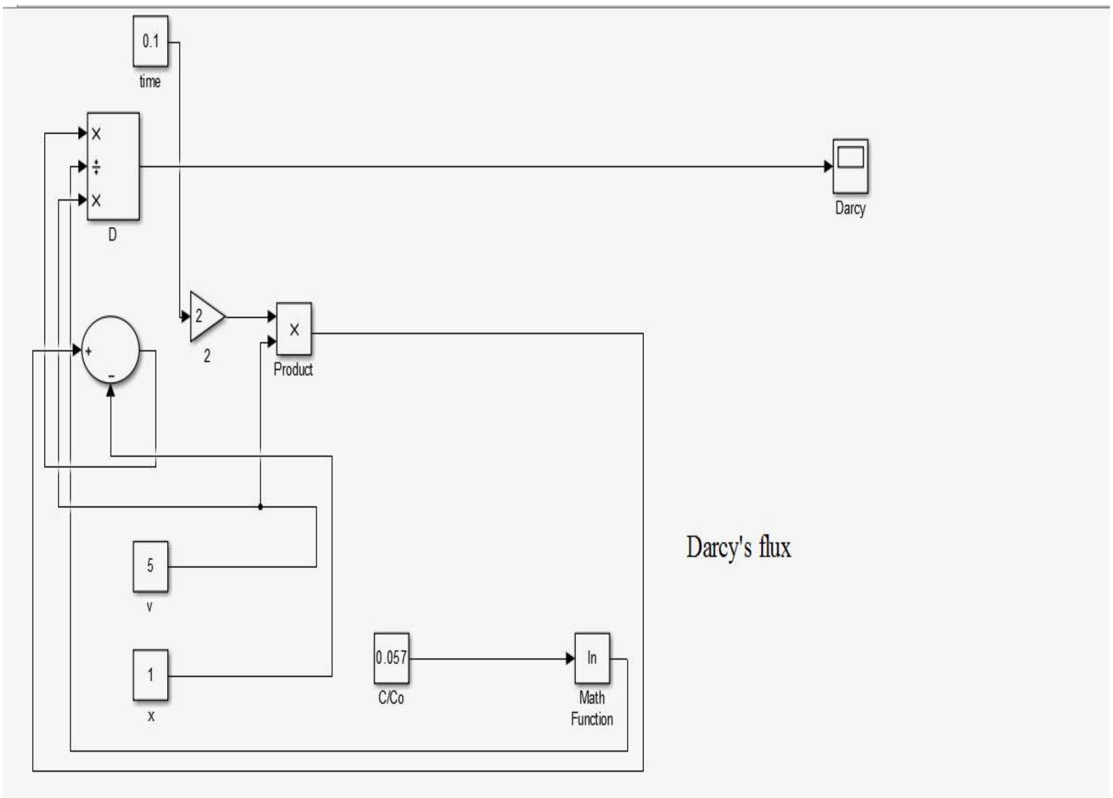


Figure 2.9: Simulink model to estimate the dispersion parameter

We can see from the suggested model that x dot and x are both inputs and outputs of the model. Therefore, we need to use a feedback loop for each of these signals. Before running the model is sure to include a Scope block for viewing the results. Now that the model is complete, we are ready to run the simulation and look at the results.

Note that Simulink model illustrated in Figure (2.9) can be used to estimate the dispersion parameter D for any type of soil in any time.

Depending on this value we design Simulink model to estimate the concentration of heavy metals and to measure the effect of rainwater.

This model can be considered as an important model to give concentrations of heavy metals without spending in traditional laboratory inspecting. Thus, the comparison between the result of suggested model equation (2.1) before and after the rain gives the effect of the rain for the contaminated soil. Depending on those, we design Simulink model illustrated in Figure (2.10).

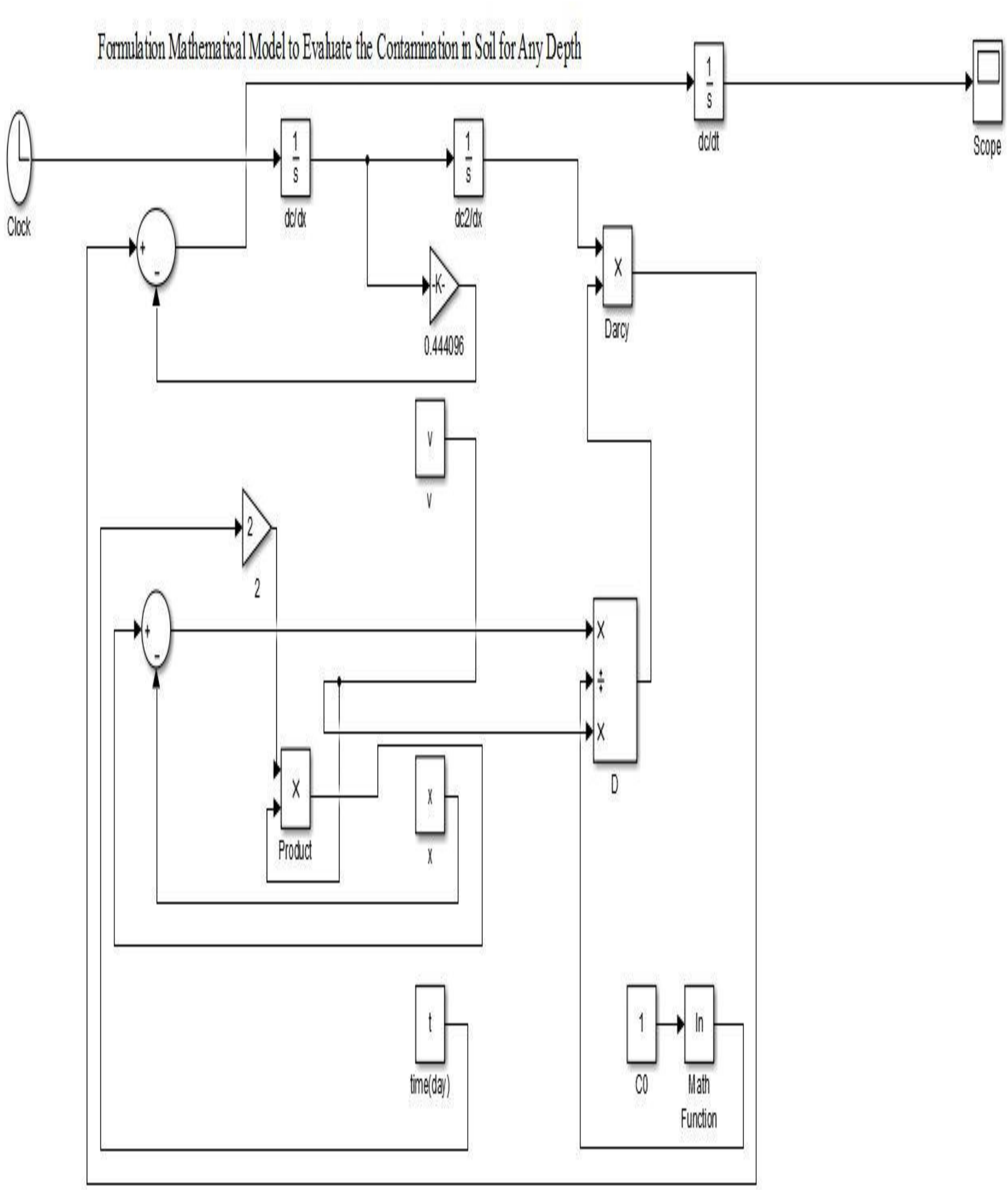


Figure 2.10: Design Simulink model for suggested problem

CHAPTER THREE

WELL-POSED PROBLEM AND ERROR ESTIMATION

CHAPTER THREE

WELL-POSED PROBLEM AND ERROR ESTIMATION

3.1. Introduction

This chapter illustrates how the suggested method is well-suited for the described model, by discussing the global error. This thesis, suggests a new modification of the error estimation which helps to reduce the computational time of the classical estimation of error it also helps to perform well for a given data and existed samples

3.2. Well-Posed Problem

A data represented by mathematical model in a domain is said to be well-posed if the following three properties hold:

i. Existence: There exists at least one representation of the problem.

ii. Uniqueness: There is at most one representation of the problem.

iii. Stability: The unique representation depends continuously on given data, i.e., a slight change in the data leads to only a small change in the representation.

Otherwise, the problem is said to be ill-posed problem. [33]

Roughly speaking, well-posed problems are those for which the given data determines physically reasonable representation. Physically reasonable representation are defined relative to a mathematical class in which theorems of existence, uniqueness, and continuous dependence on the data can be proved, a given representation is compatible with some data but not with the other.

Note

1-Why well-posedness?

If the problem is well-posed, then it stands a good chance of representation on a computer using a stable algorithm. If it is not well-posed, it needs to be re-formulated for numerical treatment. Typically this involves including additional assumptions, such as smoothness of representation. This process is known as regularization.

2- In particular, a small change in the given data implies a small change to the results must be considered, a design model that has this property is said to be well-conditioned. Otherwise, the design model is said to be ill-conditioned.

Chapter one gave satisfactory answers about the first two conditions. In this chapter, the stability of numerical schemes is investigated by suggested technique.

3.3. Stability

The Stability means that the small changes in initial data lead to small changes in result behavior. Thus a method is stable if the difference between the theoretical and numerical representation remains bounded at a given value, or tends to zero.

A more general method for studying the stability regards the consideration of eigenvalues. Let the data of difference between the theoretical and numerical representation are arranged in matrix form, say, A

Definition 3.1

A representation model is said to be zero stable, if all the eigen values of the matrix A belongs to a unit circle, i.e., $|\lambda_i| \leq 1$. if the $|\lambda_i| = 1$, then λ_i is not multiple.

Definition 3.2

Representation model is said to be stable, if all the eigen values of the matrix A have negative real parts, that is belongs to left side of real axis, and all those with zero real parts are simple. Otherwise it's unstable.

Application of the above concept will be given in the next chapter.

Here we mention the method for suggested technique. Let $C_l(x_m, t_m)$ is the value of concentration of heavy metals getting from laboratory inspecting in x_m distance and t_m time, also $C_s(x_m, t_m)$ is the value of concentration of heavy metals getting from suggested design in x_m distance and t_m time.

So, the errors can be estimated by:

$$E_m = |C_l(x_m, t_m) - C_s(x_m, t_m)| \qquad (3.1)$$

For abbreviating, rewrite equation (3.1) as:

$$E_m = |C_l - C_s| \qquad (3.2)$$

We can test the stability by other way depending on condition number which be illustrated in details in the following section.

3.4. Condition Number of the Suggested Design

The condition number ƙ(A) is a measure of how bad a design is. We will see that if a design model has a bad condition number, the results are unstable with respect to small changes in data.

Is there a general relationship that exists between $\frac{\|\delta A\|}{\|A\|}$ and $\frac{\|\delta C\|}{\|C\|}$? If so, it could help us identify well-conditioned and ill conditioned system of design.

There is a relationship; will it help to get the conditioning of the matrix,

When the data are arranged in matrix form then the definition of ƙ(A) is: consider all small changes δA in A got the resulting change, δC, in C. That is the condition number can be calculated as [28]:

$$ƙ(A) = \max\left(\frac{\text{Relative error of output}}{\text{Relative error of inputs}}\right) = \max\left(\frac{\frac{\|\delta C\|}{\|C\|}}{\frac{\|\delta A\|}{\|A\|}}\right) \qquad (3.3)$$

Another way, changes in the input data get multiplied by the condition number to produce changes in the outputs. Thus a high condition number is bad. It implies that small errors in the input can cause large errors in the output.

Is there a result to the problem of bad condition numbers? Usually, bad condition numbers in applications indicate result from poor design. So, the result to bad conditioning is redesign. If ƙ(A) is small, A is said to be well conditioned; if ƙ(A) is large, A is ill conditioned. If A is singular, ƙ(A) = ∞. Then $\|A\| = \sigma_1$ and $\|A^{-1}\| = \frac{1}{\sigma_m}$, thus ƙ(A) = $\frac{\sigma_1}{\sigma_m}$. [13]

For importance of ƙ(A) we design Simulink model to calculate ƙ(A) for given data and illustrated in Figure (3.1).

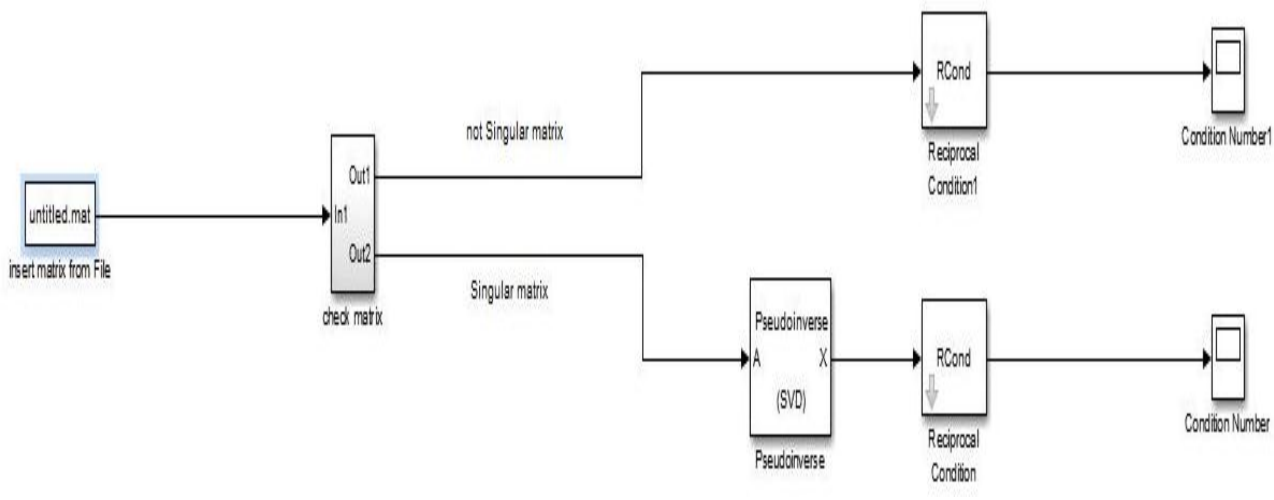


Figure 3.1: Simulink model to calculate ƙ(A)

The application of the suggested design for the given data will be introduced in the next chapter.

3.5. Error Estimate and Default Weights

Every known software package scales either the maximum relative defect or relative error. The weights used to scale the maximum defect. In this thesis, we estimate the accuracy of suggested method by using modifies this package and we say that the default weights depend on evaluate the maximum defect or error is being used and named "paytheSIM", which defined as:

$$\frac{\| C_l\,(x,t) - C_s(x,t) \|_\infty}{\| 1 - C_s(x,t) \|_\infty} \qquad (3.4)$$

when $C_s(x,t) \neq 1$

The application of this package will be given in the next chapter.

3.4. Global Error Estimation

The global error can be estimated by using the software packages COLSYS and COLNEW to assess the accuracy of the numerical results. There are many different algorithms that can be used to estimate the global error effectively; most of them performances are four algorithms: Richardson extrapolation, higher-order formulae, deferred corrections, and a conditioning constant. The first and second algorithms are modified and described in next subsections.

3.6.1. Richardson Extrapolation

Many software packages use Richardson extrapolation to estimate the global error [1]. This algorithm starts with calculate a numerical results for a given discrete mesh C_s. Next, the software determines a more accurate numerical result $C_{s/2}$ by halving all subintervals of the original mesh. Then, an estimate of norm for the global error, e_{RE}, is given by:

$$e_{RE} = \left\| \left(\frac{2^n}{2^n-1}\right)\left(C_s - C_{s/2}\right) \right\| \quad , n \neq 0 \qquad (3.5)$$

where n is the number of data in the sample

3.6.2. Higher Order Formula

For any problem; a more accurate numerical result can be determine by using Higher order formula with the same mesh as for the original result, specifically, the global error can be estimated by:

$$e_{HO} = \left\| C_s - C_l \right\|_\infty \qquad (3.6)$$

where C_l is the value of concentration of heavy metals getting from laboratory inspecting and C_s is the value of concentration of heavy metals getting from suggested design.

We apply these algorithms in the next chapter

3.7. Discussion

Our decision to use suggested technique for estimating the concentration of heavy metals to estimate the effect of rainwater in soil was motivated by its advantageous convergence properties, whereas in presence of a laboratory inspecting with classic calculating become inefficient, expensive. Moreover, it is variable with the time.

The main reason for choosing the global error introduced in section (3.6), instead of the local error is the order reductions it suffers form.

CHAPTER FOUR

APPLICATIONS

CHAPTER FOUR

Application

4.1. Introduction

In this chapter, the application about the suggested Simulink model designed in chapter two will be introduced with reasoning of accuracy and efficiency which be studied in details in chapter three. We choose the application in soil contamination and the effect of rainwater in it. That is design a Simulink model to compute the concentration of heavy metals in soil before and after the rain then compare it with its standard universal to determine the rate of contamination then compare this rate before and after the rain to determine the effect of the rain water in contaminated soil. Also, the application was chosed to show that the suggested model is economical, practical and can be used simply. The study area is Baghdad city.

4.2. Sampling

The Capital of Iraq Baghdad city (33°14'-33°25'N, 44°31'-44°17'E), is characterized by arid to semi-arid climate with dry hot summers and cold winters; the mean annual rainfall is about 151.8 mm. For the aim of collection of soil samples, the study area was divided into five main types of land use viz., residential, commercial, agricultural, main roads and industrial; and two main source areas, within each land use type viz. roadside and open areas. The sample areas are illustrated by geographic information system GIS. By using a stainless steel spatula. The samples were collected carefully from each source area in the land. They were air–dried in the laboratory, homogenized and sieved through a 2mm polyethylene sieve to remove large debris, stones and pebbles. After that, They were disaggregated with a porcelain pestle and mortar. Then these samples were stored in clean self–sealing plastic bags for further analysis. Metal determinations were done by X- Ray Fluorescence analysis (XRF). These samples represent the initial data which will be used in the suggested method in previous chapters to get the concentration of these metals for any depth and time. So, the samples were carefully chosen from each source area to get more accuracy results, where the sampling was carried out in January, February, March, April, May, June, November and December 2016 and January, February, March 2017.

Firstly, we choose the study area in Bab Al-Muadham city center of Baghdad. It has three central districts and near Mohammed Al- Kasim highway: residential land, commercial land and industrial land. Six sites were selected for study within Bab Al-Muadham area as illustrated in Figure (4.1). The distances of sampling sites from Mohammed Al- Qasim highway were (300, 600,750, 900, 900, and 950m). At each site, soil samples were collected at a range of depths (0 -5), (5-10), (10-15), (15-20), & (20-25) cm, the soil was generally taken from (0-25) cm of the topsoil because much of the nutrient uptake by plants is from this depth. Samples were preserved in cleaned polyethylene bags and finally transported directly to the laboratory; Figure (4.1) provides a sign for the character of the zones, from those samples were taken.

Figure 4.1: Location of samples in Bab Al-Muadham

Samples were collected from 28 locations, wherever 14 soil samples are collected with depth (0-10) cm, and alternative 14 soil samples are collected with depth (10-20) cm, victimization soil core, then all the samples were place in plastic baggage to be ready for inspecting to measure the concentration of heavy metals such: cadmium (Cd), nickel (Ni), and lead (Pb) .

The soil samples were taken on a dry day from numerous classes of gardens on road sides close to factories, children's playgrounds, colleges settled within the residential district, tanning animal skin factories and brick factories, within the laboratory, samples were sieved in an exceedingly 2-mm sieve to get rid of stones, glass and huge plant roots and afterward dried at temperature for three days. The dried samples were then homogenized with a mortar and a pestle. The procedure represented by [25-26] was applied to digest the samples with some modifications, and so all samples were taken to the laboratory to organize them for analyzing by XRF. The laboratory results illustrated in Table (4.1), first are used to compute the dispersion parameter by using Simulink model (2.9) in chapter two and run a simulation to get the results. Double-click on the Integrator block and set the initial data measured in laboratory dispersion test as given in Table (4.2). Then, double click on the **Gain** block and open the **Step** block. Now, run the simulation and open the **Scope** block, Hence we get the result that is the value of dispersion parameter (D_L) and close to 0.5.

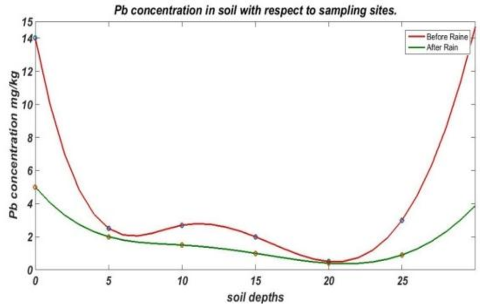
Table 4.1: average Concentrations of Pb, Ni, Cd for different zone with measure unit PPm

sites	Depth(cm)	Pb (S.M.C.=50)		Ni (S.M.C.=50)			Cd (S.M.C.=1)	
		Before Rain	After Rain	Before Rain	After Rain(1)	After Rain(2)	Before Rain	After Rain
1	0-5	14	5	3	0.4	0.4	0.0065	0.049
2	5-10	2.5	2	2.5	0.38	0.35	0.0063	0.025
3	10-15	2.7	1.5	2.3	0.33	0.2	0.0060	0.027
4	15-20	2	1	2.4	0.25	0.19	0.0058	0.023
5	20-25	0.5	0.4	1.8	0.15	0.05	0.0020	0.015
6	25-30	3	0.9	2.8	0.24	0.20	0.0055	0.030

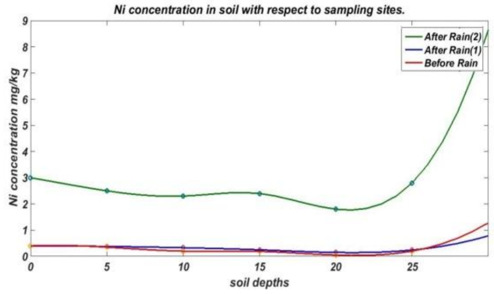
Table 4.2: Mean of the data of laboratory test

t(d)	C_0
0.1	0.057
0.15	0.259
0.2	0.5
0.25	0.691
0.3	0.891
0.35	0.898
0.4	0.943
0.45	0.967
0.5	0.983
0.55	0.991
0.6	0.995
0.65	0.997

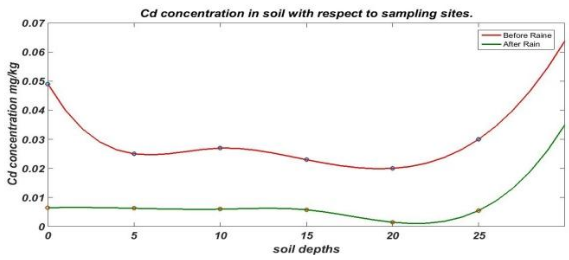
Now, the laboratory results given in Table (4.1), was applied as the initial concentrations (C_0) of heavy metals in the Simulink design (2.10) in chapter two, to get the concentrations of this heavy metals for time t(d) and depth x(m). The practical results are illustrated in Figure (4.2).



Pb



Ni



Cd

Figure 4.2: Concentration of Pb, Ni&Cd before & after the rain in Bab Al-Muadham

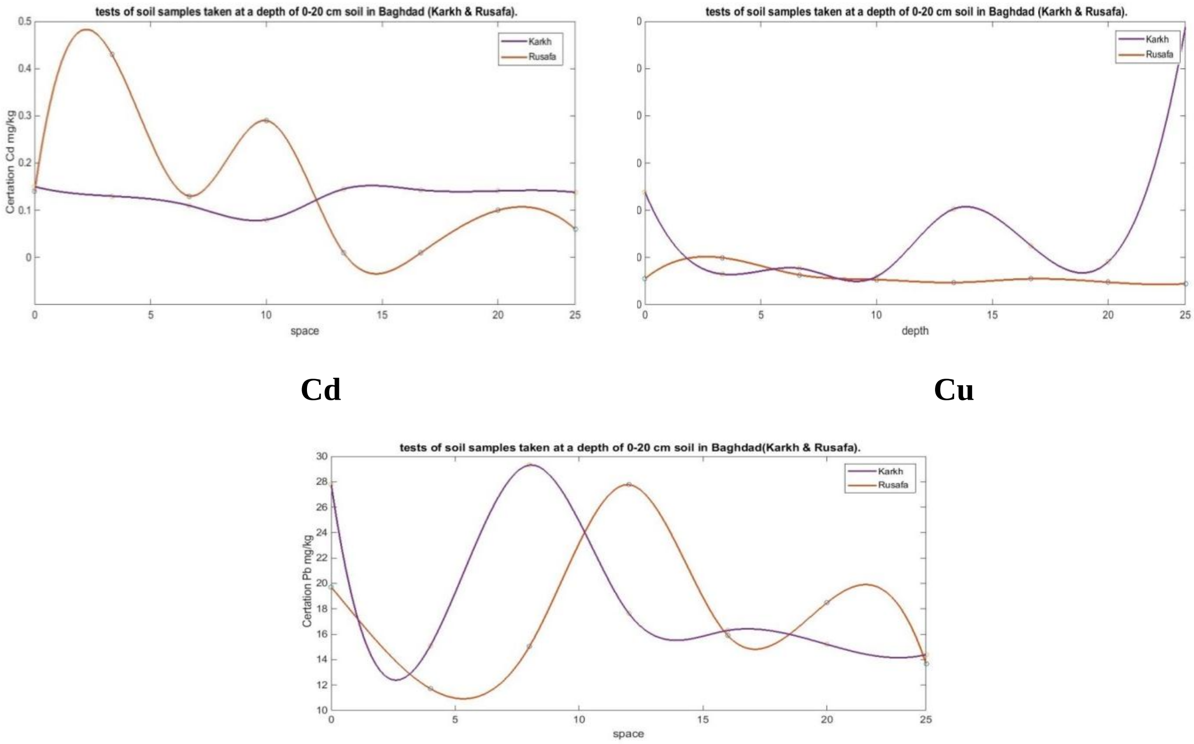
Now, we test the efficiency of the suggested method to manifesting the effect of rainwater in contaminated soil by Pb, Cu and Cd in different zones in Karkh and Rusafa; when the samples was chosen in same manner of dealing in Bab Al-Muadham with increasing data and the samples selected in (0-20) cm depth where the laboratory results for selected samples are given in Table (4.3-4.4) for and the results of Simulink method are given in Figure (4.3). While the standard universal for concentration of heavy metals in soil are gave in Table (4.5) depending on [8].

Table 4.3: Average concentrations of Pb, Cu, Cd and organic materials in Karkh soil with its Standard Universal.

No	Cd ppm (S.M.C.=1)	Cu ppm (S.M.C.=20)	Pb ppm (S.M.C.=50)	Organic material%
1	0.14	5.52	19.71	8.36
2	0.43	9.96	11.71	10.47
3	0.13	6.32	15.04	5.8
4	0.29	5.32	27.78	16.4
5	0.01	4.70	15.89	6.4
6	0.01	5.49	139.52	19.0
7	0.10	4.78	18.48	5.2
8	0.06	4.45	13.68	14.6

Table 4.4: Average concentrations of Pb, Cu, Cd and organic materials in Rusafa soil

No	Cd ppm (S.M.C.=1)	Cu ppm (S.M.C.=20)	Pb ppm (S.M.C.=50)	Organic materials%
1	0.15	23.82	27.77	5.9
2	0.13	6.49	15.10	6.1
3	0.11	7.71	29.32	6.2
4	0.08	5.96	17.65	9.3



Cd **Cu**

Pb

Figure 4.3: Concentration of Cd, Cu& Pb in Karkh and Rusafa

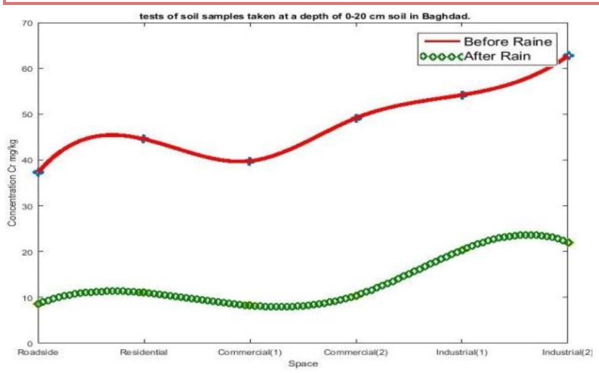
Table 4.5: Standard universal for concentration of heavy metals in soil [8]

Element	Standard Soils (mg/kg)	Critical Bound (mg/kg)	Pollution Soils (mg/kg)
Pb	50	150	600
Cd	1	5	20
Ni	50	100	500
Cr	100	250	800
Co	1	10	800
Zn	70	300	800
Cu	20	20	800
Fe	38000		

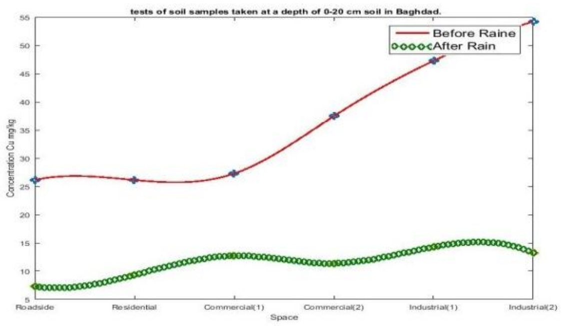
Finally, we test efficiency of suggested method for much samples selected from different zone in Baghdad with difference type of land (Commercial, Industrial, Residential, Roadside) to estimate the concentration of Cd, Cr, Cu, Fe, Mn, Ni, Pb, and Zn, before and after the rain to determine its effect. Then compared the results of suggested method with the laboratory inspecting to determine the accuracy and the practical results show the efficiency of suggested technique given in Table (4.6) and illustrated in Figure (4.4).

Table 4.6: The results of laboratory &suggested method for different zones in Baghdad

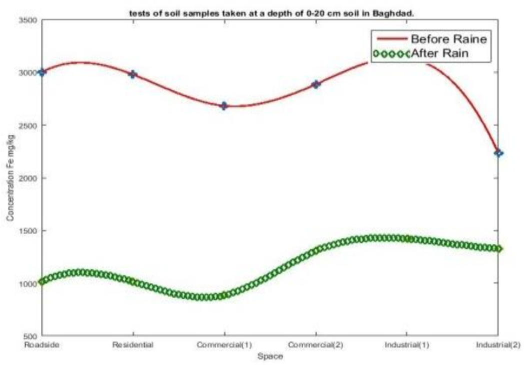
No	Site	PH	Cd (S.M.C.=1)	Cr (S.M.C.=1)	Cu (S.M.C.=1)	Fe (S.M.C.=1)	Mn (S.M.C.=1)	Ni (S.M.C.=1)	Pb (S.M.C.=1)	Zn (S.M.C.=1)
1	Roadside before rain	8.34	0.85	37.26	26.16	2998.41	581.32	155.76	400.60	149.74
-	Roadside after rain	6.89	ND	8.58	7.30	1013.13	79.00	39.20	25.34	13.70
2	Residential before rain	8.34	0.80	44.55	26.14	2977.93	541.17	136.50	450.60	78.00
-	Residential after rain	6.89	ND	11.04	9.30	1011.54	89.18	39.18	25.34	15.78
3	Commercial (1) before rain	8.30	1.00	39.76	27.28	2681.04	589.76	112.2	512.71	243.98
-	Commercial (1) after rain	6.89	0.03	8.23	12.72	884.73	153.26	36.09	27.27	18.94
4	Commercial (2) before rain	8.22	0.90	49.14	37.45	2884.73	470.60	110.15	682.71	101.60
-	Commercial (2)after rain	6.92	ND	10.40	11.36	1305.81	100.60	36.09	37.32	22.87
5	Industrial(1) before rain	8.31	1.86	54.18	47.25	3133.44	759.76	140.20	600.60	388.38
-	Industrial(1) after rain	6.78	0.05	20.38	14.29	1422.56	98.92	38.09	26.13	41.00
6	Industrial(2) before rain	8.21	1.96	62.72	54.25	2233.43	659.00	139.18	690.60	213.00
-	Industrial(2) after rain	6.67	0.04	21.98	13.25	1327.50	86.54	63.34	32.34	38.00



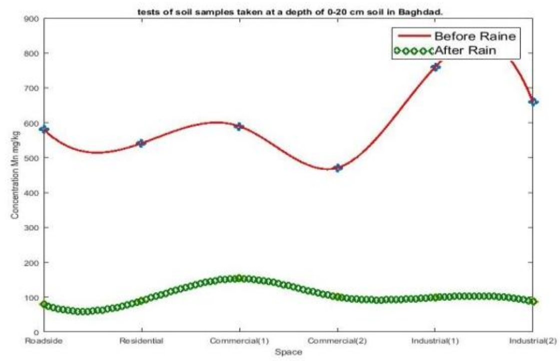
Cr



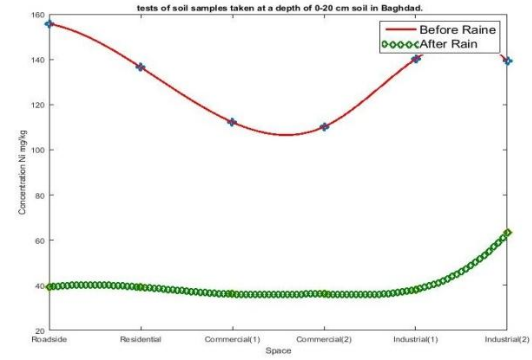
Cu



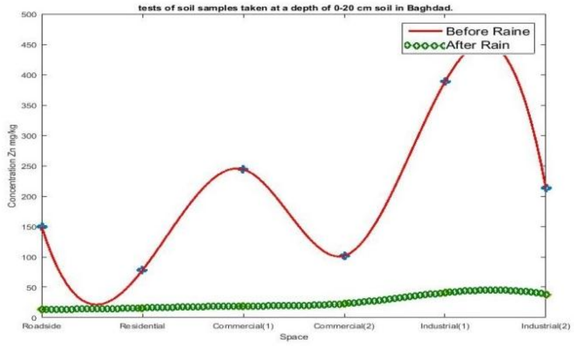
Fe



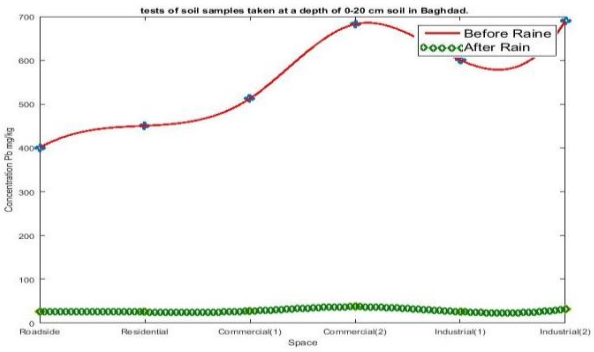
Mn



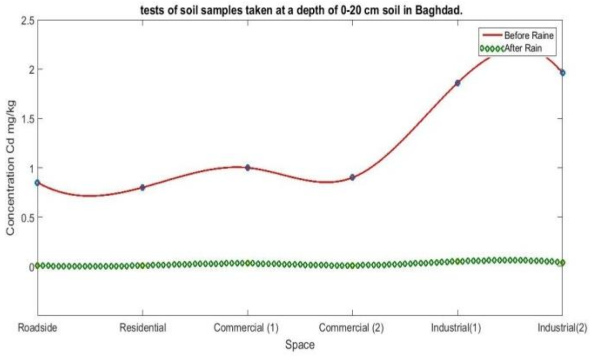
Ni



Zn



Pb



Cd

Figure 4.4: Results of suggested method for Baghdad city

To manifesting the effect of rainwater in contaminated soil we study and inspect pH in laboratory and suggested method to determine the accuracy and the results illustrated in Figure (4.5).

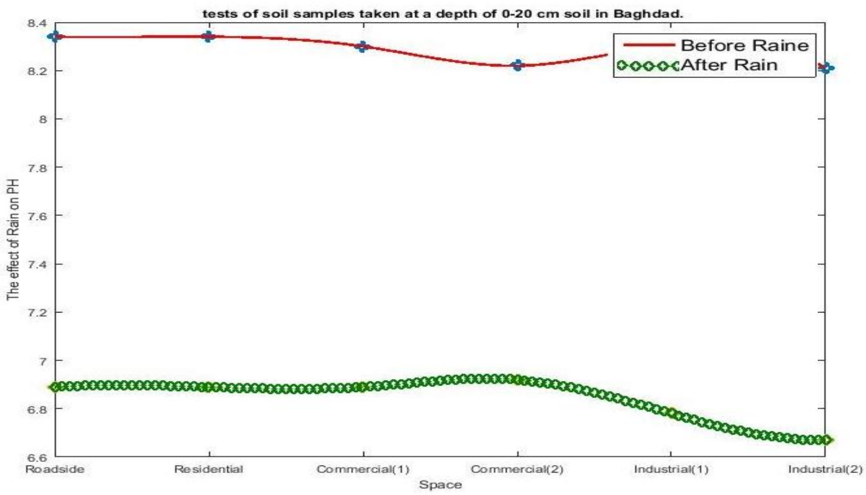


Figure 4.5: Estimate pH by suggested method

Table 4.7: The concentration (in ppm) of HM in Baghdad for 2015 -2016

Zone	Depth(cm)	Days	Pb	Cu	Zn	Ni	Cd	Fe	Mn	EC	PH
1	0-20	1	139.52	23.82	370	-	10	-	-	-	-
2	0-20	5	29.32	17.5	190	-	5	-	-	-	-
3	0-20	10	27.78	12.5	170	-	3	-	-	-	-
4	0-20	15	27.77	10	141	-	2	-	-	-	-
5	0-20	20	24.05	10	120	-	0.75	-	-	-	-
6	0-20	25	19.71	9.96	120	-	0.6	-	-	-	-
7	0-20	30	18.48	9.35	100	-	0.43	-	-	-	-
8	0-20	35	18.8	8.6	50	-	0.29	-	-	-	-
9	0-20	40	17.65	8.5	-	-	0.25	-	-	-	-
10	0-20	45	17.5	8.5	-	-	0.2	-	-	-	-
11	0-20	50	15.89	8	-	-	0.15	-	-	-	-
12	0-20	55	15.10	7.71	-	-	0.14	-	-	-	-
13	0-20	60	15.04	6.49	35	62.5	0.13	-	-	5700	8.25
14	0-20	65	13.68	6.32	24	57.5	0.11	-	-	5130	7.9
15	0-20	70	12	5.96	16	55	0.10	-	-	1300	7.9
16	0-20	75	11.71	5.52	15.5	54	0.08	-	-	868	7.7
17	0-20	80	11.6	5.32	15	43.5	0.06	-	-	803	7.68
18	0-20	85	11	4.78	14.5	36.5	0.05	-	-	346	7.6
19	0-20	90	10	4.70	11	27.3	0.05	-	-	335	7.44
20	0-20	95	10	4.49	10	27	0.05	-	-	321	7.17
21	0-20	100	4.8	4.45	9.5	26	0.01	-	-	260	7.05
22	0-20	105	2.25	0.2	0.8	20.5	0.01	-	-	97	7.0
23	0-20	121	81		47	-	0.85	-	-	-	-
24	0-20	125	90		59.1	-	1.1	-	-	-	-
25	0-20	130	101.2		63.2	-	1.5	-	-	-	-
26	0-20	135	125		71.5	-	1.7	-	-	-	-
27	0-20	140	150		78	-	1.86	-	-	-	-
28	0-20	145	192		85.5	-	1.96	-	-	-	-
29	0-20	150	300		89.5	-	2	-	-	-	-
30	0-20	155	340		90.5	-	2	-	-	-	-
31	0-20	160	410		98.2	-	2.6	-	-	-	-
32	0-20	305	690.60	54.25	388.38	155.76	8	-	-	2000	8.34
33	0-20	312	682.71	47.25	243.98	140.20	2.4	-	-	1165	8.34
34	0-20	319	600.60	37.45	213.00	139.18	1.5	-	-	576	8.31
35	0-20	326	512.71	28	149.74	136.50	1.00	-	-	515	8.30
36	0-20	333	450.60	27.28	123.70	112.2	0.90	-	-	485	8.22
37	0-20	340	400.60	26.16	101.60	110.15	0.80	-	-	471	8.21
38	0-20	347	80.3	26.14	78.00	92	0.7	-	-	369	8.03
39	0-20	354	72	18.35	41.00	67.5	0.65	-	-	356	7.3
40	0-20	361	65	17.5	38	66	0.25	-	-	263	7.27
41	0-20	368	51.1	15.65	37.7	63.34	0.05	3133.44	759.76	-	7.78
42	0-20	375	50	14.29	30.5	52	0.04	2998.41	659.00	-	7.73
43	0-20	382	39.5	13.25	25	47.75	0.03	2977.93	589.76	-	7.53
44	0-20	389	37.32	12.72	22.87	43.5	-	2884.73	581.32	-	7.47
45	0-20	396	34.5	12	20	39.20	-	2681.04	541.17	-	7.41
46	0-20	403	32.34	11.36	18.94	39.18	-	2233.43	470.60	-	6.92
47	0-20	410	27.27	11	18.8	38.09	-	1422.56	153.26	-	6.89
48	0-20	417	26.13	10.4	18	36.09	-	1327.50	100.60	-	6.89
49	0-20	424	25.34	9.7	15.78	36.09	-	1305.81	98.92	-	6.89
50	0-20	431	25.34	9.30	15	28.85	-	1013.13	89.18	-	6.78
51	0-20	438	7.65	7.30	9.7	25.6	-	1011.54	86.54	-	6.73
52	0-20	445	6.3	4	8.35	20.5	-	884.73	79.00	-	6.67
Mean	0-20		118.86	13.30	77.35	61.27	1.28	1989.5	350.75	1124.2	7.53

To illustrate the importance of suggested method economically and it's workable to estimate the concentration of heavy metals in any time and depth without using laboratory inspect we estimate the concentration of Cd, Cu, Pb, Cr, Ni, Fe, Co and Mn in Baghdad soil for the year 2017 by suggested method depending on the laboratory results of year 2016 and the results illustrated in Table (4.7) and Figure (4.6) with the value of pH illustrated in Figure (4.7), where the location of selected samples illustrated in Figure (4.8).

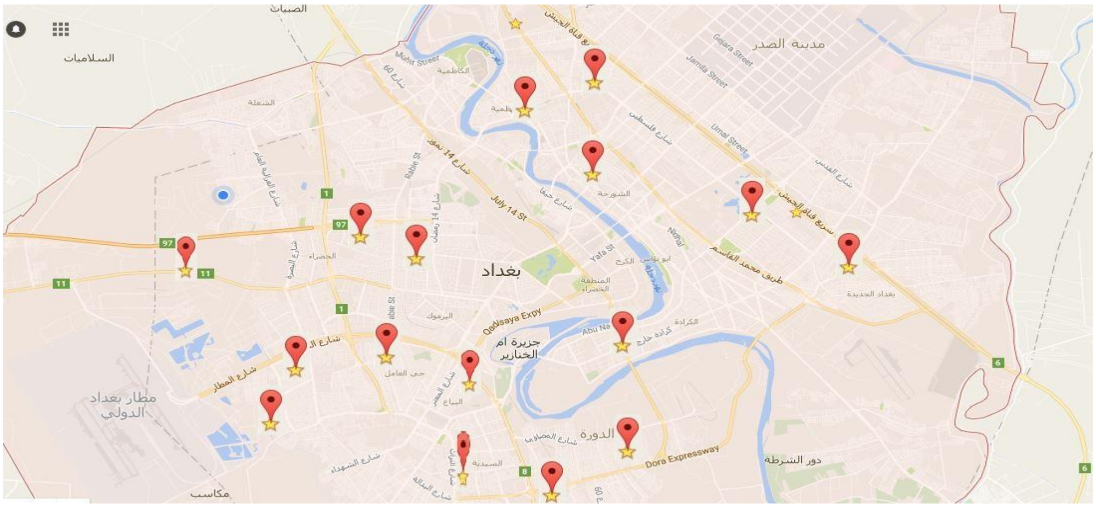
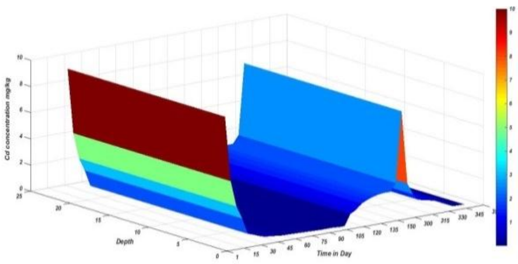
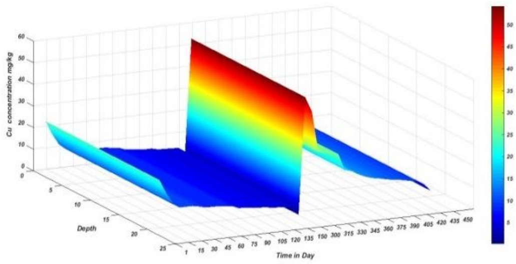


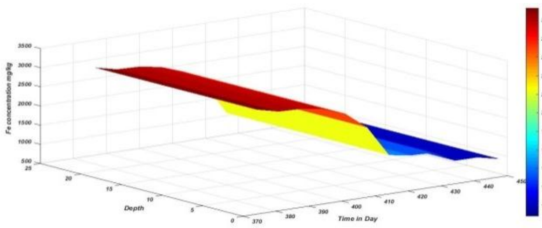
Figure 4.8: Location of samples in Babghdad



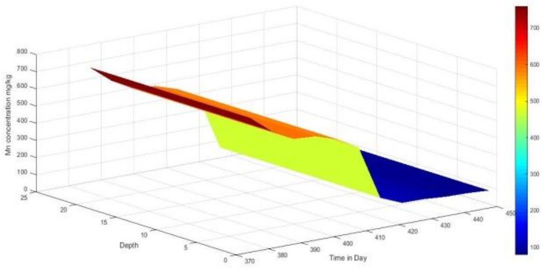
Cd



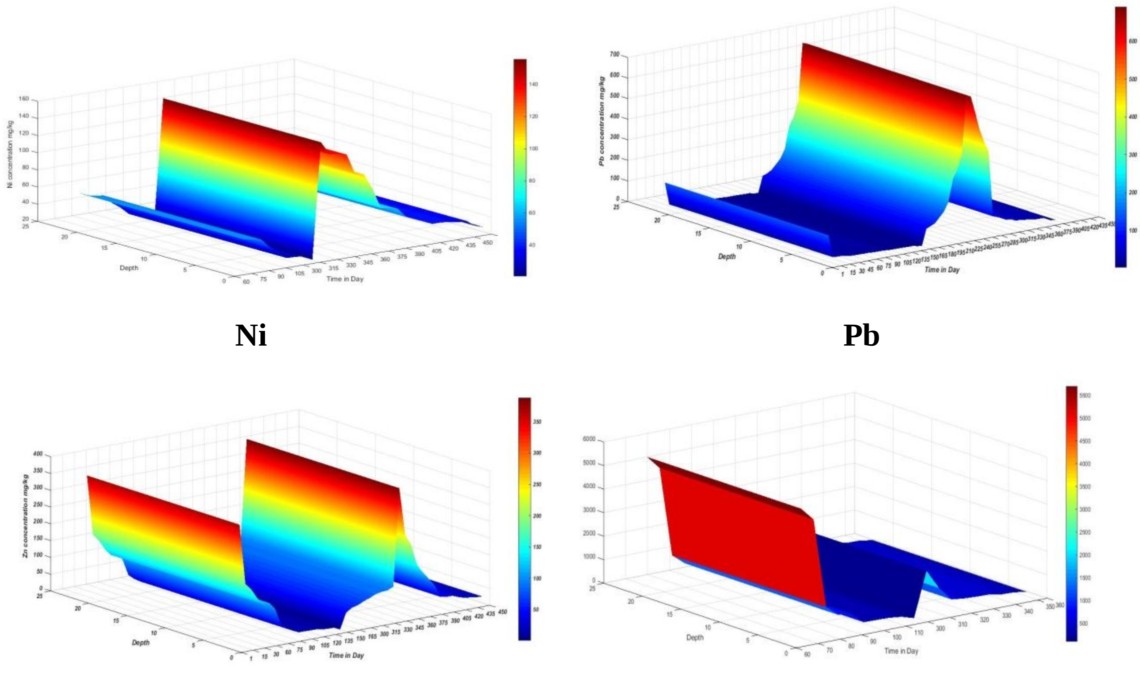
Cu



Fe



Mn



Ni **Pb**

Zn **Cr**

Figure 4.6: Concentration of HM in Baghdad soil 2017

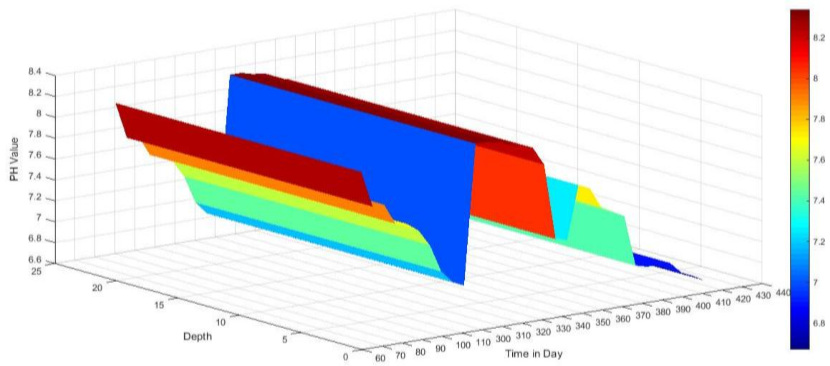


Figure 4.7: The value of pH for Baghdad soil 2017

Now, we discuss the stability of the suggested method and estimate the error to determine the accuracy of suggested method that is we give the practical application for the topics in chapter three. Table (4.9 – 4.15) computes the Defect weight, Richardson Extrapolation and global error for Pb, Cu, , Zn, Ni, Cd, Fe and Mn respectively in Baghdad soil.

Table 4.9: The accuracy of suggested method about concentration of Pb in Baghdad

Pb	(laboratory) Pb ppm	Interpolate Pb	Defect weight	Richardson Extrapolation	Higher order formulae
1	139.52	139.82	0.00213492741246797608881298035867	0.30000000000000006661338147750941	0.3
2	29.32	29.42	0.00329815303430079155672823218997	0.10000000000000002220446049250314	0.1
3	27.78	27.78	0	0	0
4	27.77	27.77	0	0	0
5	24.05	24.05	0	0	0
6	19.71	19.71	0	0	0
7	18.48	18.48	0	0	0
8	18.8	18.8	0	0	0
9	17.65	17.65	0	0	0
10	17.5	17.5	0	0	0
11	15.89	15.89	0	0	0
12	15.10	15.10	0	0	0
13	15.04	15.04	0	0	0
14	13.68	13.68	0	0	0
15	12	12	0	0	0
16	11.71	11.71	0	0	0
17	11.6	11.6	0	0	0
18	11	11	0	0	0
19	10	10	0	0	0
20	10	10.2	0.01818181818181818181818181818182	0.20000000000000004440892098500627	0.2
21	4.8	5.1	0.05172413793103448275862068965517	0.30000000000000006661338147750941	0.3
22	2.25	3.09	0.25846153846153846153846153846154	0.84000000000000018651746813702634	0.84
23	81	80.71	0.00865853658536585365853658536585	0.71000000000000015765166949677226	0.71
24	90	89.85	0.00164835164835164835164835164835	0.1500000000000000333066907387547	0.15
25	101.2	101.3	9.78473581213307240704500978473 58e-4	0.10000000000000002220446049250314	0.1
26	125	125	0	0	0
27	150	150	0	0	0
28	192	192	0	0	0
29	300	300.5	0.00166112956810631229235880398671	0.50000000000000011102230246251568	0.5
30	340	340.25	7.33137829912023460410557184750 73e-4	0.25000000000000005551115123125784	0.25
31	410	410	0	0	0
32	690.60	691	5.78368999421631000578368999421 63e-4	0.40000000000000008881784197001254	0.40
33	682.71	683	4.24156440596159190299980986090 59e-4	0.29000000000000006439293542825909	0.29
34	600.60	601	6.64893617021276595744680851063 83e-4	0.40000000000000008881784197001254	0.40
35	512.71	513	5.64520838605438866286426193766 91e-4	0.29000000000000006439293542825909	0.29
36	450.60	451	8.85739592559787422497785651018 6e-4	0.40000000000000008881784197001254	0.40
37	400.60	401	9.96015936254980079681274900398 41e-4	0.40000000000000008881784197001254	0.40
38	80.3	80	0.0036900369003690036900369	0.30000000000000006661338147750941	0.3
39	72	71.6	0.00547945205479452054794520547945	0.40000000000000008881784197001254	0.4
40	65	64.7	0.00454545454545454545454545454545	0.30000000000000006661338147750941	0.3
41	51.1	52	0.01727447216890595009596928982726	0.90000000000000019984014443252822	0.9
42	50	49.6	0.0078431372549019607843137254902	0.40000000000000008881784197001254	0.4
43	39.5	40	0.01234567901234567901234567901235	0.50000000000000011102230246251568	0.5
44	37.32	37.32	0	0	0
45	34.5	34.5	0	0	0
46	32.34	32.34	0	0	0
47	27.27	27.27	0	0	0
48	26.13	26.13	0	0	0
49	25.34	25.34	0	0	0
50	25.34	25.34	0	0	0
51	7.65	8	0.04046242774566473988439306358382	0.35000000000000007771561172376098	0.35
52	6.3	7.1	0.10958904109589041095890410958904	0.80000000000000017763568394002509	0.8

Table 4.10: The accuracy of suggested method about concentration of Cu in Baghdad

Cu	(laboratory) Cu ppm	Interpolate Cu	Defect weight	Richardson Extrapolation	Higher order formulae
1	23.82	23.82	0	0	0
2	17.5	17.5	0	0	0
3	12.5	12.5	0	0	0
4	10	10	0	0	0
5	10	10	0	0	0
6	9.96	9.96	0	0	0
7	9.35	9.35	0	0	0
8	8.6	8.6	0	0	0
9	8.5	8.5	0	0	0
10	8.5	8.5	0	0	0
11	8	8	0	0	0
12	7.71	7.71	0	0	0
13	6.49	6.49	0	0	0
14	6.32	6.32	0	0	0
15	5.96	5.96	0	0	0
16	5.52	5.52	0	0	0
17	5.32	5.32	0	0	0
18	4.78	4.78	0	0	0
19	4.70	4.70	0	0	0
20	4.49	5	0.092896174863387978142076502732224	0.51000000000000579802872380307667 6	0.51
21	4.45	5	0.1009174311926605504587155963302 8	0.55000000000000625277607468959249 4	0.55
22	0.2	0.12	0.083333333333333333333333333333333	0.10000000000000113686837721628954 4	0.1
23	54.25	55	0.013574660633484162895927601809 95	0.75000000000000852651282912217158 3	0.75
24	47.25	47.15	0.0020725388601036269430051813471 5	0.10000000000000113686837721628954 4	0.1
25	37.45	37.25	0.0052015604681404421326397919375 8	0.20000000000000227373675443257908 9	0.2
26	28	27	0.0344827586206896551724137931034 5	1.00000000000001136868377216289544	1
27	27.28	27.6	0.0113154172560113154172560113154 2	0.32000000000000363797880709212654 2	0.32
28	26.16	36.8	0.02356406480117820324005891016 2	0.64000000000000727595761418425308 4	0.64
29	26.14	26.01	0.0047899778924097273397199705232 1	0.13000000000000147792889038117640 8	0.13
30	18.35	18.1	0.0129198966408268733850129198966 4	0.25000000000000284217094304072386 1	0.25
31	17.5	17.2	0.0162162162162162162162162162162 2	0.30000000000000341060513164886863 3	0.3
32	15.65	16	0.0210210210210210210210210210210 2	0.35000000000000397903932025701340 5	0.35
33	14.29	14.08	0.0137344669718770438194898626553 3	0.21000000000000238742359215420804 3	0.21
34	13.25	13.25	0	0	0
35	12.72	12.72	0	0	0
36	12	12	0	0	0
37	11.36	11.36	0	0	0
38	11	11	0	0	0
39	10.4	10.4	0	0	0
40	9.7	9.7	0	0	0
41	9.30	9.30	0	0	0
42	7.30	7.30	0	0	0
43	4	4	0	0	0

Table 4.11: The accuracy of suggested method about concentration of Zn in Baghdad

Zn	(laboratory) Zn ppm	Interpolate Zn	Defect weight	Richardson Extrapolation	Higher order formulae
1	370	370.3	8.0862533692722371967654986522911e-4	0.30000000000000106581410364015407	0.3
2	190	190.21	0.00109947643979057591623036649215	0.21000000000000074606987254810785	0.21
3	170	170.2	0.00116959064327485380116959064327	0.20000000000000071054273576010271	0.2
4	141	141.5	0.00352112676056338028169014084507	0.50000000000000177635683940025678	0.5
5	120	120.12	9.9173553719008264462809917355372e-4	0.12000000000000042632564145606163	0.12
6	120	120.11	9.0909090909090909090909090909091e-4	0.11000000000000039079850466805649	0.11
7	100	100.14	0.00138613861386138613861386138614	0.1400000000000004973799150320719	0.14
8	50	50.1	0.00196078431372549019607843137255	0.10000000000000035527136788005136	0.1
9	35	35.09	0.0025		0.09
10	24	24	0	0	0
11	16	16	0	0	0
12	15.5	15.5	0	0	0
13	15	15	0	0	0
14	14.5	14.5	0	0	0
15	11	11	0	0	0
16	10	10	0	0	0
17	9.5	9.5	0	0	0
18	0.8	0.72	0.044444444444444444444444444444444	0.08000000000000028421709430404108	0.08
19	47	46.5	0.010416666666666666666666666666667	0.50000000000000177635683940025678	0.5
20	59.1	59.6	0.00831946755407653910149750415973	0.50000000000000177635683940025678	0.5
21	63.2	63.2	0	0	0
22	71.5	71.3	0.00275862068965517241379310344828	0.20000000000000071054273576010271	0.2
23	78	78	0	0	0
24	85.5	85.5	0	0	0
25	89.5	89.5	0	0	0
26	90.5	90.5	0	0	0
27	98.2	98.6	0.00403225806451612903225806451613	0.40000000000000142108547152020542	0.4
28	388.38	388.12	6.6772818326570445323334531819816e-4	0.26000000000000092370555648813352	0.26
29	243.98	244.9	3.2655726998122295697607967997388e-4	0.08000000000000028421709430404108	0.08
30	213.00	212.8	9.345794392523364485981308411215e-4	0.20000000000000071054273576010271	0.2
31	149.74	149	0.00490911503250630224227146079342	0.74000000000000262900812231238003	0.74
32	123.70	124.70	0.00801924619085805934242181234964	1.0000000000000035527136788005136	1
33	101.60	101.2	0.00389863547758284600389863547758	0.40000000000000142108547152020542	0.4
34	78.00	78.8	0.01012658227848101265822784810127	0.08000000000000028421709430404108	0.8
35	41.00	41.00	0	0	0
36	38	38	0	0	0
37	37.7	37.7	0	0	0
38	30.5	31.4	0.02857142857142857142857142857143	0.9000000000000031974423109204622	0.9
39	25	25	0	0	0
40	22.87	22.87	0	0	0
41	20	20	0	0	0
42	18.94	18.94	0	0	0
43	18.8	18.8	0	0	0
44	18	18	0	0	0
45	15.78	15.78	0	0	0
46	15	15	0	0	0
47	9.7	9.7	0	0	0
48	8.35	8.35	0	0	0

Table 4.12: The accuracy of suggested method about concentration of Ni in Baghdad

Ni	(laboratory) Ni ppm	Interpolate Ni	Defect weight	Richardson Extrapolation	Higher order formulae
1	62.5	62.5	0	0	0
2	57.5	27.5	0	0	0
3	55	54.5	0.008928571428571428571428571428571428	0.5000000000232830643762289846205 29	0.5
4	54	54.3	0.01272727272727272727272727272727	0.700000000325962901267205784687 4	0.7
5	43.5	43.5	0	0	0
6	36.5	36.5	0	0	0
7	27.3	27	0.010600706713780918727915194346 29	0.300000000139698386257373907723 17	0.3
8	27	27.4	0.014285714285714285714285714285 71	0.400000000186264515009831876964 23	0.4
9	26	25.6	0.014814814814814814814814814814 81	0.400000000186264515009831876964 23	0.4
10	20.5	21.3	0.037209302325581395348837209302 33	0.800000000372529030019663753928 46	0.8
11	155.76	155.7	3.82750701709619800969635110997 7e-4	0.060000000027939677251474781544 63	0.06
12	140.20	139.2	0.007082152974504249291784702549 58	1.000000000465661287524579692410 6	1
13	139.18	140	0.005849621914681124268797260664 86	0.820000000381842255770155347776 67	0.82
14	136.50	136.8	0.002197802197802197802197802197 8	0.300000000139698386257373907723 17	0.3
15	112.2	112.5	0.002650176678445229681978798586 57	0.300000000139698386257373907723 17	0.3
16	110.15	110.02	0.001169590643274853801169590643 27	0.130000000060535967378195360013 37	0.13
17	92	92.4	0.004301075268817204301075268817 2	0.400000000186264515009831876964 23	0.4
18	67.5	67.5	0	0	0
19	66	66	0	0	0
20	63.34	63.44	0.001554243083618277898663350948 09	0.100000000046566128752457969241 06	0.1
21	52	52.4	0.007547169811320754716981132075 47	0.400000000186264515009831876964 23	0.4
22	47.75	47.75	0	0	0
23	43.5	43.5	0	0	0
24	39.20	39.20	0	0	0
25	39.18	39.18	0	0	0
26	38.09	38.09	0	0	0
27	36.09	36.09	0	0	0
28	36.09	36	0.002426530062011323806956052844 43	0.090000000041909515877212172316 95	0.09
29	28.85	29	0.005025125628140703517587939698 49	0.150000000069849193128686953861 59	0.15
30	25.6	26	0.015037593984962406015037593984 96	0.400000000186264515009831876964 23	0.4
31	20.5	20	0.023255813953488372093023255813 95	0.500000000232830643762289846205 29	0.5

Table 4.13: The accuracy of suggested method about concentration of Cd in Baghdad

Cd	(laboratory) Cd ppm	Interpolate Cd	Defect weight	Richardson Extrapolation	Higher order formulae
1	10	10	0	0	0
2	5	5	0	0	0
3	3	3	0	0	0
4	2	2	0	0	0
5	0.75	0.8	0.028571428571428571428571428571428571 43	0.0500000000232830643762289846205 3	0.05
6	0.6	0.5	0.0625	0.10000000004656612875245796924106	0.1
7	0.43	0.43	0	0	0
8	0.29	0.29	0	0	0
9	0.25	0.25	0	0	0
10	0.2	0.2	0	0	0
11	0.15	0.15	0	0	0
12	0.14	0.14	0	0	0
13	0.13	0.13	0	0	0
14	0.11	0.11	0	0	0
15	0.10	0.10	0	0	0
16	0.08	0.08	0	0	0
17	0.06	0.06	0	0	0
18	0.05	0.05	0	0	0
19	0.05	0.05	0	0	0
20	0.05	0.05	0	0	0
21	0.01	0.01	0	0	0
22	0.01	0.01	0	0	0
23	0.85	1	0.081081081081081081081081081081081 08	0.15000000006984919312868695386159	0.15
24	1.1	1.6	0.238095238095238095238095238095 24	0.50000000023283064376228984620529	0.5
25	1.5	1.41	0.036	0.09000000004190951587721217231695	0.09
26	1.7	1.7	0	0	0
27	1.86	1.86	0	0	0
28	1.96	1.96	0	0	0
29	2	2	0	0	0
30	2	2.1	0.03333333333333333333333333333333	0.10000000004656612875245796924106	0.1
31	2.6	2.3	0.08333333333333333333333333333333	0.30000000013969838625737390772317	0.3
32	8	8.2	0.02222222222222222222222222222222	0.20000000009313225750491593848212	0.2
33	2.4	2.3	0.02941176470588235294117647058824	0.10000000004656612875245796924106	0.1
34	1.5	1.7	0.08	0.20000000009313225750491593848212	0.2
35	1.00	0.9	0.05	0.10000000004656612875245796924106	0.1
36	0.90	1	0.05263157894736842105263157894737	0.10000000004656612875245796924106	0.1
37	0.80	0.75	0.02777777777777777777777777777778	0.05000000002328306437622898462053	0.05
38	0.7	0.7	0	0	0
39	0.65	0.65	0	0	0
40	0.25	0.25	0	0	0
41	0.05	0.05	0	0	0
42	0.04	0.04	0	0	0
43	0.03	0.04	0.00970873786407766990291262135922	0.01000000000465661287524579692411	0.01

Table 4.14: The accuracy of suggested method about concentration of Fe in Baghdad

Fe	(laboratory) Fe ppm	Interpolate Fe	Defect weight	Richardson Extrapolation	Higher order formulae
1	3133.44	3133.84	1.2761450211202000995393116473756e-4	0.4000976800976800976800976800976 8	0.4
2	2998.41	2998.41	0	0	0
3	2977.93	2977.93	0	0	0
4	2884.73	2884.73	0	0	0
5	2681.04	2681	1.4914020670832649774052586836885e-5	0.0400097680097680097680097680097 7	0.04
6	2233.43	2233	1.9244281539363506576621330719691e-4	0.43010500610500610500610500610501	0.43
7	1422.56	1422.95	2.8801034027368006968445306133918e-4	0.41010012210012210012210012210012	0.41
8	1327.50	1327.01	3.6883703424915318027850959729018e-4	0.49011965811965811965811965811966	0.49
9	1305.81	1305.2	3.7495886930770349170881765520619e-4	0.49011965811965811965811965811966	0.49
10	1013.13	1013.93	6.0150079378383441965034068610533e-4	0.61014896214896214896214896214896	0.61
11	1011.54	1011	5.3331226420684614928792936575345e-4	0.54013186813186813186813186813187	0.54
12	884.73	884.03	7.9030855904169442155058539283 98e-4	0.70017094017094017094017094017094	0.7

Table 4.15: The accuracy of suggested method about concentration of Mn in Baghdad

Mn	(laboratory) Mn ppm	Interpolate Mn	Defect weight	Richardson Extrapolation	Higher order formulae
1	759.76	759	9.99000999000999000999000999001e-4	0.760185592185592185592185592185559	0.76
2	659.00	659.8	0.00119402985074626865671641791045	0.800195360195360195360195360195536	0.8
3	589.76	590	4.06256347755433678651228925451196e-4	0.240058608058608058608058608058605861	0.24
4	581.32	582	0.00116774282181618354169528781426	0.680166056166056166056166056166056606	0.68
5	541.17	542	0.00153046172001770172591827703201	0.830202686202686202686202686202686620269	0.83
6	470.60	470	0.00127226463104325699745547073791	0.600146520146520146520146520146520146652	0.60
7	153.26	153.9	0.00414883962141838454557241021652	0.640156288156288156288156288156288815629	0.64
8	100.60	100.1	0.00492125984251968503937007874016	0.500122100122100122100122100122100121001221	0.5
9	98.92	98.92	0	0	0
10	89.18	89.18	0	0	0
11	86.54	86.54	0	0	0
12	79.00	79.00	0	0	0

4.3. Results and Discussion

The practical results of suggested method shown as follows :

1. The average of the concentrations of heavy metals in soil for different zones in Baghdad city decreases after the rain that is, the effect of rain is clear, especially, if pH of the rain is near or equal to 0.5.
2. The average of the concentrations of heavy metals in soil for different zones in Baghdad city increases with time, posing a great risk to the environment contamination.
3. For the comparison among the concentrations of various regions: residential, industrial, commercial and agricultural regions, it's found that:

soils agricultural < soils residential < soils commercial < soils industrial

that is, the agricultural regions are the lowest while the industrial regions are the highest for the concentrations of heavy metals.

4. For comparing upon the depth of soil, we see the effect of depth on the concentration of heavy metals, that is, while the depth increase, The concentrations of heavy metals are decrease, i.e., the concentration of heavy metals in depth (0–20) cm is larger than the concentration in depth (20–40) cm for the same soil.
5. There are different causes for increasing the concentrations of heavy metals in soil such as: the big traffic jams resulting from the great number of cars lately

which use gasoline that contains a lot of Fourth Lead Ethylene which cause big problem to the environment. This creates dangers to human beings. In addition, the increase in the amount of litter and how to get rid of industry waste in sewerage and the decrease in the green region which participate in lessening the damage of waste on the environment.

6. From Figure (4.6) we see that:

❖ For spreading the metallic element Cu, represents Peakons wave that rise or descend from one straight line state to by days a different and approaches a relentless at days in year.

❖ For spreading the Pb, represents kink waves in the first part which defined on the interval [2016, 2017] and represents peak on waves in the mid defined (November) on remainder interval.

❖ For spreading the Zn, represents Peakons are peaked solitary wave solution, in this case, the wave resolution is sleek aside from a peak at a corner of its crest or for bottom. Peakons are the points at that abstraction by-product changes sign in order that peakons have a finite jump in figuring of the answer C(x,t), this suggests that peakons have discontinuities within the x-derivative however each one-sided derivatives exist and dissent only by a signal.

❖ For spreading the Ni, represents kink waves within the 1st half that outlined on the interval (60-375) days and represents peak on waves within the second half outlined on remainder interval.

- ❖ For spreading the Cd, represents first kink wave's high focus on (1-80) days and second in the non-rainy months are cuspons.
- ❖ For spreading the Fe, represents kink waves.
- ❖ For spreading the Mn, represents kink waves.
- ❖ For spreading the PH, represents Periodic solutions are travelling wave solutions.

CHAPTER FIVE

CONCLUSIONS

&

FUTURE WORK

CHAPTER Five

Conclusions &Future Works

5.1. Conclusions

This thesis demonstrates Simulink concept and used it to design a model equation that describes the effect of rainwater in contaminated soil by estimate the concentration of heavy metals in soil for any depth and time. . Thus the results provided many of the features, such as:

1- The Simulink model can be considered to be a good representation of that estimate the concentrations of heavy metals in the soil.

2- The practical results showed that the predictive models established in this thesis are much more efficient compared to those established by the traditional methods.

3- Simulink environment requires very good programming skills.

4- Inexpensive because the existing compilers will be used for development.

5- Source code is available; therefore it can be modified and upgraded at no cost.

6- The program is usually written for an obsessive simulation problem.

7- As a result of increase in the population during the late years which results in converting the regions of vegetation to residential regions and the technological development which causes contamination because of the prolife ration of plants and workshops scattered everywhere. Add to all this, wars and their great contamination which are considered the most dangerous contaminates of the soil and environment.

8- All these types of contaminates cause high rate of concentration of waste which exceeds the normal amount in soil, the increase of these metals has different types of danger on human health.

9- The plants absorb these dangerous materials which in its turn go to human beings through food consumption which they acquire because of eating these plants that have the dangerous metals.

5.2. Future Works

The studies for future investigations are suggested below:

- Used of Simulink environment in applied studies is a simple and high-precision technique.
- Simulink environment can be used to solve many types of Differential equation.
- The error analysis and convergence for Simulink environment must be student in future.
- Simulink environment may be used to solve delay differential equation.
- Simulink environment may be used to solve stiffness problem.

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APPENDIX

Appendix

Contamination

1. Introduction

This appendix consists of a quick summary of environmental contamination, soil, heavy metals that deal with throughout this thesis.

2. Environment Contamination

A contamination is defined typically as the presence of any material within the surroundings in quantities that results directly or indirectly, alone or reacting with different materials, harmful effects on health of organisms [5].

Environmental impact on human health is that the most blatant side that's thought-about in each study, as well as the standard of life and what changes within the setting and dangerous on human life, victimisation biological, physical, chemical, and psychosocial factors to indicate damage however way it will go.

The three main components that researchers partition the setting area unit water, air, and soil, all of them represent a good field that wants extended studies to be lined because it deserves. At the tip the three main components don't seem to be separated and any sort of pollution in anyone of those three components can have an effect on the opposite two components.

3. Soil Contamination

The contamination in soil, is defined as the presence of metals however they build up and their movement quickness within the soil that depends on several factors. Some contaminants endure chemical changes specially those organic (carbon- based), they additionally could degrade within the soil which can modification their toxicity negatively or absolutely, compared with the initial compound. Some chemical components (such as metals) could have some changes in their characteristics, though they cannot break down, however these changes would facilitate to form them uptake by plants and animals simply.

Different contaminants vary in their tendency to:

♦ find yourself in water command within the soil or within the underlying groundwater (by natural action through the soil).

♦ volatilise (evaporate) into the air; or

♦ Bind tightly to the soil.

The characteristics of the soil additionally have an effect on the fate of contaminants and whether or not they may be without delay concerned by plants or animals. web site management and land use (such as farming practices) will have an effect on some soil characteristics. Vital soil characteristics which will have an effect on the behavior of contaminants include:

♦ Soil geology and clay content (soil texture)

♦ pH (acidity) of the soil;

♦ quantity of organic matter within the soil;

♦ wet levels;

♦ Temperature; and

♦Presence of different chemicals [2].

During this thesis, we have a tendency to target soil contamination by heavy metals.

4. What are the Heavy Metals? ([6], [3])

Means all metals that increase density of soil by 5 g/cm3, and at density less it is referred to as lightweight metals. A number of these metals play a very important role within the lives of the living and have totally different biological effects. For instance the iron well-known enzymes within the blood and also the installation of importance area unit all of the elements (Mn), Ze, Pb and Co protein catalysts. However, these metals area unit harmful and dangerous to be in bound concentrations. Adding to it the seriousness of those metals, It is unacceptable to be analyzed by bacterium and alternative natural processes. Moreover because the legitimacy of that modify them to cover long distances for ton or sources sites.

May be the foremost dangerous factor is owing to status to every bio-accumulate within the tissues and organs of living organisms within the surroundings water or land. Additionally, some significant metals function hot isotopes, therefore, these metals are going to be charged double the danger to the surroundings in terms of being harmful and hot at a similar time, as is that the case in sixty five of hot Zn, U 235.

Heavy metal contamination in soil might create risks and hazards to people at large. Excessive concentrations of some heavy metals in biological systems, particularly animals (human beings in particular) area unit extremely dangerous to human health, and should even cause death. for instance, heavy metals such as: cadmium (Cd), Nickel (Ni) and Arsenic (As) area unit malignant neoplastic disease. Table (A.1) provides an outline of some dangerous significant metals that area unit usually gift in farm soils and their health impacts to human beings [4].

Table A.1: Some dangerous HMs and their health impacts to human beings[1]

Heavy Metal	Health Impact/s
Pb	Mental lapse or even death
Cr	Allergic dermatitis
As	Skin damage, cancer, affects kidney and central nervous system
Zn	Zinc shortages can cause birth defects
Cd	Affects kidney, liver and GI tract
Cu	Anemia, liver and kidney damage, and stomach/intestinal irritation
Ni	Various kinds of cancer

5. The Importance of Water Harvest Home

Rain collected from rooftops and native establishments will create a vital contribution to the availability of potable water. And contributes to the improvement of the extent of groundwater and also the increase of inexperienced areas in urban areas. Water collected from the surface of the planet, generally from specially ready areas.

Rainwater harvest home systems is cheap native materials and square measure doubtless to achieve success in most housing sites. Combined water might not be directly drinkable and should have to be compelled to be treated before consumption. as a result of their mixed with some pollutants, like mercury from burning coal from near buildings, or bird stool.

Rain surpluses are wont to enhance the extent of groundwater. This method is termed "recharge of groundwater"

6. Rainwater Options

There are many options of rainwater such:

- Doesn't contain deposits and high percentages of salts.
- Don't contain disinfectants like chlorine et al.
- It contains high nitrogen levels, that are required for growth.
- Will be accustomed drink directly.

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APPENDIX

References Appendix

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المستخلص

الهدف من هذه الرسالة هو تصميم نموذج سميولنك لتخمين تأثير مياه الأمطار في التربة الملوثة بالمعادن الثقيلة من خلال تخمين تركيز النحاس ،الرصاص ،الزنك ،الكادميوم ،الكوبالت ، النيكل ، الكروم والحديد. لذا، يمكن اتخاذ هذا النموذج كطريقة لتحديد مستوى التلوث من قبل بعض العناصر الثقيلة مع امكانية تحديد مصدر التلوث وتأثيرهم.

ويقترح نهج جديد لتخمين معلمة التشتت على أساس بيئة سميولنك.

الهدف من استخدام بيئة سميولنك هو حل المعادلات التفاضلية بشكل منفرد وكبيانات معطاة بالتوازي مع نهج الرياضيات التحليلي. بشكل عام، تم صياغة و حل نماذج رياضية لانتشار المعادن الثقيلة في التربة وتأثير مياه الأمطار عليها للتنبؤ بسلوك النظام في ظل شروط مختلفة.

الرسالة تقترح تقنية تستخدم لحل نموذج معادلة باستخدام بيئة سميولنك . الكود الرئيسي الذي استخدم وقدم هو MATLAB/ode45 لتمكين حل نموذج معادلة وتجربة استجابة النظم الهندسية بشروط تطبيقية مختلفة.

أخيرا، قورنت أداء النموذج المقترح مع الطرق التقليدية، في تخمين تركيز المعادن الثقيلة قبل وبعد المطر، ثم تطبيقها على التربة في بغداد بعد تصنيفها إلى مناطق مختلفة مثل: سكني، صناعي، تجاري، زراعي والطرق الرئيسية. ثم قورن نتائج الطرق المقترحة مع النتائج التي تم الحصول عليها من قبل الفحص المختبري باستخدام جهاز الأشعة السينية لتحديد نسبة الدقة. نتائج هذا العمل اثبتت أن النموذج المقترح يمكن تطبيقه بنجاح لتخمين سريع و دقيق لتركيز المعادن الثقيلة.

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تحديد تأثير مياه الأمطار في التربة الملوثة في مدينة بغداد باستخدام الطرق الرياضية

رسالة
مقدمة الى كلية التربية للعلوم الصرفة / ابن الهيثم – جامعة بغداد
وهي جزء من متطلبات نيل شهادة ماجستير علوم
في الرياضيات

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