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Modified Numerical Simulation Technique for Solving Nonlinear Epidemic Models

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Prepared by

Mahdi Abdulridha Sabaa

Supervised by

Asst. Prof. Dr. Maha Abduljabbar Mohammed

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بسم الله الرحمن الرحيم

((وَلَمَّا دَخَلُواْ مِنْ حَيْثُ أَمَرَهُمْ أَبُوهُم مَّا كَانَ يُغْنِي عَنْهُم مِّنَ اللَّهِ مِن شَيْءٍ إِلاَّ حَاجَةً فِي نَفْسِ يَعْقُوبَ قَضَاهَا وَإِنَّهُ لَذُو عِلْمٍ لِّمَا عَلَّمْنَاهُ وَلَكِنَّ أَكْثَرَ النَّاسِ لاَ يَعْلَمُونَ ﴿٦٨﴾))

صدق الله العلي العظيم

سُورَة يُوسُف - الآية (٦٨)

الاهداء

الى من علمني النجاح و الصبر . . الى من دعائه هو سر نجاحي . . أبي الغالي

الى من تتسابق الكلمات لتخرج معبره عن مكنونها . . الى من افتقدها

اليوم وهي حاضره في كل خطواتي نحو النجاح . . أمي الغالية

الى من أحبهم وأعشقهم . . سر سعادتي . . زوجتي وأطفالي

الى من علمني الكثير استاذتي . . أ . م . د مها عبد الجبار

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الى من تذوقت معهم أجمل اللحظات . . أخوتي

الى الشموع التي تحترق لتضيئ الاخرين . . زملائي و زميلاتي

الى كل من علمني حرفا اصبح شمعة مضيئة تضيء لي الطريق أمامي

أساتذتي

اهدي لكم هذا الجهد . . في كل تواضع

مع خالص تقديري

مهدي

Supervisor's Certification

I certify that this thesis was prepared under my supervision at the department of mathematics, college of education for pure science / Ibn Al-Haitham, university of Baghdad as a partial fulfillment of the requirements for the degree of master of science in mathematics.

Signature:

Name: **Asst. Prof. Dr. Maha Abduljabbar Mohammed**

Date: 17 / 11 / 2019

In view of the available recommendation, I forward this thesis for debate by the examining committee.

Signature: Y. Y. Yousif

Name: **Asst. Prof. Dr. Yousif Yaqoub Yousif**

Head of Department of Mathematics

Date: 20 / 11 / 2019

Committee Certification

We certify that we have read this thesis entitled " **Modified Numerical Simulation Technique for Solving Nonlinear Epidemic Models** " and as examining committee, we examined the student **(Mahdi Abdulrida Sabaa)** in its contains and what is connected with it, and that in our opinion it is adequate for the partial fulfillment of the requirement for the degree of Master of Science in Mathematics.

(Member and Supervisor)

Signature:

Name: **Asst. Prof. Dr. Maha A. Mohammed**

Date: 17/11/ 2019

(Member)

Signature:

Name: **Asst. Prof. Dr. Basim N. Abood**

Date: 17 / 11/ 2019

(Chairman)

Signature: A. N. Salman

Name: **Prof. Abbas Najim Salman**

Date: 17 /11/ 2019

(Member)

Signature: Ghada

Name: **Dr. Ghada Hassan Ibraheem**

Date: 17/11 / 2019

Approved by the University Committee of graduate studies.

Signature: Firas A. Alsharidy

Name: **Asst. Prof. Dr. Firas Abdulhameed Abdullatif**

The dean of Education College for Pure Science/ Ibn AL Haitham University of Baghdad

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ABSTRACT

The aim of this thesis is to solve the nonlinear autonomous system of initial value problem (IVP) for ordinary differential equations (ODE) of the first order that has multi variables and multi parameters, these parameters are random variables. This study uses a modified numerical simulation process that is more suitable to solve some models. The new approach mixes between a random process which is Monte Carlo technique and a numerical method which is Runge-Kutta (RK). The new process is called Mean Monte Carlo Runge-Kutta (MMCRK). It is applied to solve two epidemic models which are alcohol consumption model and smoking habit model under study. Four approximate methods which are Analytic methods, Adomian decomposition (ADM) method, Variational iteration method (VIM), and Numerical methods, finite difference (FD) method and Rung-Kutta of 4^{th} order method (RK4) are applied on the two models under study in this thesis to verify the solutions of these models. The difference measure error and mean square error are used for comparison between the numerical simulation solutions of modified method MMCRK and the predicted values of the previous study. The comparison between MMCRK and Mean Monte Carlo Finite Difference (MMCFD) that was used in one study, numerical simulation methods has been made, the MMCRK method has been approached to the predicted values of the previous studies with the alcohol consumption and smoking habit models. Three softwares are used for computing the presented results in this thesis which are Mathematica.11 and Matlab softwares 2013, the figures have been sketched by the Magic Plot software.

PUBLICATIONS AND SUBMITED PAPERS

Publication Papers:

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LIST OF SYMBOLS AND ABBREVIATIONS

ADM	Adomian Decomposition Method
BFD	Backward Finite Difference Method
BVP	Boundary Value Problem
CFD	Central Finite Difference Method
Eq.	Equation
Eqs.	Equations
IVP	Initial Value Problem
FD	Finite Difference Method
ODEs	Ordinary Differential Equations
RK	Runge-Kutta Method
RK4	Runge-Kutta of the fourth order Method
MC	Monte Carlo method
MMCFD	Mean Monte Carlo Finite Difference Method
MMCRK	Mean Monte Carlo Runge Kutta Method
MSE	Mean square error
VIM	Variational Iteration Method
SEIR	Susceptible-Exposed-Infectious-Recovered

	model type
SIR	Susceptible-Infectious-Recovered model type
SIS	Susceptible-Infectious-Susceptible model type
A_n	Adomian polynomials
$a(t)$	Non-drink alcohol people in the first model (alcohol consumption model)
$a(t)$	Social class who never smoke from the total Population in the second model (smoking habit model)
$b(t)$	Social class of people who smoke less than 20 cigarettes per day
$c(t)$	Social class who smoke more than 20 cigarettes per day
$d(t)$	The social class of ex-smokers
$m(t)$	Non-risk-drink alcohol people
$r(t)$	Risk-drink alcohol people
f	Function
F	Cummelative disterbution Function
F^{-1}	The inverse of function F
p^{th}	Percentiles
h	Step size
$\lvert E_p \rvert$	The difference measure error
λ	Lagrange Multiplier in correction fuctional of VIM.

λ	The rate of normal smokers who stop smoking in the second model (smoking habit model)
ε	Continuous random variable that distributes
α	Birth rate in alcohol consumption model in the
α	first model (alcohol consumption model)
	The rate of smokers who are excessively and who are becoming a normally smoker by reducing the number of cigarettes per day in the second model (smoking habit model).
β	The rate at which a risk-drink alcohol people becomes a non-drink alcohol people in the first model (alcohol consumption model).
β	The transmission of smoke infection because of social pressure to adopt smoking habit in the second model (smoking habit model).
μ	Death rate in alcohol consumption model in the first model (alcohol consumption model).
μ	Rate of births Spain in smoking habit model in the second model (smoking habit model).
ξ	Transmission rate because social pressure that leads to increased the alcohol drinking.
σ	Growing death rate due to alcohol drinking.
η	A rate that transmits a non-risk-drink alcohol people move to the risk-drink alcohol people.
γ	The rate of smokers who are normally and who are becoming an excessive smoker by reducing the number of cigarettes per day.
δ	The rate of excessive smokers who stop smoking.

LIST OF APPENDICES

CHAPTER 1:

INRODECTION

AND

PRIMARY CONCEPTS

1.1 Introduction:

Ordinary differential equation (ODE) is an equation for unknown functions of dependent variable and its derivatives. If the independent variable of ODE is in terms of a variety of time and does not appear explicitly, then the system is called an autonomous system or sometimes, a time-invariant system. The highest derived to the differential equation is the order of it [80]. The power of the highest order derivative in the equation is its degree [83]. The initial value problems of the nonlinear autonomous system of the first-order ordinary differential equations in this thesis is to raise a campaign about alcohol consumption and smoking habit in Spain as epidemiological model. The behavior of the autonomous dynamic system of the thesis applications are studied [42].

The mathematical model is a description of a natural phenomenon either as a deterministic model or stochastic model. The stochastic model provides multiply results. These results include one or more random variables. These variables are solved by randomly, such as Mont Carlo simulation [44, 51, 81]. On the other hand, a deterministic model does not contain a random variable and in this case the solution is unique and in a specified period of time. These models are solved by deterministic methods such as Runge-Kutta, finite-difference, finite volume, finite-element etc. [48]. There is also a randomized-deterministic modeling approach, where it represents a specific random model and is considered a third type of method. Where the Chemical Master Equation (CME) application is an example of stochastic deterministic modeling, where is mixed of deterministic and stochastic deterministic model. This shows us the time development in the

probability density function of the system in the case of the system. Unfortunately, a few bounds only to solve the route approach of the CME for general system [50, 52]. In the present thesis, stochastic-deterministic models show us general behavior of the real social epidemic models. The epidemic models are the extensive applications of nonlinear autonomous stochastic-deterministic models [9, 25, 41], that specialized in this study.

The most epidemiological models can be represented in the form of a system of ordinary differential equations, where this system depends on the independent time t. Using the simulation because of these models have parameters that have random distribution in nature, some of these systems are resolved, see [16].

1.2 Epidemic Models:

The epidemic model is a model which deals with an epidemic that spreads rapidly in a large size of population, where the epidemic models are considered as stochastic-deterministic models that can be formulated as a system of differential equations from the first order. Analysis of epidemic behavior either decays, grows or remains in the population with the time [25].

Epidemic model is divided according to the weakness of humans toward the disease. Susceptible (S), Exposed (E), Infectious (I), Recovered (R). Susceptible (S) is the group people with them who has been infected, Exposed (E) is the group people infected but not move the infection. Infectious (I) are people transform the disease while Recovered (R) are the individuals who have immunity from the disease

and cannot infect others. The profile of a disease that can be represented by Susceptible-Exposed-Infection-Recovered (SEIR) type is known as epidemic model, which is used in this thesis. There are also others simple types of the disease models such as Susceptible-Infectious-Susceptible (SIS) type and Susceptible-Infectious-Recovered (SIR) type. The preliminaries of SIS, SIR and SEIR dynamic models are outlined by [25].

The stability of the epidemic models were also evaluated in recent year. A SIR model of a nonlinear autonomous system of ODE was discussed by [7].

The basic reproduction number (R_0) is an important tool to see the stability of the behavior of the epidemic model. R_0 is threshold quantity and considered as a tool to determine whether an epidemic occurs or the disease simply dies out. This value determines the probability of transmission [25]. The disease is non-transporting in infectious time in his infectious period, if R_0 less than one, therefore the infectious will be away in the future. There is an epidemic in the population, if R_0 more than one. If R_0 equal to one, the disease becomes a settler in society and a consistent rate, such that each infected individual transmits the disease to other susceptible individuals [8, 15, 46, 59].

1.3 Analytical Methods:

In this section, the ADM and VIM have been used to solve the system of nonlinear ordinary differential equations and may provide the exact solution, as these methods will be shown in detail in Chapter 2 and

Chapter 3. In this thesis, the system of the nonlinear ordinary differential equation of epidemic models is solved.

1.3.1 Adomian Decomposition Method (ADM):

The Adomian decomposition method (ADM), is analytic method that was introduced and developed by George Adomian 1976 [83], ADM is a reliable method to solve many various kinds of problems, so it is a trusty method which emerging in applied science. This method has been used by many researchers as well as it has extensive applications of linear and nonlinear ordinary differential equations, partial differential equations and integral equations [5, 83]. It consists of the sum of an infinite number of components of decomposition the unknown function $u(x)$ of any equation and which is written in a series of the decomposition:

$$u(x) = \sum_{n=0}^{\infty} u_n\,(x) \tag{1.1}$$

or equivalently

$$u(x) = u_0\,(x) + u_1(x) + u_2(x) + \cdots$$

where the linear components $u_n(x)$, $n \geq 0$ are evaluated in a recursive manner. The decomposition method concerns itself with finding the components u_0, u_1, u_2 ,….etc. The zero component is determined by all terms that are not included under the integral sign. Consequently, by setting the recurrence of these components $u_i(x)$, $i \geq 1$ for the unknown function $u(x)$ identified.

Consider the following nonlinear differential equation

$$Lu + Ru = g(x), \tag{1.2}$$

where L and R are linear and nonlinear operators, respectively, and $g(x)$ is the source inhomogeneous term. The ADM introduces for Eq. (1.2) in the form

$$u_0(x) = g(x),$$

$$u_{n+1}(x) = \int_0^x (\sum_{n=0}^{\infty} u_n(t))dt, \quad n \geq 0 \tag{1.3}$$

For the nonlinear solution $u(x)$, the infinite series of polynomials becomes:

$$u(x) = \sum_{n=0}^{\infty} A_n (u_0, u_1, \ldots., u_n). \tag{1.4}$$

Recurrently the components $u_n(x)$ of the solution $u(x)$ determined and the Adomian polynomial (A_n) which are obtained from the formula of the nonlinear terms [4].

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[g\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, \qquad n = 1,2,\ldots \tag{1.5}$$

The formulas of the first several Adomian polynomials from A_0 to A_4, have been listed below as given in [4].

$A_0 = g(u_0),$

$A_1 = u_1 g'(u_0),$

$A_2 = u_2 g'(u_0) + \frac{1}{2!} u_1^2 g''(u_0),$

$A_3 = u_3 g'(u_0) + u_1 u_2 g''(u_0) + \frac{1}{3!} u_1^3 g'''(u_0),$

$$A_4 = u_4 g'(u_0) + \left(\frac{1}{2!}u_2^2 + u_1 u_3\right) g''(u_0) + \frac{1}{2!}u_1^2 u_2 g'''(u_0) + \frac{1}{4!}u_1^4 g''''(u_0),$$

⋮

and so on.

1.3.2 Variational Iteration Method (VIM):

The Variational Iteration Method (VIM), is an iterative analytic method that was established by Ji-Huan. This method is used widely in scientific and engineering applications, where it is used to solve linear equations, nonlinear, homogeneous and inhomogeneous. VIM is an effective and reliable method and has a quick solution approach the exact solution. The VIM differs from the ADM, where it does not require specific treatment of nonlinear problems as in the Adomian method [83]. If the exact solution is not possible, then the obtained series can be used for numerical purposes. In order to define the basic concepts of the VIM, is considered the following nonlinear equation [39].

$$Lu(x) + Ru(x) = g(x), \qquad x > x_0,\ x_0 \in R \tag{1.6}$$

where, L is a linear operator, R a nonlinear operator and $g(x)$ is the source of the inhomogeneous term. The VIM introduces functional for Eq. (1.6) in the form:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(t)\left(Lu(x) + R\tilde{u}_n(x) - g(x)\right)dt, \tag{1.7}$$

where λ is a general Lagrange multiplier which can be identified optimally via the variational theory, and $\tilde{u}_n$ as a restricted variation. The Lagrange multiplier λ may be constant or a function and it is given by the general formula [82].

$$\lambda(t) = (-1)^n \frac{1}{(n-1)!}(t-x)^{n-1} \tag{1.8}$$

However, for fast convergence, the function $u_o(x)$ should be selected by using the initial condition for ODE as follows:

$u_0(x) = u(0)$, for first order.

$u_0(x) = u(0) + xu'(0)$, for second order.

$u_0(x) = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0)$, for third order

⋮

and so on.

The successive approximations $u_{n+1}(x)$, $n \geq 0$ of the solution $u_n(x)$ will be ready immediately, when using a selective function $u_0(x)$. Consequently, the solution is given by

$$u(x) = \lim_{n\to\infty} u_n(x) \tag{1.9}$$

1.4 Numerical Methods:

In this section, we study two numerical methods which are finite difference and Runge-Kutta that give results may converge to the exact solution.

1.4.1 Finite Difference (FD) Method:

Finite difference (FD) method is one of the approximate methods that used to solve the differential equations. In general, where the solution is accurate and necessary for the technology intended solution required [76]. FD is a numerical method to solve initial value problem. The result of FD represents the discrete numerical values that approximate the exact solution. The system of differential equations which have time-dependent coefficients can be solved numerically by FD method [23]. Sometimes, this method is called the method of lines and can be considered as a discretization method [36]. FD is an iteration process to solve differential equations [29]. It considered as an approximation of the derivative of the differential equation.

There are three types of finite approximation methods; finite difference, finite volume and finite element. Finite difference (FD) method is the oldest numerical method to solve differential equations. It approximates the derivatives of differential equations and deals with the points where the solution domain is treated as a grid system. Finite volume (FV) method deals with the integral form and approximates surface and volume integrals when the solution domain is subdivided into a limited number of neighboring volumes. The surface and volume integrals are approximated by appropriate quadrature formulae. Finite element (FE) method has the most properties of the FV method. The domain is divided into a collection of discrete volumes of finite

elements. The solution is an approximation by linear function [27]. A system of finite difference equations is stable when the cumulative effect of all the rounding error is negligible. In some cases, it is quite possible to develop this system [47].

The general form of FD for an ordinary differential equation (ODE) can be written as follows:

$$y' = f(t, y), \qquad c \leq t \leq d, \tag{1.10}$$

with initial value

$$y(c) = c_{0,}$$

FD discrete the time t in (c, d) into m sub intervals which is equal to endpoints, $t_i = c + ih,$ for $i = 1,2, \dots, m,$ where m is the maximum number of iterations and $h = \frac{d-c}{m}$ is step size.

The step size $h = 1,$ (per day, week or year) is chosen in our study since FD is solving the real social epidemic model estimated on a time basis. Therefore, Eq. (1.10) becomes:

$$y'(t_i) = f\big(t_i, y(t_i)\big). \tag{1.11}$$

By using the central difference formula of FD, the equation [27]

$$y_i' = f(y_i), \tag{1.12}$$

becomes:

$$\frac{dy}{dt} \approx \frac{y_{i+1} - y_{i-1}}{2h}, \qquad i = 1,2, \dots m. \tag{1.13}$$

Suppose that a step size h is a fixed positive number. Some of higher powers for approximations of a function using Taylor series, approach to zero. The term of error for the central FD is [31].

$$O(h^2) = -\frac{1}{6}h^2 y'''(t) \tag{1.14}$$

We can explain the deriving of the central finite difference formula by using a Taylor theorem as follows:

The expression of the Taylor series of order m at h is

$$y(t+h) = y(t) + hy'(t) + \frac{1}{2}h^2 y''(t) + \frac{1}{6}h^3 y'''(t) + \cdots + \frac{1}{m!}h^m y^{(m)}(t). \tag{1.15}$$

The approximate function at $(t+h)$ and $(t-h)$ lead to

$$y(t+h) = y(t) + hy'(t) + \frac{1}{2}h^2 y''(t) + \frac{1}{6}h^3 y'''(t) + \cdots, \tag{1.16}$$

$$y(t-h) = y(t) - hy'(t) + \frac{1}{2}h^2 y''(t) - \frac{1}{6}h^3 y'''(t) + \cdots, \tag{1.17}$$

we obtain by subtraction

$$y(t+h) - y(t-h) = 2hy'(t) + \frac{2}{3!}h^3 y'''(t) + \frac{2}{5!}h^5 y''''(t) + \cdots \tag{1.18}$$

The central finite difference formula with its truncation error can be written as

$$y'(t) = \frac{1}{2h}[y(t+h) - y(t-h)] - \frac{1}{6}h^2 y'''(\delta). \tag{1.19}$$

where δ is a value in the interval of a function y and $-\frac{1}{6}h^2 y'''(\delta)$ is the error term [19].

To construct a formula of finite difference method to solve differential equations, let first mention that our study considers the function f in Eq. (1.20) is a nonlinear of y, h is the step size, m is an integer and $t = 1,2,3,\dots,m$. Suppose that

$$t \approx t_m, y(t) \approx y_m\,, f(y) \approx f_m \text{ and } f_m \approx f(y_m). \tag{1.20}$$

The first derivative form in calculus is given for the forward, backward and central differential schemes is given as

$$\frac{dy}{dt} = \lim_{h\to 0} \begin{cases} \frac{y(t+h)-y(t)}{h}, \\ \frac{y(t)-y(t-1)}{h}, \\ \frac{y(t+h)-y(t-h)}{2h}, \end{cases} \rightarrow \begin{cases} \frac{y_{m+1}-y_m}{h}, \\ \frac{y_m-y_{m-1}}{h}, \\ \frac{y_{m+1}-y_{m-1}}{2h}, \end{cases} \tag{1.21}$$

Substitute the corresponding Eqs. (1.16) and (1.17) in Eq. (1.10) to obtain the finite difference schemes of Eq. (1.10);

$$\frac{y_{m+1}-y_m}{h} = f_m, \tag{1.22}$$

$$\frac{y_m-y_{m-1}}{h} = f_m\,, \tag{1.23}$$

$$\frac{y_{m+1}-y_{m-1}}{2h} = f_m. \tag{1.24}$$

The Equations expressions (1.22), (1.23) and (1.24) are called the respectively forward Euler, backward Euler, and central finite difference schemes [53].

A finite difference approximation satisfies the consistency condition and stability that is the necessary and sufficient condition for the convergence solutions [14].

1.4.2 Runge-Kutta (RK) Method:

Runge-Kutta (RK) method is provided an approximate solution for a system of ordinary differential equations with known initial conditions [47]. It is a numerical technique that used to solve the ordinary differential equation only of the first order. This method is used for high accuracy and at the same time decrease the errors [77]. RK has constructed a four-stage with respect fourth-order (RK4) method [43].

The study shows that the solution of the system of nonlinear ODE is feasible by the Runge-Kutta method; it yields more accurate results than that obtained by finite difference methods. The use of Runge-Kutta methods to solve problems of this type is a novel approach. It is anticipated that this technique can be utilized to solve other complex problems of similar nature [47].

The general form of the first order ODE given in Eq. (1.10), the initial value $y(t_0) = y_0$, with the interval $t_0 \leq t \leq t_m$, where t is the independent variable, y is the dependent variable, m is the number of points;

t_0 is given, $t_1 = t_0 + h$, $t_2 = t_1 + 2h$, $t_m = t_0 + mh$,

where h is a fixed step size, the unknown function $y_1, y_2, y_3, \ldots, m$ can be solved by using the RK4 method [79] as the following:

$$y_{i+1} = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1.25}$$

where

$$k_1 = hf(t_i, y_i) \tag{1.26}$$

$$k_2 = hf(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}) \tag{1.27}$$

$$k_3 = hf(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \quad (1.28)$$

$$k_4 = hf(t_i + h, y_i + k_3) \quad (1.29)$$

The classical fourth order Runge-Kutta method is solves the IVP and gives a more accurate result. This method convergent when the difference between the exact solution and the solution of differential equation at k^{th} step satisfies the condition.

$$\lim_{h\to 0}(\max_{1\le k\le m}|y(t_k) - y_k|) = 0, \quad (1.30)$$

where m is the number of iterations of RK.

The stability of a numerical method ensures that small changes in the initial conditions should not lead to large changes in the solutions [14]. RK numerical iteration method with different orders such as RK2, RK4, RK45 and RK78 [58]. In this subsection, the Runge-Kutta of order four method is used for solving some nonlinear system of ODEs.

1.5 Simulation Methods:

The use of random numbers in the statistics after taking the random samples of experimental units without exact characteristics. Now uses in the simulation studies of the stochastic processes, this area is called simulation, Monte Carlo and resampling [30]. Monte Carlo (MC) method indicates a sampling by traditional technical by generating random numbers to sample from a probability distribution [69]. After the development of computer system, MC method is an international method. Monte Carlo is a derived name of a city in Principality of Monaco [76]. The Monte Carlo algorithm using computer programs creates random numbers with a probability density function that equal to

one if the numbers between 0 and 1, and in another place equal to zero. These numbers are considered as random variables a distributed uniformly on (0,1) [68].

The MC simulation process generates uniform random numbers. The following MC procedure is used in our study [17]: firstly, we generate random numbers in the interval (0,1) such that each of these random numbers is considered as a random variable that has a uniform distribution on the interval (0,1) (standard uniform distribution). Then the inverse transform method is used to transform the random variables which have the standard uniform distribution, into random variables that have specific distribution [19]. The inverse transform method (inversion method) has the following formula:

Let X be a random variable and $F(x)$ a cumulative distribution function. Suppose that F^{-1} is the inverse of function F and let ε be a continuous random variable that distributes uniformly on interval (0,1), such as [19].

$$P(F^{-1}(\varepsilon) \leq x) = P\big(\varepsilon \leq F(x)\big) = F(x). \tag{1.31}$$

ε is a continuous random variable then $P(\varepsilon = F(x)) = 0$ and by taking F^{-1} for $\varepsilon = F(x)$, the random variable X is equal to $F^{-1}(\varepsilon)$ and it has been written as:

$$X = F^{-1}(\varepsilon). \tag{1.32}$$

Box-Muller transformation is an example of a method that uses the inverse transform to convert two uniform random variables into normally distributed random variables.

The probability density function of uniform distribution on (a, b), where a and b are lower and upper bounds is

$$f(x) = \begin{cases} \frac{1}{b-a}, & for\ a \leq x \leq b, \\ 0 & otherwise. \end{cases}, \tag{1.33}$$

and the cumulative distribution function G of the uniform distribution on (a, b) is

$$F(x) = \begin{cases} 0 & for\ x < a, \\ \frac{x-a}{b-a} & for\ a \leq x \leq b, \\ 1 & for\ x > b. \end{cases} \tag{1.34}$$

Then the inverse of the uniform cumulative function F^{-1} has the following formula:

$$X = F^{-1}(\varepsilon) = a + (b-a)\varepsilon, \tag{1.35}$$

where ε has the standard uniform distribution [68].

The importance of the Monte Carlo and simulation methods in the past years have increased in various sciences. The simulation methods have a central role in the scientific developments as the physical sciences, the computational life sciences, and the other computational sciences. The developed approach to the simulation system as well as the development of computers is making it a tool for a substance for the

processing of the various natural sciences, together with theory and traditional experimentally. At the kernel of Monte Carlo simulation is random number generation. The parameter values can be used for any value required in simulation applications. This is one of the features that make the Monte Carlo simulation method is very useful [76]. The types of random sampling are Monte Carlo and Latin hypercube sampling.

The reason of using a simulation technique in our study belong to the nature of parameters of the models under study, are random variables and not available, the simulation technique generates the data of these parameters.

1.6 Numerical Simulation Method:

Numerical simulation methods can solve the system of differential equation using a numerical method and simulation processes. The numerical simulation method is considered more appropriated to solve such systems that have randomness in their coefficients, these coefficients depend on the variable time, and they are treated by the simulation process. The Monte Carlo finite difference (MMCFD) is a numerical simulation method that merges between Monte Carlo simulation process (MC) and finite difference numerical iteration method (FD). This method was suggested at the first time by Mohammed. M, et al, in 2019 [55]. This mixed method MMCFD simulates firstly the parameters of a model firstly when these parameters as random variables. Then the system is solved numerically m times using FD with the first simulated estimation parameters. The m numerical simulated results have been gotten. The last numerical

iteration result has been selected which is FD of order m iteration result (FD_m) that is called the final solution. This process is returned with the second simulated estimation parameters, and so on until the last number of simulations. Finally, the mean of the n time simulations for the final solutions is called Monte Carlo finite difference (MMCFD). MMCFD is one an established approximate method using to solve the system of differential equation numerically. This method was applied on obesity model [55] and cocaine consumption [57]. To more understand, see Figure 1.1.

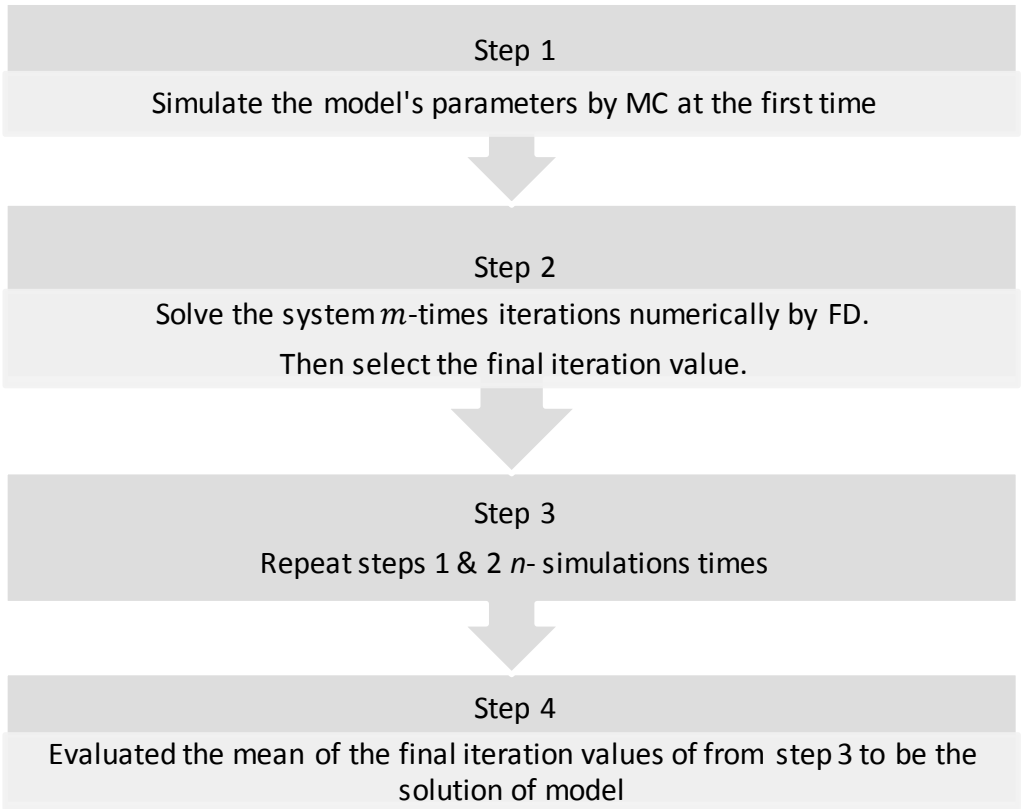

Figure 1.1: The steps of MMCFD process

1.7 Prediction Interval and Percentile:

A prediction Interval is one of the statistical indicators to describe the data. It is also describes the solution for numerical simulation, in this study. The proposed modified method in Chapter four is useful to determine the prediction interval when the random distribution of the numerical solution is necessary for estimation of real epidemiological models. Prediction interval is an interval contains upper and lower bounds for predicted values of the distribution for each subpopulation of a model. It can be obtained by using the p^{th} percentiles (P%) to give upper and lower limits. A percentile is a value within a distribution that divided ordered predicted values into two or more parts by a straight-line between these values. It belongs to the vector distribution of random variables. As a consequence, the p^{th} percentile of the predicted values is inside a population. The percentile value is equal to or less than the number that required to calculate it when $0 < p < 100$. Since, the index becomes $\big((n) \times (p \div 100)\big)$ when n represent the total number of predicted values in the distribution and represents for the p^{th} percentile value within the population distribution [64, 86].

1.8 Errors:

Two types of errors can be used in this thesis which are difference measure error and mean square error. The uses of these errors, is to purpose the comparison of the methods used in our study.

1.8.1 Absolute and Relative Errors:

There are many types of numerical errors, where the following species are the most common, the absolute error, relative error, and truncation error. Let a and b are two values, one the exact value (a) and the other approximate value (b). The general formula for absolute error is $|a-b|$, and the relative error is $\frac{|a-b|}{|a|}$ [19].

1.8.2 Difference Measure Error ($|E_p|$):

The difference measure error $|E_p|$ is the difference between the approximate (either analytic or numerical) solutions and the predicted values that propose in our study [55].

1.8.3 Mean Square Error (MSE):

The MSE is the quality of a predictor (random variable), or an estimator (an estimate of a parameter of the population from which data is sampled).

If a vector of n predictions generated from a sample of n data points on all variables, and x is the vector of observed values and $\hat{x}$ is the variable being predicted, then denoted by MSE is computed as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2. \qquad (1.32)$$

MSE is mean $(\frac{1}{n}\sum_{i=1}^{n})$ of squares of the errors $(x_i - \hat{x}_i)^2$ [62].

1.9 Review of Literature:

In this section, we remember the researchers who use analytical methods such as adomian decomposition method (ADM) and variational iteration method (VIM), and who used numerical methods such as finite difference (FD) and Runge-Kutta (RK). Some researchers were talking about some epidemiological models.

The modified ADM with its applications on some equations have been given, see examples in [1, 2, 21, 60]. Some researchers used the ADM to solve a system of ordinary differential equations [13] and apply on the epidemic model [12, 49]. As well as, ADM solved a system of integral–differential equations [11]. Recently, the accuracy of nonlinear singular initial value problems was discussed using a semi-analytic [80]. ADM is applied to solve fuzzy fractional order differential algebraic equations [10], modified adomian decomposition method was applied on Integro-Differential Inequality [66].

Many works to solve nonlinear problems using VIM [39], with autonomous ordinary differential systems [40] and to solve differential equations that have fractional order [61].

The Runge-Kutta (RK) methods provide an approximate solution for a system of ordinary differential equations with known initial conditions. Runge-Kutta (RK) method is a powerful tool for the solution of ordinary differential equations (ODE). Most of the research has been oriented towards improving the accuracy or the flexibility (to accommodate problems of diverse nature) of the classical Runge-Kutta method [47]. The solution of the system of nonlinear ordinary

differential equations *(ODE)* is obtained by using this method. A similar approach has been taken by [57].

Previously, an inverse problem for nonlinear parabolic was solved by finite difference scheme jointed with Monte Carlo algorithm and the unknown diffusion coefficient was estimated using polynomial format [26]. Finite difference method was integrated with Monte Carlo simulation process in order to predict the behavior of the dam [67], the random variables which were generated by Monte Carlo method in this problem have a Gaussian distribution. In recent year, the elliptical partial differential is analyzed by stochastic finite element method [66]. Nonlinear random differential equations were solved by generalized polynomial chaos method [22].

The social epidemic is known as the spread of bad habits through social pressure as the cocaine, obesity, smoking and alcohol consumption. Some epidemiological models have been studied to understand the dynamics of phenomena which become better. [73] predicted the future behavior of alcohol consumption in the Spanish population by estimating the parameters of the model and by fitting the model to real data. [33] studied the effect of the smoke-free law on the evolution of smoking habits in Spain, before and after applying this law from during 2006 to 2009. Predicted the effect of this law on the growth of the smoking habit in the Spanish population.

Recently, [20] estimated epidemic model parameters using least-square fitting. [87] analyzed a mathematical model of epidemics of seasonal influenza in Australia using the likelihood-based method. [45] studied the optimal control strategies of influenza epidemic model in

Korea. There are other researchers who analyzed the behavior of some mathematical epidemic models recently. [18] discussed Ebola synthetic epidemics. Chowell in 2017 discussed dynamic of epidemic outbreaks and estimated the parameters using fitting approach [20].

In 1983, the inversion of nonlinear stochastic operators is studied by Adomian and Rach [5]. In 1976, the nonlinear stochastic differential equations are studied by Adomian [3].

In 2004, the mathematical model in biology is studied by Allman [9]. In 2006, solution of the epidemic model by Adomian decomposition method are studied by Biazar [12]. In 2001, the mathematical models in population biology and epidemiology is studied by Brauer [15]. In 2010, the Monte Carlo simulation via a numerical algorithm for solving a nonlinear inverse problem are studied by Farnoosh and Ebrahimi [26]. In 2000, the variational iteration method for autonomous ordinary differential systems is studied by Ji-Huan [40]. In 1991, the numerical methods for ordinary differential systems of the initial value problem is studied by Lambert [48]. In 2012, the combining Monte Carlo and finite difference methods for effective simulation of dam behavior is studied by Rohaninejad and Zarghami [67].

1.10 Problem Statement:

A social epidemic is a bad habit that is moving community by social pressure. Works researchers represent the mathematical model in order to control the spread the epidemic. These epidemiological models can be represented in the form of a nonlinear systems of ODEs. These models have parameters and these parameters are random distribution in nature.

Because of the first one missing some real data in real model, the simulation technique help to generate random variables.

The simulation technique itself, may be appropriate to such these models for some reasons: through it, we get to better understand through the detailed control of the system and to analyze the phenomenal changes and the effects of the information under study. The simulation system sometimes design the experience of a new system. The simulation can also be used to analyze a dynamic system with their real time [68].

The importance of this study comes, in fact from some real models of nonlinear systems of ODEs that are made up of random variables. To resolve these systems, use suitable numerical simulation methods such as MMCFD and the new proposed method MMCRK, where these methods support the expected solutions.

1.11 Research Objectives:

This research is to achieve the following objectives:

- To use some analytical methods as ADM and VIM and some iterative numerical methods as FD and RK4 and to solve two epidemic models which are alcohol consumption model and smoking habit model.
- Create a modified method which is Mean Monte Carlo Runge-Kutta (MMCRK) for solving models in the form of nonlinear systems of ordinary differential equations and compare it with another numerical simulation method which is MMCFD for the purpose of comparisons.
- To apply the new method MMCRK under study on selected social epidemic models.

- To compare the simulation results obtained from the modefied numerical simulation method with the results of analytical and numerical methods with predicted and stochastic-deterministic solutions.
- To compare between methods under study by some indicators such as the difference measure error of numerical and analytical solutions and Mean square error for numerical simulation solutions.
- To analysis the analytical and numerical simulation results obtained graphically and tabularly towards the solutions of the epidemic models.

1.12 Scope of Research:

Social epidemic models with numerical simulation technique are considered in our study. These models are treated as deterministic problems with a probability process that can be programmed into computers to save time, effirte and cost [69]. Chose, two models of social epidemics which are alcohol consumption model and the of smoking habit model to prove modified numerical simulation method which is MMCRK method in the present study.

1.13 Thesis Outline:

This research embarks on finding the alternative modified methods of simulation technique approaches in order to supply numerical simulation solutions for some real stochastic-deterministic nonlinear epidemic systems as well as to give prediction ranges of these solutions.

This thesis is divided into five chapters; Chapter 1, introduction and the preliminaries and concepts of this research are outlined briefly in the

subsections of introduction, research objectives, literature review, problem statement and scop of the research, some concepts about our study. Chapter 2 provides a brief literature review ideas and concepts of ordinary differential equations, epidemic model of alcohol consumption specified with their applications, analytic methods as ADM and VIM, also numerical iteration methods as FD and RK4. In Chapter 3, epidemic model of smoking habit that has solved by analytic methods as ADM and VIM, also numerical iteration methods as FD and RK4.

Next in Chapter 4, a modified approach between Monte Carlo simulation and Runge-Kutta method, namely Mean Monte Carlo Runge-Kutta (MMCRK) method, is applied to solve two epidemic models which are alcohol consumption and the smoking habit. MMCRK has been compared with a numerical simulation method which is MMCFD some indicators. Finally, Chapter 5 is the overall findings and conclusion of the research are provided. In addition, some future works are added to extend the present study.

CHAPTER 2:
Applications of Some Analytic and Numerical Methods on Alcohol Consumption Mode

2.1 Introduction:

Alcohol consumption habit is considered as a social disease that spread out rapidly by social pressure or social contact. Recently, the rate of alcohol consumption has increased more with the developing countries, so alcohol consumption represented a big problem that effirte not only on the human health, but also on the community economy. When the number of people who are suffering from such diseases increased, because it is very expensive, regard to the health effects of chloride that impact on the healthy body, the chloride can damage some parts of the body such as heart and liver, influence also on the other functions. An addition that, the cost of alcohol affects the economy [28, 73]. Alcohol problem in Ireland and the United Kingdom, this problem was discussed and the data was reported for the first year of 2002 until the end of 2014 [24]. In Spain, the effects of the different usage of alcohol between the female and male in Spanish university alumni was studied [32].

The type of epidemiological models had been used to of many social diseases. In the recent years, several researchers were interested to study and analyze the social epidemics about ecstasy or heroin addiction [70, 85], smoking habit evaluation in Spain [33, 34, 37], a cocaine abuse in Spain [35, 57, 71], campaigns on reducing excess weight in Valencia [56, 57, 72].

In this chapter, we try to apply some classic analytic methods such as ADM and VIM and some numerical methods like FD and RK4, on social epidemic model which is Alcohol consumption in Spain. This application is used to compare between the numerical and analytical solutions using the difference measure error, since ADM, VIM, FD, and RK4 are confident methods.

2.2 Mathematical Model of Alcohol Consumption:

The mathematical model of alcohol consumption was explained and described in the current study by Santonja et al., (2010) [73]. This model consists of three subpopulations of Spanish population who have about 15-64 years old from 1997 to 2007 years that represented as the nonlinear system of three ordinary differential equations of the first order. This system is referred to analyze the changing in social epidemic stages (non-drink alcohol people, non-risk-drink alcohol people and risk-drink alcohol people), see Table 2.1. The parameters of this model are described in Table 2.2.

The model is described as

$$a'(t) = \alpha + \beta r(t) - \mu a(t) - \xi a(t)(m(t) + r(t)) - a(t)(\alpha - \mu a(t) - \sigma m(t) - \sigma r(t)) \quad (2.1)$$

$$m'(t) = \xi a(t)(m(t) + r(t)) - \eta m(t) + \mu a(t)m(t) - \sigma a(t)m(t) - \alpha m(t) \quad (2.2)$$

$$r'(t) = \eta m(t) - \beta r(t) + \mu a(t)r(t) - \sigma a(t)r(t) - \alpha r(t) \quad (2.3)$$

These parameters are transitional links that connect the different groups of society to move people from one stage to another stage of the epidemic.

The initial conditions of equations (2.1), (2.2) and (2.3) in 1997 are $a(t=0)=0.362,\quad m(t=0)=0.581,\quad r(t=0)=0.057,$ with the predicted parameters are given as: $\alpha=0.01$, $\beta=0.0014$, $\mu=0.008$, $\xi=0.0284534$, $\sigma=0.009$, and $\eta=0.000110247$, [73].

Table 2.1: Variables of alcohol consumption model [73]

$a(t)$	Non-drink alcohol people are subpopulations who never drink alcohol in their life.
$m(t)$	Non-risk-drink alcohol people are subpopulations who drink a little liquid of alcohol that means the men who drink less than 50 cc and women who drink less than 30 cc of alcohol every day.
$r(t)$	Risk-drink alcohol people are subpopulations who drink a lot of alcohol that means the men who drink more than 50 cc and the women who drink more than 30 cc of alcohol every day.

Table 2.2: Parameters of alcohol consumption model [73]

α	Birth rate in Spain
β	The rate at which a risk-drink alcohol people becomes a non-drink alcohol people
μ	Death rate in Spain
ξ	Transmission rate because social pressure that leads to increase the alcohol drinking
σ	Growing death rate due to alcohol drinking
η	A rate that transmits a non-risk-drink alcohol people move to the risk-drink alcohol people

2.3 Problem Solution using Analytical Methods:

In this section, two analytic methods have been used which are ADM and VIM to solve the epidemic model of alcohol consumption.

2.3.1 Adomian Decomposition Method (ADM):

The nonlinear system of equations $(2.1), (2.2)$ and (2.3) can be solved by using the Adomian decomposition method with the initial condition and the given parameters. Let l be an operator that is given by $l = \frac{d}{dt}$ and the inverse of this operation is $l^{-1} = \int_0^t (.)dt$, then by applying l^{-1} for both sides of equations (2.1), (2.2) and (2.3) we obtain:

$$a(t) - a(0) = l^{-1}\big(\alpha + \beta r(t) - \mu a(t) - \xi a(m(t) + r(t)) - a(t)(\alpha - \mu a(t) - \sigma m(t) - \sigma r(t))\big),$$

where $a_0 = 0.362$.

Similarity,

$$m(t) - m(0) = l^{-1}\big(\xi a(t)\big(m(t) + r(t)\big) - \eta m(t) + \mu a(t)m(t) - \sigma a(t)m(t) - \alpha m(t)\big),$$

where $m_0 = 0.581$, and

$$r(t) - r(0) = l^{-1}\,(\eta m(t) - \beta r(t) + \mu a(t)r(t) - \sigma a(t)r(t) - \alpha r(t)),$$

where $r_0 = 0.057$.

The above equations equivalent the following Eqs. (2.4), (2.5) and (2.7):

$$a_{k+1} = l^{-1}(\alpha + \beta r_k - \mu a_k - \xi A_k - \xi B_k - \alpha a_k + \mu C_k + \sigma A_k + \sigma B_k),$$
$$k \geq 0. \quad (2.4)$$

$$m_{k+1} = l^{-1}(\xi(A_k + B_k) - \eta m_k + \mu A_k - \sigma A_k - \alpha m_k),\ k \geq 0. \quad (2.5)$$

$$r_{k+1} = l^{-1}(\eta m_k - \beta r_k + \mu B_k - \sigma B_k - \alpha r_k),\ k \geq 0. \quad (2.6)$$

The general form of the nonlinear borders A_k, B_k and C_k have to be:

$$A_k = (\sum_{n=0}^{2} a_n)(\sum_{n=0}^{2} m_n),\ k = 0,1,2$$
$$A_k = (a_0 + a_1 + a_2)(m_0 + m_1 + m_2),$$
$$= a_0 m_0 + a_0 m_1 + a_0 m_2 + a_1 m_0 + a_1 m_1 + a_1 m_2 + a_2 m_0 + a_2 m_1 + a_2 m_2$$
$$B_k = (\sum_{n=0}^{2} a_n)(\sum_{n=0}^{2} r_n),\ k = 0,1,2$$
$$B_k = (a_0 + a_1 + a_2)(r_0 + r_1 + r_2),\ k = 0,1,2$$
$$= a_0 r_0 + a_0 r_1 + a_0 r_2 + a_1 r_0 + a_1 r_1 + a_1 r_2 + a_2 r_0 + a_2 r_1 + a_2 r_2$$
$$C_k = (\sum_{n=0}^{2} a_n),^2\ k = 0,1,2$$
$$C_k = (a_0 + a_1 + a_2)^2,$$
$$C_k = a_0^2 + a_0 a_1 + a_0 a_2 + a_1 a_0 + a_1^2 + a_1 a_2 + a_2 a_0 + a_2 a_1 + a_2^2$$

The nonlinear borders of A_0, B_0 and C_0 as the following:

$$A_0 = a_0 m_0,\ B_0 = a_0 r_0,\ C_0 = (a_0)^2$$

Substituting for all a_0, r_0, A_0, B_0 and C_0 by Eq.(2.4), to obtain:

$$a_1 = -0.00009881t,$$

Substituting for all m_0, A_0 and B_0 by Eq.(2.5), to find

$$m_1 = 0.00048711t,$$

Substituting for all m_0, r_0 and B_0 by Eq.(2.6), to get

$r_1 = -0.00060638t,$

Now we find a_2, m_2 and r_2.

The nonlinear borders of A_1, B_1 and C_1 are given in the following formula:

$A_1 = a_0 m_1 + m_0 a_1,$

$B_1 = a_0 r_1 + r_0 a_1,$

$C_1 = 2a_0 a_1,$

Substituting for all a_1, r_1, A_1, B_1 and $C_1$by Eq.(2.4), to obtain:

$a_2 = 1.2117815 \times 10^{-8} t^2$

Substituting also for all m_1, A_1 and B_1 by Eq.(2.5), to get

$m_2 = -0.00000403t^2,$

Substituting for all $m_1, r_1 and\, B_1$by Eq.(2.6), to have

$r_2 = 0.00000359t^2,$

At the same previous steps, the nonlinear borders of A_2, B_2 and C_2 are given as:

$A_2 = a_0 m_2 + a_1 m_1 + m_0 a_2,$

$B_2 = a_0 r_2 + a_1 r_1 + r_0 a_2,$

$C_2 = 2a_0 a_2 + (a_1)^2.$

Substituting for all $a_2, r_2. A_2, B_2$ and C_2 by Eq.(2.4), to be:

$a_3 = 2.55439073 \times 10^{-11} t^3,$

Substituting for all m_2, A_2 and B_2 by Eq.(2.5), to find

$m_3 = 1.27758808 \times 10^{-8} t^3,$

Substituting for all $m_2(t), r_2(t)$ and $B_2(t)$ by Eq.(2.6), to get

$r_3 = -1.42663059 \times 10^{-8} t^3.$

The Adomian decomposition method assumes that the unknown functions $a(t)$, $m(t)$ and $r(t)$ that can be written by series as follows:

$a(t) = \sum_{k=0}^{\infty} a_k, \quad m(t) = \sum_{k=0}^{\infty} m_k, \qquad r(t) = \sum_{k=0}^{\infty} r_k,$

$$a(t) = \sum_{k=0}^{\infty} a_k = a_0 + a_1 + a_2 + a_3 \ldots$$

$$a(t) = 0.362 - 0.00009881t + 1.21178155 \times 10^{-8} t^2 + 2.55439073 \times 10^{-11} t^3 + \cdots \quad (2.7)$$

$$m(t) = \sum_{k=1}^{\infty} m_k = m_0 + m_1 + m_2 + m_3 + \cdots$$

$$m(t) = 0.581 + 0.00048711t - 0.00000403t^2 + 1.27758808 \times 10^{-8} t^3 + \cdots \quad (2.8)$$

$$r(t) = \sum_{k=1}^{\infty} r_k = r_0 + r_1 + r_2 + r_3 + \cdots$$

$$r(t) = 0.057 - 0.00060638t + 0.00000359t^2 - 1.42663059 \times 10^{-8} t^3 + \cdots \quad (2.9)$$

$\vdots$

$a(t)$, $m(t)$ and $r(t)$ of ADM results are unsettled terms.

2.3.2 Variational Iteration Method (VIM):

The VIM gives a better approximate solution by constructing a correctional functional that uses an initial function. Where Lagrange multiplier considers the key of the correction functional which can be specified via variation theory [83].

The nonlinear system of alcohol model under study can be solved by using the VIM with given initial condition and parameters [73]. The correction functional of the system of equations (2.1), (2.2) and (2.3) becomes:

$$a_{k+1} = a_k + \int_0^t \lambda \Big(a'_k - \big(\alpha + \beta r_k - \mu a_k - \xi a_k (m_k + r_k) - a_k (\alpha - \mu * a_k - \sigma m_k - \sigma r_k) \big) \Big) dt, \text{ for all } k \geq 0. \quad (2.10)$$

$$m_{k+1} = m_k + \int_0^t \lambda \big(m'_k - (\xi a_k (m_k + r_k) - \eta m_k + \mu a_k m_k - \sigma a_k m_k - \alpha m_k) \big) d, \text{ for all } k \geq 0. \quad (2.11)$$

$$r_{k+1} = r_k + \int_0^t \lambda \big(r'_k - (\eta m_k - \beta r_k + \mu a_k r_k - \sigma a_k r_k - \alpha r_k) \big)\, dt, \quad k \geq 0. \quad (2.12)$$

The Lagrange multiplier is $\lambda = -1$ in equations (2.10), (2.11) and (2.12). By substituting this value in Eqs. (2.10), (2.11) and (2.12). The zero borders become:

$$a_0 = 0.362,\ m_0 = 0.581,\ r_0 = 0.057$$

In equations (2.10), (2.11) and (2.12) if we substitute (k=0), we obtain the following $a_1(t), m_1(t)$ and $r_1(t)$.

$a_1 = 0.362 + 0.00011927t,$

$m_1 = 0.581 + 0.00048711t,$

$r_1 = 0.057 - 0.00060638t,$

By the same way, if we have (k=1), in equations (2.10), (2.11) and (2.12), we get the following $a_2(t), m_2(t)$ and $r_2(t)$.

$$a_2 = 0.362 + 0.00011927t - 0.00000147t^2 + 1.30183484 \times 10^{-10}t^3,$$

$$m_2 = 0.581 + 0.00048711t - 0.00000212t^2 - 1.54291666 \times 10^{-10}t^3,$$

$$r_2 = 0.057 - 0.00060638t + 0.00000359t^2 + 2.41081825 \times 10^{-11}t^3,$$

Continuing in the same manner when (k=2), we can a achieved the following $a_3(t), m_3(t)$ and $r_3(t)$.

$$a_3 = 0.362 + 0.00011928t - 0.00000147t^2 + 1.04339442 \times 10^{-8}t^3 - 2.97475940 \times 10^{-12}t^4 + 1.20789692 \times 10^{-14}t^5 - 1.75446808 \times 10^{-18}t^6 + 6.64675814 \times 10^{-23}t^7 \quad (2.13)$$

$$m_3 = 0.581 + 0.00048710t - 0.00000212t^2 + 3.66528836 \times 10^{-9}t^3 + 3.38204598 \times 10^{-12}t^4 - 1.31514568 \times 10^{-14}t^5 + 1.82643463 \times 10^{-18}t^6 - 6.60192260 \times 10^{-23}t^7 \quad (2.14)$$

$$r_3 = 0.057 - 0.0006064t + 0.00000359t^2 - 1.40992325 \times 10^{-8}t^3 - 4.07286579 \times 10^{-13}t^4 + 1.07248763 \times 10^{-15}t^5 - 7.19665484 \times 10^{-20}t^6 - 4.48355311 \times 10^{-25}t^7 \quad (2.15)$$

And so on, continue in order to get better approximations:
$a(t) = \lim_{k\to\infty} a_k(t)$, $m(t) = \lim_{k\to\infty} m_k(t)$ and $r(t) = \lim_{k\to\infty} r_k(t)$.

2.4 Numerical Methods:

In this section, two numerical methods have been used which are FD and RK4 to solve the epidemic model of alcohol consumption.

2.4.1 Finite Difference (FD) Method:

The nonlinear system of equations (2.1), (2.2) and (2.3) of the alcohol consumption model under study can be solved by using the finite difference with the given initial conditions:
a_0, r_0 and m_0 and the given parameters in Table 2.2 and the real step size h=1, 0.5, 0.25 where $h=\frac{Upper\ bound - Lower\ bound}{m}$, m=10 refers to the number of years from 1997 to 2007. In the same time, m refers to the number of iterations for the FD numerical method
The zero terms becomes: a_0=0.362, m_0=0.581, and r_0=0.057.

In order to find a_1, m_1 and r_1, Backward Finite Difference (BFD) method can use as follows:

$$a_1 = a_0 + h\big(\alpha + \beta r_0 - \mu a_0 - \xi a_0(m_0 + r_0) - a_0(\alpha - \mu a_0 - \sigma m_0 - \sigma r_0)\big), \quad (2.16)$$

$$m_1 = m_0 + h\big(\xi a_0(m_0 + r_0 - \eta m_0 + \mu a_0 m_0 - \sigma a_0 m_0 - \alpha m_0)\big), \quad (2.17)$$

$$r_1 = r_0 + h(\eta m_0 - \beta r_0 + \mu a_0 r_0 - \sigma a_0 r_0 - \alpha r_0), \quad (2.18)$$

The a_1, m_1 and r_1 are calculated from Eqs. (2.16), (2.17) and (2.18) to obtain the following values $a_1 = 0.34550505$, $m_1 = 0.59661796$ and $r_1 = 0.05263699$, respectively.

Now, the Central Finite Difference (CFD) method can be used to find the next steps and so on for m time, follows:

$$a_{i+1} = a_{i-1} + 2h\big(\alpha + \beta r_i - \mu a_i - \xi a_i(m_i + r_i) - a_i(\alpha - \mu a_i - \sigma m_i - \sigma r_i)\big), \quad (2.19)$$

$$m_{i+1} = m_{i-1} + 2h\big(\xi a_i(m_i r_i - \eta m_i + \mu a m_i - \sigma a_i m_i(t) - \alpha m_i)\big), \quad (2.21)$$

$$r_{i+1} = r_{i-1} + 2h(\eta m_i - \beta r_i + \mu a_i r_i - \sigma a_i r_i - \alpha r_i), \quad (2.21)$$

for all $i = 1,2,\dots,m$. To find a_1, a_2, …, a_m, m_1, m_2, …, m_m and r_1, r_2, …, r_m that consider as numerical solutions for alcohol consumption model.

2.4.2 Runge-Kutta of 4th Order (RK4) Method:

RK4 is one of the most accurate iteration numerical methods. The nonlinear system of equations (2.1), (2.2) and (2.3) of the alcohol consumption model can be solved by using Fourth order Runge-Kutta (RK4) method with the given initial condition a_0, m_0 and r_0 with the given parameters in Table 2.2.

For the general form of RK in Eq.(1.24) in Chapter 1, where

$$a_{i+1} = a_i + \frac{1}{6}(ka_1 + 2ka_2 + 2ka_3 + ka_4)h, \tag{2.22}$$

$$m_{i+1} = m_i + \frac{1}{6}(km_1 + 2km_2 + 2km_3 + km_4)h, \tag{2.23}$$

$$r_{i+1} = m_i + \frac{1}{6}(kr_1 + 2kr_2 + 2kr_3 + kr_4)h, \tag{2.24}$$

Firstly, we must find ka_1, km_1 and kr_1 for the first term of RK4 as follows:

$$ka_1 = f_1(t_i, a_i, m_i, r_i),$$
$$ka_1 = \alpha + \beta r_i - \mu a_i - \xi a_i(m_i + r_i) - a_i(\alpha - \mu a_i - \sigma m_i - \sigma r_i), \tag{2.25}$$

$$km_1 = f_2(t_i, a_i, m_i, r_i),$$
$$km_1 = \xi a_i(m_i + r_i) - \eta m_i + \mu a_i m_i - \sigma a_i m_i - \alpha m_i, \tag{2.26}$$

$$kr_1 = f_3(t_i, a_i, m_i, r_i),$$
$$kr_1 = \eta m_i - \beta r_i + \mu a_i r_i - \sigma a_i r_i a - \alpha r_i, \tag{2.27}$$

Secondly, to complete the second term of RK4 ka_2, km_2 and kr_2 must be found as follows:

$$ka_2 = f_1\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, m_i + \frac{1}{2}hkm_1, r_i + \frac{1}{2}hkr_1\right)$$

$$ka_2 = \alpha + \beta(r_i + 0.5kr_1) - \mu(a_i + 0.5ka_1) - \xi(a_i + 0.5ka_1)(m_i + 0.5km_1 + r_i + 0.5kr_1) - (a_i + 0.5ka_1)(\alpha - \mu * (a_i + 0.5ka_1) - \sigma(m_i + 0.5km_1) - \sigma(r_i + 0.5kr_1)), \quad (2.28)$$

$$km_2 = f_2\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, m_i + \frac{1}{2}hkm_1, r_i + \frac{1}{2}hkr_1\right),$$

$$km_2 = \xi(a_i + 0.5ka_1)((m_i + 0.5km_1 + r_i + 0.5kr_1) - \eta(m_i + 0.5km_1) + \mu(a_i + 0.5ka_1)(m_i + 0.5km_1) - \sigma(a_i + 0.5ka_1)(m_i + 0.5km_1) - \alpha(m_i + 0.5km_1)), \quad (2.29)$$

$$kr_2 = f_3\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, m_i + \frac{1}{2}hkm_1, r_i + \frac{1}{2}hkr_1\right),$$

$$kr_2 = \eta(m_i + 0.5km_1) - \beta(m_i + 0.5kr_1) + \mu(a_i + 0.5ka_1)(r_i + 0.5kr_1) - \sigma(a_i + 0.5ka_1)(r_i + 0.5kr_1) - \alpha(r_i + 0.5kr_1), \quad (2.30)$$

Now, ka_3, km_3 and kr_3 are calculated:

$$ka_3 = f_1\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, m_i + \frac{1}{2}hkm_2, r_i + \frac{1}{2}hkr_2\right),$$

$$ka_3 = \alpha + \beta(r_i + 0.5kr_2) - \mu(a_i + 0.5ka_2) - \xi(a_i + 0.5ka_2)(m_i + 0.5km_2 + r_i + 0.5kr_2) - (a_i + 0.5ka_2)(\alpha - \mu(a_i + 0.5ka_2) - \sigma(m_i + 0.5km_2) - \sigma(r_i + 0.5kr_2)), \quad (2.31)$$

$$km_3 = f_2\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, m_i + \frac{1}{2}hkm_2, r_i + \frac{1}{2}hkr_2\right),$$

$$km_3 = \xi(a_i + 0.5ka_2)((m_i + 0.5km_2 + r_i + 0.5kr_2) - \eta(m_i + 0.5km_2) + \mu(a_i + 0.5ka\ \ )(m_i + 0.5km_2) - \sigma(a_i + 0.5ka_2)(m_i + 0.5km_2) - \alpha(m_i + 0.5km_2)), \quad (2.32)$$

$$kr_3 = f_3\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, m_i + \frac{1}{2}hkm_2, r_i + \frac{1}{2}hkr_2\right),$$

$$kr_3 = \eta(m_i + 0.5km_2) - \beta(m_i + 0.5kr_2) + \mu(a_i + 0.5ka_2)(r_i + 0.5kr_2) - \sigma(a_i + 0.5ka_2)(r_i + 0.5kr_2) - \alpha(r_i + 0.5kr_2), \quad (2.33)$$

Now, to find ka_4, km_4 and kr_4 as follows:

$$ka_4 = f_1(t_i + h, a_i + hka_3, m_i + hkm_3, r_i + hkr_3),$$

$$ka_4 = \alpha + \beta(r_i + kr_3) - \mu(a_i + ka_3) - \xi(a_i + ka_3)(m_i + km_3 + r_i + kr_3) - (a_i + ka_3)(\alpha - \mu * (a_i + ka_3) - \sigma(m_i + km_3) - \sigma(r_i + kr_3)), \quad (2.34)$$

$$km_4 = f_2(t_i + h, a_i + hka_3, m_i + hkm_3, r_i + hkr_3),$$

$$km_4 = \xi(a_i + ka_3)((m_i + km_3 + r_i + kr_3) - \eta(m_i + km_3) + \mu(a_i + ka_3)(m_i + km_3) - \sigma(a_i + ka_3)(m_i + km_3) - \alpha(m_i + km_3)), \quad (2.35)$$

$$kr_4 = f_3(t_i + h, a_i + hka_3, m_i + hkm_3, r_i + hkr_3),$$

$$kr_4 = \eta(m_i + km_3) - \beta(m_i + kr_3) + \mu(a_i + ka_3)(r_i + kr_3) - \sigma(a_i + ka_3)(r_i + kr_3) - \alpha(r_i + kr_3). \quad (2.36)$$

For substituting Eqs. (2.25), (2.28), (2.31) and (2.34) in Eq. (2.22) to get the numerical solutions of a_i, substituting Eqs. (2.26), (2.29), (2.32) and (2.35) in Eq. (2.23) to compute the numerical solutions of m_i, in the same proses, substituting Eqs. (2.27), (2.30), (2.33) and (2.36) in equation (2.24) to obtain the numerical solutions of r_i, for all i-iterations, $i = 0,1,\dots,m$.

2.5 Results and Discussion:

The approximate solutions for nonlinear alcohol consumption model in Spain are analyzed and discussed in this section then listed in Table 2.3. The predicted values of variables $a(t)$, $m(t)$ and $r(t)$ for alcohol consumption model [73] had been given. Since the exact solution is not available for the current model, the predicted values are used to compare between the current approximate solutions of ADM and VIM with the predicted values [73] in the interval (0,10) from 1997 to 2007. For comparison purpose, the corresponding difference measure error of $a(t)$, $m(t)$ and $r(t)$ for ADM and VIM methods are shown numerically in Table 2.4, where the difference measure error in this study is the absolute value of the difference between the analytic solutions and the predicted values. In Table 2.4, the difference measure error of ADM for $a(t)$ have smaller values from 2003 to 2005 and 2007 than the VIM, while the difference measure error of ADM for $m(t)$ are smaller than VIM from 1999 untill 2001 and 2007. For $r(t)$, the difference measure

error of ADM in the interval (0,10) from 1997 to 2007 is oscillatory with VIM error.

The Figures describe the behavior of alcohol drinking habit from 1997 to 2007. Figure 2.1 (a) of $a(t)$ shows the ADM and VIM obtained results near to some predicted values in 2001 until 2005. While Figure 2.1 (b) of $m(t)$ shows the predicted values around both ADM and VIM obtained results. Regarding Figure 2.1 (c) of $r(t)$, both ADM and VIM curves obtained results converge to the predicted values in 2001 until 2005.

For Figure 2.1 (a) that related to non-drink alcohol people $a(t)$, the ADM curve, there is small deacresing from 1997 to 2007. More other, there exists a variation between them such that the VIM curve is higher level than ADM curve. On the other hand, both ADM and VIM curves of non-risk-drink alcohol people $m(t)$ have higher that appears during the ten years from 1997 till 2007 in Figure (b). Figure (c) illustrates the decrease in the risk-drink alcohol people $r(t)$ through the ten years under study for both ADM and VIM curves. The results are calculated by Mathematica software, the Figures are drawn by the Magic Plot program. Finally, the percentage of non-drink alcohol people $a(t)$ and the risk-drink alcohol people $r(t)$ are almost decreasing, but increase with the non-risk-drink alcohol people $m(t)$.

Table 2.3: Approximate solutions and predicted values [73] of the alcohol consumption model

Sub. pop	Method	1997	1999	2001	2003	2005	2007
$a(t)$	Predicted Values	0.362	0.383	0.363	0.359	0.354	0.400
	ADM	0.362	0.36180243	0.36160497	0.36140759	0.36121033	0.36101316
	VIM	0.362	0.36223274	0.36245419	0.36266487	0.36286526	0.36305586
$m(t)$	Predicted Values	0.581	0.578	0.581	0.588	0.591	0.566
	ADM	0.581	0.58195819	0.58288472	0.58378022	0.58464529	0.58548056
	VIM	0.581	0.58196578	0.58291479	0.58384724	0.58476327	0.58566309
$r(t)$	Predicted Values	0.057	0.039	0.056	0.053	0.055	0.034
	ADM	0.057	0.05580150	0.05463109	0.05348808	0.05237178	0.05128152
	VIM	0.057	0.05580148	0.05463101	0.05348789	0.05237147	0.05128105

Table 2.4: Difference measure error for ADM and VIM solutions as relative the predicted values

Sub. pop	Difference measure error	1999	2001	2003	2005	2007
$a(t)$	ADM	0.02119757	0.00139503	0.00240759	0.00721033	0.03898684
	VIM	0.02076726	0.00054581	0.00366487	0.00886526	0.03694414
$m(t)$	ADM	0.00395819	0.00188472	0.00421978	0.00635471	0.01948056
	VIM	0.00396578	0.00191479	0.00415276	0.00623673	0.01966309
$r(t)$	ADM	0.01680150	0.00136891	0.00048808	0.00262822	0.017281517
	VIM	0.01680148	0.00136899	0.00048789	0.00262853	0.01728105

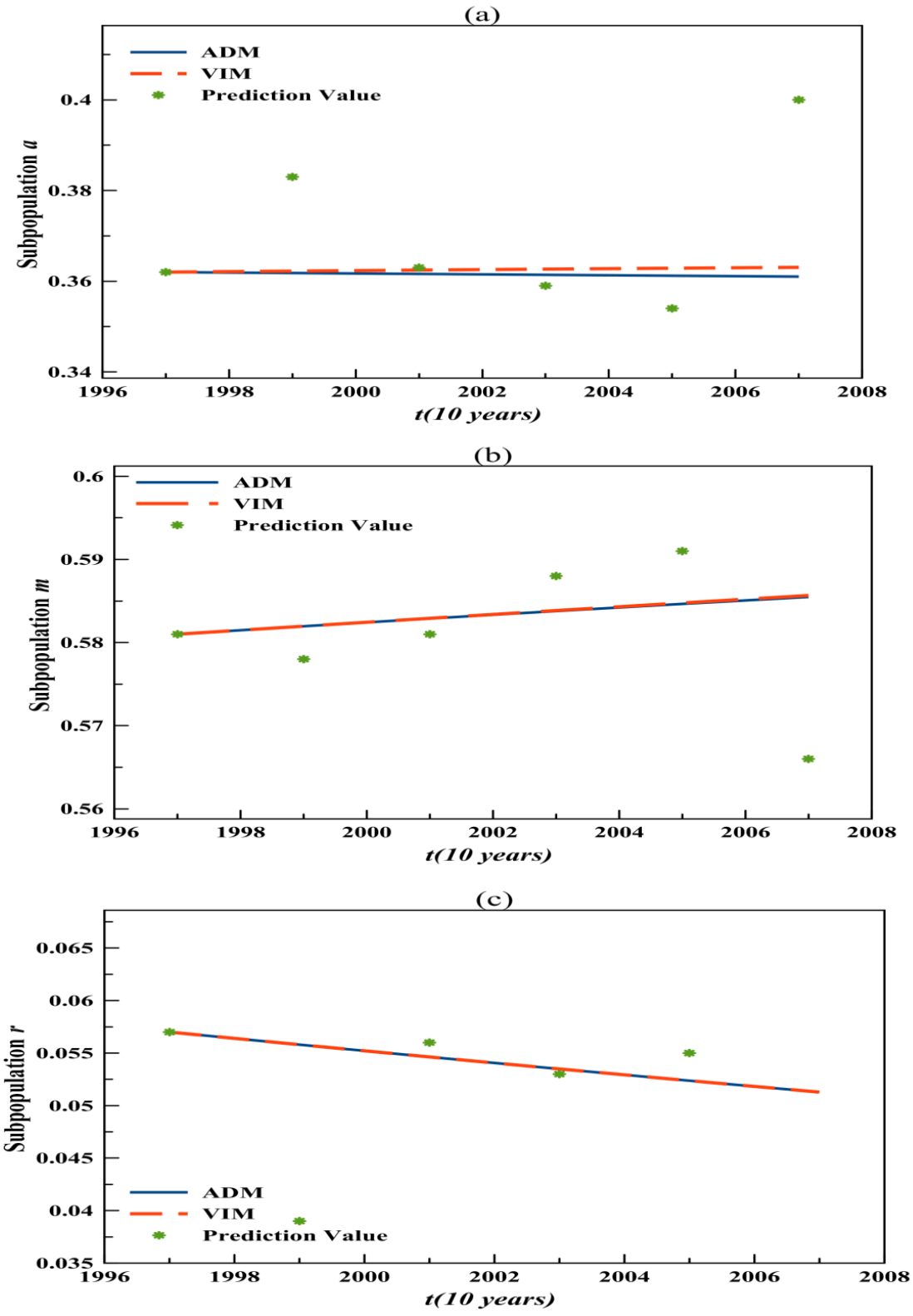

Figure 2.1: Variation of analytic solutions for ADM and VIM around predicted values [73] of (a) $a(t)$, (b) $m(t)$ and (c) $r(t)$ from 1997 to 2007 years

The numerical solutions for nonlinear alcohol consumption model in Spain are analyzed and discussed in this section then listed in Table 2.5. The predicted values of variables $a(t)$, $m(t)$ and $r(t)$ for alcohol consumption model, [73] had been given. In the current study, the exact solution is not available, therefore the predicted values have been treated to compare between the current numerical solutions of FD or RK4 and the predicted values [73] in the interval (0,10) from 1997 to 2007.

For comparison purpose, the corresponding difference measure error of $a(t)$, $m(t)$ and $r(t)$ for FD and RK4 methods are shown numerically in Table 2.6, where the difference measure error in this study is the absolute value of the difference between the numerical solutions and the predicted values. The difference measure error of FD for $a(t)$ have the smallest value (0.13147033) when (h=1) and it is smaller than the other method Moreover RK4, the difference measure error of FD for $m(t)$ are the smallest (0.12525489) when (h=1) and it is smaller than RK4. For $r(t)$, the smallest difference error for $a(t)$ is (0.00437743) with RK4 when (h=0.25) in the interval (0,10).

The Figuer 2.2 when h=1 (real step size) and m=10 (number 0f iteration) describe the behavior of alcohol drinking habit from 1997 to 2007. Figure 2.2 (a) of $a(t)$ shows the FD and RK4 obtained results near to some predicted values in 2001 until 2005. While Figure 2.2 (b) of $m(t)$ shows the predicted values around both FD and RK4 obtained results from 1997 until 2005. Regarding to Figure 2.2 (c) of $r(t)$, both FD and RK4 curves obtained results converge to the predicted values in 2001 until 2005.

For Figure 2.2 (a) that related to non-drink alcohol people $a(t)$, the curves for both FD and RK4 are decreasing from 1997 to 2007. Moreover, there is not exists a variation between them and the two curves are keep the same level. Both FD and RK4 curves of non-risk-drink alcohol people $m(t)$ have higher that appears during the ten years from 1997 until 2007 in Figure2.2 (b). On the other hand Figure2.2 (c) of the risk-drink alcohol people $r(t)$ through the ten years under study for both FD and RK4 curves have the same level. The results are calculated by the Matlab 2013 software for numerical method FD and RK4, the Figures are drawn by the Magic Plot program. Finally, the curves of non-drink alcohol people $a(t)$ are decreasing gradually, but increase with the non-risk-drink alcohol people $m(t)$ and the risk-drink alcohol people $r(t)$ have the same level.

Table 2.5: Numerical solutions for the alcohol consumption model from 1997 to 2007 (when t =10)

Model Variables	Predicted Values [73]	Step Size, h (year)	FD	RK4
$a(t)$	0.400	1	0.23052967	0.23038835
		0.5	0.23040252	0.23038853
		0.25	0.23037078	0.23038863
$m(t)$	0.566	1	0.70625489	0.70634659
		0.5	0.70637209	0.70634614
		0.25	0.70640135	0.70634591
$r(t)$	0.034	1	0.06321544	0.06139526
		0.5	0.06322538	0.06138333
		0.25	0.06322787	0.06137743

Table 2.6: Difference measure error $|E_p|$ for FD and RK4 solutions with the predicted values [73] from 1997 to 2007 (when t =10)

Model Variables	Predicted Values [73]	Step Size, h(year)	FD (10 iter.)	RK4 (10 iter.)
$a(t)$	0.400	1	0.13147033	0.13161165
		0.5	0.13159747	0.13161147
		0.25	0.13162922	0.13161137
$m(t)$	0.566	1	0.12525489	0.12534659
		0.5	0.12537208	0.12534614
		0.25	0.12540135	0.12534591
$r(t)$	0.034	1	0.00621543	0.00439526
		0.5	0.00622538	0.00438333
		0.25	0.00622786	0.00437743

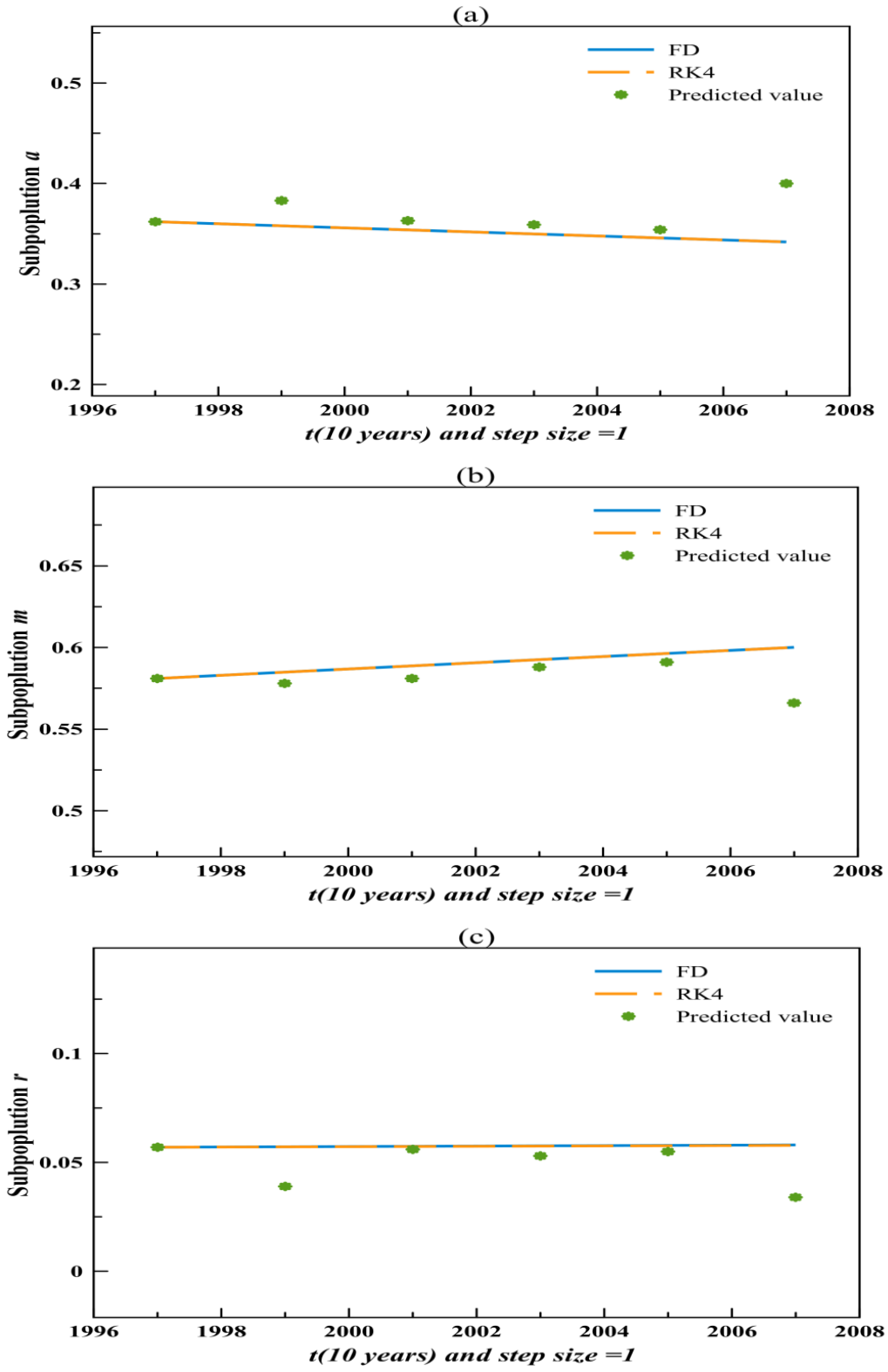

Figure 2.2: Variation of numerical solutions by using FD and RK4 around predicted values [73] of (a) $a(t)$, (b) $m(t)$ and (c) $r(t)$ from 1997 to 2007 years when h=1

2.6 Results Analysis:

In the current study, the purpose of using analytic and numerical methods is to solve such difficult nonlinear system that do not have available exact solution and to see the convergence of obtained results to the predicted values.

The convergence of the results for the analytic methods which are Adomian decomposition and variational iteration methods are examined in the nonlinear case. These methods have been known as a powerful device for solving a system of ordinary, partial differential equations or Integral equations and so on. In our work, they are used for solving a system of nonlinear ordinary differential equations. The behavior of unhealthy social habit is (alcohol consumption in Spain) is analyzed, based on the epidemiological model through ten years under study. The ADM and VIM methods help to analyze the effects of the unhealthy social habit of alcohol consumption. The obtained results are shown there is increasing in alcohol consumption with non-risk-drink consumers and declining the risk-drink consumers during the ten years under study. For the number of the non-drink consumers has a small increase with the VIM and maintains its level with respect to the ADM. The most predicted values [73] around the ADM and VIM curves. Other analytical methods can solve such system under study like homotopy perturbation method, and Semi Analytical Iterative method Temimi and Ansari.

There is a convergence of the results for the numerical methods which are FD and RK4 has been noted in the nonlinear case. The FD and RK4 methods help to analyze the effects of bad social habit. The obtained results have been shown that there is increasing in alcohol consumption

with non- risk-drink consumers, there is increasing gradually on the of risk-drink consumers and decreasing gradually with the non-drink consumers during the ten years from 1997 until 2007 under study. The most predicted value [73] around the FD and RK4 curves. The most closer numerical results to the predicted values for the non-drink and non-risk-drink consumer are with FD while the most closer numerical results for the risk consumer are with RK4.

CHAPTER 3:
Application of Some Analytical and Numerical Methods on Smoking Habit Model

3.1 Introduction:

Epidemiological models are used to study the epidemiological processes as infectious diseases. When a bad habit is spreading rapidly, the model that is being established from spread this bad habit is called a social epidemic model. Some researchers studied such these models like smoking habit [33], cocaine consumption [71], alcohol consumption [73] or obesity epidemics [74]. Lung cancer affects smoking by 10 times more than smokers as one of the ten-day smoke dying with lung Cancer. In Spain, the smoking habit is estimated that around 55,000 deaths each year are attributable to smoking [63].

Epidemiological models are studied to analyze epidemic stages and infectious diseases. Many researchers analyzed the social habits models, such as Guerrero, Santonja and. Villanueva in 2006, studied to analyze the Spanish smoke-free legislation of 2006 [33]. In 2011, Sánchez, et al. predicted the cocaine consumption in Spain [71]. In 2018, Mohammed, et al. A non-conventional hybrid numerical approach to solve the multi-dimensional random sampling for cocaine abuse in Spain [57]. In 2010, the economic cost of alcohol consumption was studied in Spain by Santonja, et al. [73]. The mathematical modeling of the social obesity epidemic in the region of Valencia in Spain was being modeled in 2010 by Santonja, et al. [74]. In 2015, Mohammed, et al. Solved the weight reduction model due to health campaigns in Spain numerically using several types of Runge-Kutta method [57]. In the purely hyperbolic

case, an adequate definition of the numerical viscosity required by the WENO scheme was provided in 2013 when capillary effects are exist [34].

Some classic analytic methods such as ADM and VIM and some numerical methods like FD and RK4 have been applied to a social epidemic model which is a smoking habit, in this chapter. Since ADM, VIM, FD and RK4 are the most suitable analytic and numerical methods to get accurate results for nonlinear autonomous system that has multi-variable and multi-parameters. Therefore, they are used in this study to solve such system that is difficult to find its exact solution. Moreover, ADM, VIM, FD and RK4 are easy and efficient methods that can give the most reliable for the solutions.

3.2 Mathematical Model of Smoking Habit:

The current model has been used successfully to predict the evolution of the smoking habit in Spain after the Spanish smoke-free law in 2006 was applied [33]. The population consists of four types of individuals, whose proportions are denoted by a (non-smokers), b (normal smokers), c (excessive smokers) and d (ex-smokers). All of them are functions of time. Four ordinary differential equations of the first order can describe the nonlinear smoking habit model in Eqs. (3.1), (3.2), (3.3) and (3.4) as follows:

$$a'(t) = \mu\big(1 - a(t)\big) - \beta a(t)\big(b(t) + c(t)\big) \quad (3.1)$$

$$b'(t) = \beta a(t)\big(b(t) + c(t)\big) + \rho d(t) + \alpha c(t) - (\gamma + \lambda + \mu)b(t) \quad (3.2)$$

$$c'(t) = \gamma\, b(t) - (\alpha + \delta + \mu)c(t) \quad (3.3)$$

$$d'(t) = \lambda b(t) + \delta c(t) - (\rho + \mu)d(t) \quad (3.4)$$

The initial conditions of Eqs. (3.1), (3.2), (3.3) and (3.4) are: $a(t=0)$=0.5045, $b(t=0)$=0.2059, $c(t=0)$=0.1559, $d(t=0)$=0.1337, with the predicted parameters that are given as: $\mu = 0.01$, $\beta = 0.0381$, $\rho = 0.0425$, $\alpha = 0.1244$, γ =0.11750, λ =0.0498 and δ =0.0498 [33].

Table 3.1: Variables of smoking habit model, [33]

$a(t)$	Social class who never smokes from the total population.
$b(t)$	Social class of people who smoke less than 20 cigarettes per day.
$c(t)$	Social class who smoke more than 20 cigarettes per day.
$d(t)$	The social class of ex-smokers.

Table 3.2: Parameters of smoking habit model, [33]

μ	Rate of births Spain.
β	The transmission of smoke infection because of social pressure to adopt smoking habit.
ρ	The rate of returns to smoking.
α	The rate of smokers who are excessively and who are becoming a normally smoker by reducing the number of cigarettes per day.
γ	The rate of smokers who are normally and who are becoming an excessive smoker by reducing the number of cigarettes per day.
λ	The rate of normal smokers who stop smoking.
δ	The rate of excessive smokers who stop smoking.

3.3 Problem Solution using Analytical Methods:

In this section has been used two Analytical methods which are ADM and VIM to solve the epidemic model of smoking habit.

3.3.1 Adomian Decomposition Method (ADM):

The nonlinear system of Eqs. (3.1), (3.2), (3,3) and (3.4) of the smoking habit model can solve by the Adomian decomposition method with the given initial condition. Let l be an operator that is given by $l = \frac{d}{dt}$ and the inverse of this operation is $l^{-1} = \int_0^t (.)dt$, then by applying l^{-1} for both sides of Eqs. (3.1), (3.2), (3.3) and (3.4), to obtain:

$$\mathrm{a(t)} - \mathrm{a(0)} = l^{-1}\left(\mu\big(1 - \mathrm{a(t)}\big) - \beta \mathrm{a(t)}\big(\mathrm{b(t)} + \mathrm{c(t)}\big)\right),$$

where $a_0 = 0.5045$. Similarity,

$$b(t) - b(0) = l^{-1}\Big(\beta * \mathrm{a(t)}\big(\mathrm{b(t)} + \mathrm{c(t)}\big) + \rho \mathrm{d(t)} + \alpha \mathrm{c(t)} - (\gamma + \lambda + \mu)\mathrm{b(t)}\Big),$$ where $b_0 = 0.2059$.

Also,

$$c(t) - c(0) = l^{-1}\big(\gamma \mathrm{b(t)} - (\alpha + \delta + \mu)\mathrm{c(t)}\big),$$

where $c_o = 0.1559$, and

$$d(t) - d(0) = l^{-1}(\lambda \mathrm{b(t)} + \delta \mathrm{c(t)} - (\rho + \mu)\mathrm{d(t)}),$$

where $d_0 = 0.1337$.

The above equations can generate as follows with k iterations, $k \geq 0$.

$$a_{k+1} = l^{-1}(\mu(1 - a_k) - \beta A_k - \beta B_k), \text{ for all } k \geq 0 \quad (3.5)$$

$$b_{k+1} = l^{-1}(\beta A_k + \beta B_k + \rho d_k + \alpha c_k - (\gamma + \lambda + \mu)b_k), \text{ for all } k \geq 0 \quad (3.6)$$

$$c_{k+1} = l^{-1}(\gamma\, b_k - (\alpha + \delta + \mu)c_k), \text{ for all } k \geq 0 \quad (3.7)$$

$$d_{k+1} = l^{-1}(\lambda b_k + \delta c_k - (\rho + \mu)d_k), \text{for all } k \geq 0. \quad (3.8)$$

The general form of the nonlinear terms $A_k(t)$ and $B_k(t)$ have to be:

$A_k = (\sum_{n=0}^{2} a_n)(\sum_{n=0}^{2} b_n)$, for all $k = 0,1,2$

$$A_k = (a_0 + a_1 + a_2)(b_0 + b_1 + b_2),$$
$$= a_0b_0 + a_0b_1 + a_0b_2 + a_1b_0 + a_1b_1 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2$$

$B_k = (\sum_{n=0}^{2} a_n)(\sum_{n=0}^{2} c_n)$, for all $k = 0,1,2$

$$B_k = (a_0 + a_1 + a_2)(c_0 + c_1 + c_2)$$
$$= a_0c_0 + a_0c_1 + a_0c_2 + a_1c_0 + a_1c_1 + a_1c_2 + a_2c_0 + a_2c_1 + a_2c_2.$$

The nonlinear terms of A_0 and B_0 explain as the following:

$A_0 = a_0b_0$, $B_0 = a_0c_0$.

Substituting for all a_0, A_0 and B_0 by Eq.(3.5), to get

$a_1 = -0.00199932t$,

by substituting for all b_0, c_0, d_0, A_0 and B_0 by Eq.(3.6), to obtain

$b_1 = -0.00447553t$,

As well as, substituting for all b_0 and c_0 by Eq.(3.7), to find

$c_1 = -0.00452353t$,

In the same ways, substitute all b_0, c_0 and d_0 by Eq.(3.8), to get

$e_1 = 0.01099838t$,

Now a_2, b_2, c_2 and d_2 can be found. First, the nonlinear terms of A_1 and B_1 are given in the following formula:

$A_1 = a_0b_1 + b_0a_1$,

$B_1 = a_0c_1 + c_0a_1$.

Then by substituting all a_1, A_1 and B_1 by Eq. (3.5), to obtain

$a_2 = 0.01t + 0.00011026t^2$,

and for substituting all b_1, c_1, d_1, A_1 and B_1 by Eq.(3.6), to find

$b_2 = 0.00029629t^2$,

by the same method substituting for all b_1 and c_1 by Eq.(3.7), we get

$c_2 = 0.00015368t^2$,

similarity substituting for all b_1, c_1 and d_1 by Eq.(3.8), we get

$d_2 = -0.000512785t^2$,

To obtain a_3, b_3, c_3 and d_3, at the same previous steps, the nonlinear terms of A_2 and B_2 are given as:

$A_2 = a_0 b_2 + a_1 b_1 + b_0 a_2$

$B_2 = a_0 c_2 + a_1 c_1 + c_0 a_2$

Substituting all $a_2, A_2 \; and \; B_2$ by Eq.(3.5), we obtain

$a_3 = 0.01t - 0.00011892t^2 - 0.00000399t^3$.

When substituting all b_2, c_2, d_2, A_2 and B_2 by Eq.(3.6), to find

$b_3 = 0.00006892t^2 - 0.00001618t^3$.

similarity substituting b_2 and c_2 by Eq.(3.7), we get

$c_3 = 0.00000216t^3$.

Substituting also all b_2, c_2 and d_2 by Eq.(3.8), to find

$d_3 = 0.0498t + 0.00011028t^3$.

The Adomian decomposition method assumes that the unknown functions $a(t)$, $b(t)$, $c(t)$ and $d(t)$ can be expressed by an infinite series as a polynomial of the form:

$a(t) = \sum_{k=0}^{\infty} a_k$, $b(t) = \sum_{k=0}^{\infty} b_k$, $c(t) = \sum_{k=0}^{\infty} c_k$, $d(t) = \sum_{k=0}^{\infty} d_k$,

$$a(t) = \sum_{k=0}^{\infty} a_k = a_0 + a_1 + a_2 + a_3 \ldots$$

$$a(t) = 0.5045 + 0.01800068t - 0.00000866t^2 - 0.00000399t^3 + \cdots \quad (3.9)$$

$$b(t) = \sum_{k=0}^{\infty} b_k = b_0 + b_1 + b_2 + b_3 + \cdots$$

$$b(t) = 0.2059 - 0.00447553t + 0.00036521t^2 - 0.00001618t^3 + \ldots \quad (3.10)$$

$$c(t) = \sum_{k=0}^{\infty} c_k = c_0 + c_1 + c_2 + c_3 + \cdots$$

$$c(t) = 0.1559 - 0.00452353t + 0.00015368t^2 + 0.00000216t^3 + \cdots \quad (3.11)$$

$$d(t) = \sum_{k=0}^{\infty} d_k = d_0 + d_1 + d_2 + d_3 + \cdots$$

$$d(t) = 0.1337 + 0.06079838\text{t} - 0.000512785t^2 + 0.00011028t^3 + \cdots \quad (3.12)$$

3.3.2 Variational Iteration Method (VIM):

The nonlinear system of the smoking habit model can be solved by the VIM with given initial condition. The correction functional for the system of Eqs. (3.1), (3.2), (3.3) and (3.4) become:

$$a_{k+1} = a_k + \int_0^t \lambda \left(a'_k - \left(\mu(1 - a_k) - \beta a_k (b_k + c_k) \right) \right), \; k \geq 0 \quad (3.13)$$

$$b_{k+1} = b_k + \int_0^t \lambda \left(b'_k - (\beta a_k (b_k + c_k) + \rho d_k + \alpha c_k - (\gamma + \lambda + \mu) b_k) \right), \quad k \geq 0 \quad (3.14)$$

$$c_{k+1} = c_k + \int_0^t \lambda \left(c'_k - (\gamma b_k - (\alpha + \delta + \mu) c_k) \right), \; k \geq 0 \quad (3.15)$$

$$d_{k+1} = d_k + \int_0^t \lambda \left(d'_k - (\lambda b_k + \delta c_k - (\rho + \mu) d_k) \right), \; k \geq 0. \quad (3.16)$$

The Lagrange multiplier is $\lambda = -1$. By substituting this value in equations (3.13), (3.14), (3.15) and (3.16), the zero terms become: a_0 =0.5045, b_0=0.2059, c_0=0.1559, d_0=0.1337.

Now, to find a_1, b_1, c_1 and d_1, in Eqs. (3.13), (3.14), (3.15) and (3.16), when substituting (k =0), the following has been obtained:

$$a_1 = 0.5045 - 0.00199932t,$$
$$b_1 = 0.2059 - 0.00447554t,$$
$$c_1 = 0.1559 - 0.00452354t,$$
$$d_1 = 0.1337 + 0.01099838t,$$

By the same way, if we have (k =1), in Eqs. (3.13), (3.14), (3.15) and (3.16) the following has been gotten:

$$a_2 = 0.5045 - 0.00199932t + 0.00011026t^2 - 2.28498716 \times 10^{-7}t^3,$$
$$b_2 = 0.2059 - 0.00447554t + 0.00024884t^2 + 2.28498716 \times 10^{-7}t^3,$$
$$c_2 = 0.1559 - 0.00452353t + 0.00015368t^2,$$
$$d_2 = 0.1337 + 0.01099838t - 0.00051279t^2,$$

Continuing in the same way, when (k=2), the following can be achieved:

$$a_3 = 0.5045 - 0.00199932t + 0.00011026t^2 - 0.00000527t^3 + 1.99202672 \times 10^{-8}t^4 - 5.52507601 \times 10^{-10}t^5 + 4.65135788 \times 10^{-13}t^6 + 8.52541875 \times 10^{-16}t^7 \quad (3.17)$$

$$b_3 = 0.2059 - 0.00447554t + 0.00024884t^2 - 0.00001228t^3 - 2.69344475 \times 10^{-8}t^4 + 3.50390346 \times 10^{-10}t^5 - 4.2405435 \times 10^{-13}t^6 - 2.84180625 \times 10^{-16} \quad (3.18)$$

$$c_3 = 0.1559 - 0.00452354t + 0.00015368t^2 + 3.10383361 \times 10^{-7}t^3 + 6.71214979 \times 10^{-9}t^4 \quad (3.19)$$

$$d_3 = 0.1337 + 0.01099838t - 0.00051279t^2 + 0.00001566t^3 + 2.84480901 \times 10^{-9}t^4 \quad (3.20)$$

And so on, continue in order to get better approximations:

$a(t) = \lim_{k\to\infty} a_k(t)$, $b(t) = \lim_{k\to\infty} b_k(t)$, $c(t) = \lim_{k\to\infty} c_k(t)$ and $d(t) = \lim_{k\to\infty} d_k(t)$.

3.4 Numerical Methods:

In this section has been used two numerical methods which are FD and RK4 to solve the epidemic model of smoking habit.

3.4.1 Finite Difference (FD) Method:

The nonlinear system of Eqs. (3.1), (3.2), (3.3) and (3.4) of the smoking habit model can solve using the finite difference method with the initial conditions: a_0 =0.5045, b_0=0.2059, c_0=0.1559 and d_0=0.1337, and the predicted parameters that are given in Table 3.2, and the real step size $h = 1,\ 0.5,\ 0.25$ where $h = \frac{Upper\ bound - Lower\ bound}{m}$, in this study, m=16 refers to numbers of years from 2006 to 2022. In the same time, m refers to the number of iterations.

In order to find a_1, b_1, c_1 and d_1, backward finite difference (BFD) can be used as follows:

$$a_1 = a_0 + h\big(\mu(1 - a_0) - \beta a_0(b_0 + c_0)\big), \quad (3.21)$$

$$b_1 = b + h\big(\beta a_0(b_0 + c_0) + \rho d_0 + \alpha c_0 - (\gamma + \lambda + \mu)\big), \quad (3.22)$$

$$c_1 = c_0 + h(\gamma b_0 - (\alpha + \delta + \mu)c_0), \quad (3.23)$$

$$d_1 = d_1 + h(\lambda b_0 + \delta c_0 - (\rho + \mu)d_0), \quad (3.24)$$

The a_1, b_1, c_1 and d_1 are calculated from Eqs. (3.21), (3.22), (3.23) and (3.24) to obtain the following values: a_1 =0.50250068, b_1 =0.20142446, c_1 =0.15137647 and d_1 =0.14469839, respectively.

Now, the central finite difference (CFD) method can be used to find the next steps and so on for m times follows:

$$a_{i+1} = a_{i-1} + 2h\big(\mu(1 - a_i) - \beta a_i(b_i + c_i)\big), \quad (3.25)$$

$$b_{i+1} = b_{i-1} + 2h\big(\beta a_i(b_i + c_i) + \rho d_i + \alpha c_i - (\gamma + \lambda + \mu)\big), \quad (3.26)$$

$$c_{i+1} = c_{i-1} + 2h(\gamma b_i - (\alpha + \delta + \mu)c_i), \quad (3.27)$$

$$d_{i+1} = d_{i-1} + 2h(\lambda b_i + \delta c_i - (\rho + \mu)d_i), \quad (3.28)$$

for $i = 1,2,\dots,m$, to find $a_1, a_2, \dots, a_m$, $m_1, m_2, \dots, m_m$ and $r_1, r_2, \dots, r_m$ that consider as numerical solutions for smoking habit model.

3.4.2 Runge-Kutta of 4th Order (RK4) Method:

RK4 is one of the most accurate iteration numerical methods. The nonlinear system of Eqs. (3.1), (3.2), (3.3) and (3.4) of the smoking habit model can be solved by RK4 with initial conditions: a_0, b_0, c_0 and d_0, with the predicted parameters in Table 3.2.

For the general form of RK in Eq.(1.24) in Chapter 1, where

$$a_{i+1} = a_i + \frac{1}{6}(ka_1 + 2ka_2 + 2ka_3 + ka_4)h \quad (3.29)$$

$$b_{i+1} = b_i + \frac{1}{6}(kb_1 + 2kb_2 + 2kb_3 + kb_4)h \quad (3.30)$$

$$c_{i+1} = c_i + \frac{1}{6}(kc_1 + 2kc_2 + 2kc_3 + kc_4)h \quad (3.31)$$

$$d_{i+1} = d_i + \frac{1}{6}(kd_1 + 2kd_2 + 2kd_3 + kd_4)h \quad (3.32)$$

Now, we must find ka_1, kb_1, kc_1 and kd_1as follows:

$$ka_1 = f_1(t_i, a_i, b_i, c_i, d_i)$$
$$ka_1 = \mu(1 - a_i) - \beta a_i(b_i + c_i), \quad (3.33)$$
$$kb_1 = f_2(t_i, a_i, b_i, c_i, d_i)$$
$$kb_1 = \beta a_i(b_i + c_i) + \rho d_i + \alpha c_i - (\gamma + \lambda + \mu)b_i, \quad (3.34)$$
$$kc_1 = f_3(t_i, a_i, b_i, c_i, d_i)$$
$$kc_1 = \gamma b_i - (\alpha + \delta + \mu)c_i, \quad (3.35)$$
$$kd_1 = f_4(t_i, a_i, b_i, c_i, d_i)$$
$$kd_1 = \lambda b_i + \delta c_i - (\rho + \mu)d_i, \quad (3.36)$$

Also, to find ka_2, kb_2, kc_2 and kd_2 as follows:

$$ka_2 = f_1\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$
$$ka_2 = \mu\big(1 - (a_i + 0.5ka_1h)\big) - \beta(a_i + 0.5ka_1h))(b_i + 0.5kb_1h) + (c_i + 0.5kc_1h), \quad (3.37)$$
$$kb_2 = f_2\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$
$$kb_2 = \beta(a_i + 0.5ka_1h)\big((b_i + 0.5kb_1h) + (c_i + 0.5kc_1h)\big) + \rho(d_i + 0.5kd_1h) + \alpha(c_i + 0.5kc_1h) - (\gamma + \lambda + \mu)(b_i + 0.5kb_1\text{h}), \quad (3.38)$$
$$kc_2 = f_3\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right),$$

$$kc_2 = \gamma(b_i + 0.5kb_1h) - (\alpha + \delta + \mu)(c_i + 0.5kc_1h), \quad (3.39)$$

$$kd_2 = f_4\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_1, b_i + \frac{1}{2}hkb_1, c_i + \frac{1}{2}hkc_1, d_i + \frac{1}{2}hkd_1\right)$$

$$kd_2 = \lambda(b_i + 0.5kb_1h) + \delta(c_i + 0.5kc_1h) - (\rho + \mu)(d_i + 0.5kd_1h), \quad (3.40)$$

To get ka_3, kb_3, kc_3 and kd_3 as follows:

$$ka_3 = f_1\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$ka_3 = \mu\big(1 - (a_i + 0.5ka_2h)\big) - \beta(a_i + 0.5ka_2h))(b_i + 0.5kb_2h) + (c_i + 0.5kc_2h), \quad (3.41)$$

$$kb_3 = f_2\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kb_3 = \beta(a_i + 0.5ka_2h)\big((b_i + 0.5kb_2h) + (c_i + 0.5kc_2h)\big) + \rho(d_i + 0.5kd_2h) + \alpha(c_i + 0.5kc_2h) - (\gamma + \lambda + \mu)(b_i + 0.5kb_2\mathrm{h}), \quad (3.42)$$

$$kc_3 = f_3\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kc_3 = \gamma(b_i + 0.5kb_2h) - (\alpha + \delta + \mu)(c_i + 0.5kc_2h), \quad (3.43)$$

$$kd_3 = f_4\left(t_i + \frac{1}{2}h, a_i + \frac{1}{2}hka_2, b_i + \frac{1}{2}hkb_2, c_i + \frac{1}{2}hkc_2, d_i + \frac{1}{2}hkd_2\right),$$

$$kd_3 = \lambda(b_i + 0.5kb_2h) + \delta(c_i + 0.5kc_2h) - (\rho + \mu)(d_i + 0.5kd_2h), \quad (3.44)$$

To obtain ka_4, kb_4, kc_4 and kd_4 as follows:

$$ka_4 = f_1(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$

$$ka_4 = \mu\big(1 - (a_i + ka_3h)\big) - \beta(a_i + ka_3h))(b_i + kb_3h) + (c_i + kc_3h), \quad (3.45)$$

$$kb_4 = f_2(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$

$$kb_4 = \beta(a_i + ka_3h)\big((b_i + kb_3h) + (c_i + kc_3h)\big) + \rho(d_i + kd_3h) + \alpha(c_i + kc_3h) - (\gamma + \lambda + \mu)(b_i + kb_3\mathrm{h}), \quad (3.46)$$

$$kc_4 = f_3(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$
$$kc_4 = \gamma(b_i + kb_3h) - (\alpha + \delta + \mu)(c_i + kc_3h), \quad (3.47)$$

$$kd_4 = f_4(t_i + h, a_i + hka_3, b_i + hkb_3, c_i + hkc_3, d_i + hkd_3),$$
$$kd_4 = \lambda(b_i + kb_3h) + \delta(c_i + kc_3h) - (\rho + \mu)(d_i + kd_3h). \quad (3.48)$$

For substituting Eqs. (3.33), (3.37), (3.41) and (3.45) in Eq. (3.29) to get the numerical solutions of a_i, By the same way, substituting Eqs (3.34), (3.38), (3.42) and (3.46) in Eq. (3.30) we get the numerical solutions of b_i, As well as, substituting Eqs. (3.35), (3.39), (3.43) and (3.47) in Eq. (3.31) we obtain the numerical solutions of c_i. And substituting Eqs. (3.36), (3.40), (3.44) and (3.48) in Eq. (3.32) to get the numerical solutions of d_i, $i = 0,1,\dots,m$.

3.5 Results and Discussion:

Approximate and numerical solutions for nonlinear smoking habit model in Spain are discussed and analyzed in this section. Table 3.3 is to validate the real and predicted values (2006-2009) [33] with approximate solutions. Where h=1, 0.5 and 0.25 are the step size and m=3 is the number of iterations.

The predicted values of variables $a(t)$, $b(t)$, $c(t)$ and $d(t)$ for smoking habit model, had been given [33]. The exact solution is not available in the current model. Therefore, a comparison between the predicted values and the real data with the expected approximate solutions of the analytic solutions for ADM and VIM methods, moreover, the expected approximate solutions of the numerical solutions for FD and RK4 methods, in the interval of years (0,16) from 2006 to

2022, has been done in Table 3.4. Where h=1,0.5 and 0.25 are the step size and m=16, is the number of iterations.

For the purpose of comparison, the difference measure error for $a(t)$, $b(t)$, $c(t)$ and $d(t)$ between the predicted value [33] and the ADM, VIM, FD and RK4 methods from 2006 to 2009 which are shown numerically in Table 3.5, where the difference measure error $|E_p|$ in Table 3.5, in this study, is the difference between the analytic solutions and the predicted values or the difference between the numerical solutions and the predicted values. Notes the difference measure error for $a(t)$ of FD method has the smallest value when (h=0.5) than with step size (h =1 and 0.25) and compared with the other methods under study with the different step size (h =1, 0.5 and 0.25). As well as, the difference measure errors of $b(t)$, $c(t)$ and $d(t)$ in VIM have the smallest errors that compared with ADM, FD and RK4 methods when (h =1, 0.5 and 0.25).

Figure 3.1, when h=1 (real step size) describes the behavior of smoking habit from 2006 to 2022. In Figure 3.1 (a) that is related to non-smoke people $a(t)$, the curve of ADM rises, this mean the people who do not smoke are increase through 16 years to 2022, while there is stable with the other methods VIM, FD and RK4, because these methods have the same nature which is iterative. These methods VIM, FD and RK4 agree with the previous study (Figure. 2, page 249) in [33].

Figure 3.1 (b) that related to normal smoke people $b(t)$, show us the curves of the four methods ADM, FD, VIM and RK4 under study are near to the predicted values from 2006 until 2013. After that, the curves

ADM, FD, VIM and RK4 gradually decrease a yearly until 2022. Only the curve of VIM is more decrease from 2013 until 2022 than the other curves.

While Figure 3.1 (c) that related to excessive smokers $c(t)$, the curve of all methods are decreasing. Observe that the numerical methods (FD and RK4) are more decreasing than the analytical methods (ADM and VIM). These results agree with previous study [33].

Figure 3.1 (d) that related to ex-smokers $d(t)$, notes that, there is increasing from 2006 to 2022 for all curves. The curves of analytical methods (ADM and VIM) are increasing more than the curves of the numerical methods (FD and RK4) from 2013 until 2022. The nature of ex-smokers in the current study is to agree with the previous study [33].

In Figure 3.2, when h=0.5 (real step size) explain the behavior of smoking habit through sixteen years from 2006 to 2022. In Figure 3.2 (a), the curve of ADM is increasing, this means, the people who do not smoke are increasing through 16 years, while there is stable with the other methods VIM, FD and RK4, these methods VIM, FD and RK4 are agree with the previous study (Figure. 2, page 249) in [33].

Figure 3.2 (b), the curves of the methods ADM, FD, VIM and RK4 under study are near to the predicted values from 2006 until 2013. The curves of ADM, FD, VIM and RK4 are decreased step by step. But the curve of VIM is more decrease from 2013 until 2022 than the other curves.

While Figure 3.2 (c), the curve of all methods are decreasing. We can be noted that the analytical methods (ADM and VIM) are more decreasing than the numerical methods (FD and RK4). These results agree with a previous study [33].

Figure 3.2 (d), can be noted that, there is increasing from 2006 to 2022 for all curves. The nature of ex-smokers in the current study agrees with the previous study [33].

Table 3.3 a: Approximate solutions of the smoking habit model from 2006 to 2009 (when t =3)

Model Variables		Real Data [33]		Predicted Values [33]		ADM	VIM (3 iter.)	Step Size, h (year)	FD (3 iter.)	RK4 (3 iter.)
		in 2006	in 2009	in 2006	in 2009					
$a(t)$	$b(t)$									
0.4997	0.1856	0.4997	0.4835	0.5041	0.5049	0.55832470	0.49939633	1	0.49929684	0.49940233
								0.5	0.49941218	0.49940232
								0.25	0.49940479	0.49940232
	0.1902					0.19500158	0.19437918	1	0.19416778	0.19442567
								0.5	0.19446101	0.19442561
								0.25	0.19443454	0.19442561

Table 3.3 b: Approximate solutions of the smoking habit model from 2006 to 2009 (when t =3)

Model Variables		Real Data [33]		Predicted Values [33]		ADM	VIM (3 iter.)	Step Size, h (year)	FD (3 iter.)	RK4 (3 iter.)
		in 2006	in 2009	in 2006	in 2009					
$c(t)$	0.1094	0.1094	–	0.1264	0.1240	0.14372090	0.14372145	1	0.14356661	0.14369647
								0.5	0.14369579	0.14369651
								0.25	0.14369626	0.14369651
$d(t)$	0.2053	0.2053	0.2017	0.1773	0.1805	0.16250281	0.16250304	1	0.16296876	0.16247554
								0.5	0.16243102	0.16247556
								0.25	0.16246441	0.16247556

Table 3.4: Expected approximate solutions of the smoking habit model from 2006 to 2022 (when t = 16)

Model Variables	ADM	VIM (16 iter.)	Step Size, h (year)	FD (16 iter.)	RK4 (16 iter.)
$a(t)$	0.77521396	0.48643685	1	0.49049314	0.49025842
			0.5	0.49031746	0.49025841
			0.25	0.49027319	0.49025841
$b(t)$	0.16532272	0.14627349	1	0.17438741	0.17011593
			0.5	0.17128695	0.17011592
			0.25	0.17041579	0.17011591
$c(t)$	0.12413672	0.12457661	1	0.11159163	0.11499751
			0.5	0.11404419	0.11499750
			0.25	0.11475214	0.11499750
$d(t)$	0.24252661	0.24427130	1	0.22352783	0.22462814
			0.5	0.22435139	0.22462817
			0.25	0.22455887	0.22462817

Table 3.5: Difference measure error $|E_p|$ for ADM, VIM, FD and RK4 solutions as relative to the predicted values [33] from 2006 to 2009 (when t = 3)

Model Variables	ADM in 2009	VIM (3 iter.) in 2009	Step Size, h (year)	FD (3 iter.) in 2009	RK4 (3 iter.) in 2009
$a(t)$	0.0534247	0.00550367	1	0.00560316	0.00549767
			0.5	0.00548782	0.00549768
			0.25	0.00549521	0.00549767
$b(t)$	0.00440158	0.00377918	1	0.30869684	0.30880233
			0.5	0.30881218	0.30880232
			0.25	0.30880479	0.30880232
$c(t)$	0.0197209	0.01972145	1	0.37529684	0.37540233
			0.5	0.37541217	0.37540232
			0.25	0.375404789	0.37540232
$d(t)$	0.01799719	0.01799696	1	0.31879684	0.31890233
			0.5	0.31891218	0.31890232
			0.25	0.31890479	0.31890232

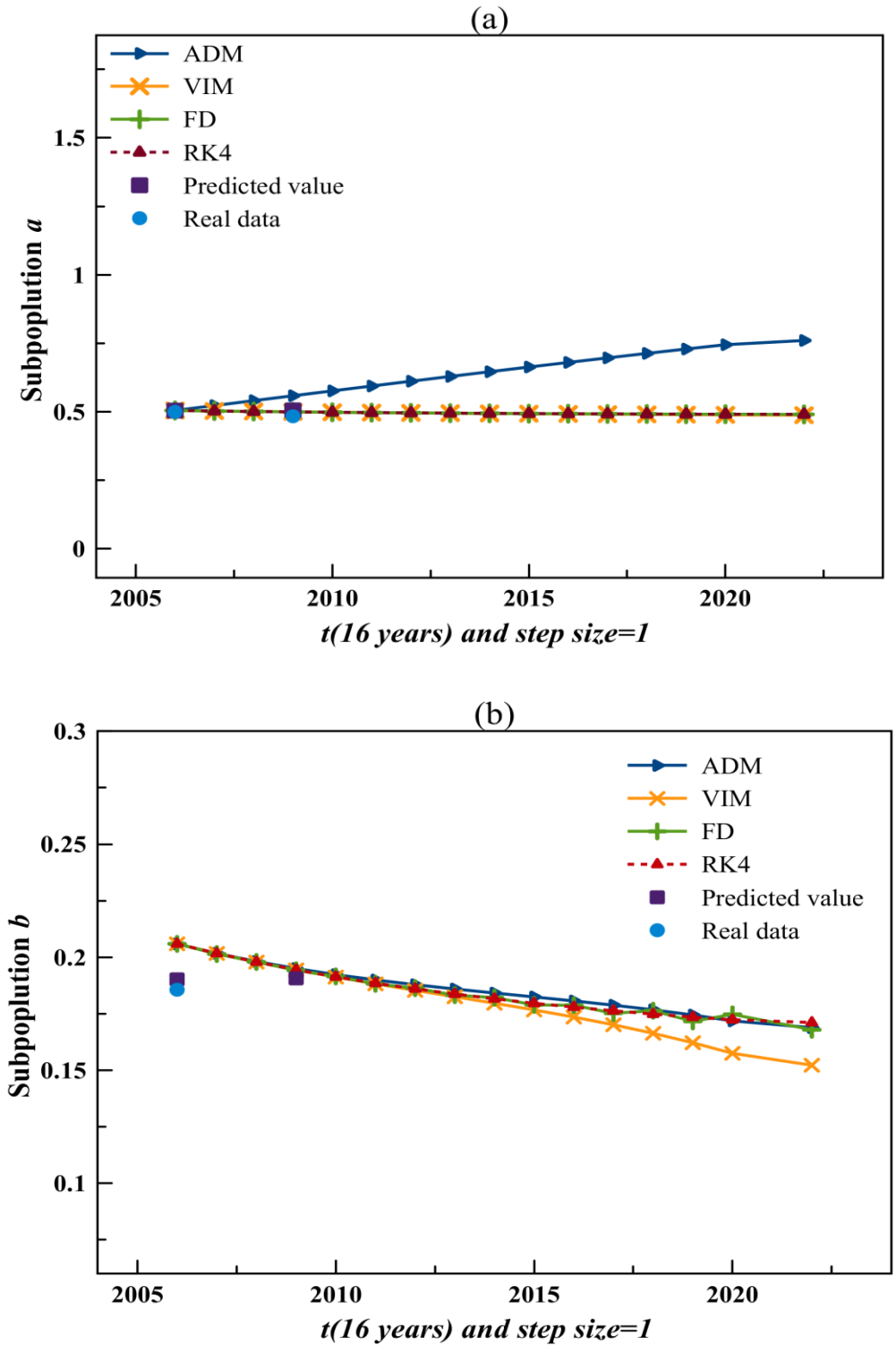

Figure 3.1 (a, b): Variation of approximate and numerical solutions by using ADM, VIM, FD and RK4 around predicted values [33] of (a) $a(t)$ and (b) $b(t)$ from 2006 to 2022 years when h=1

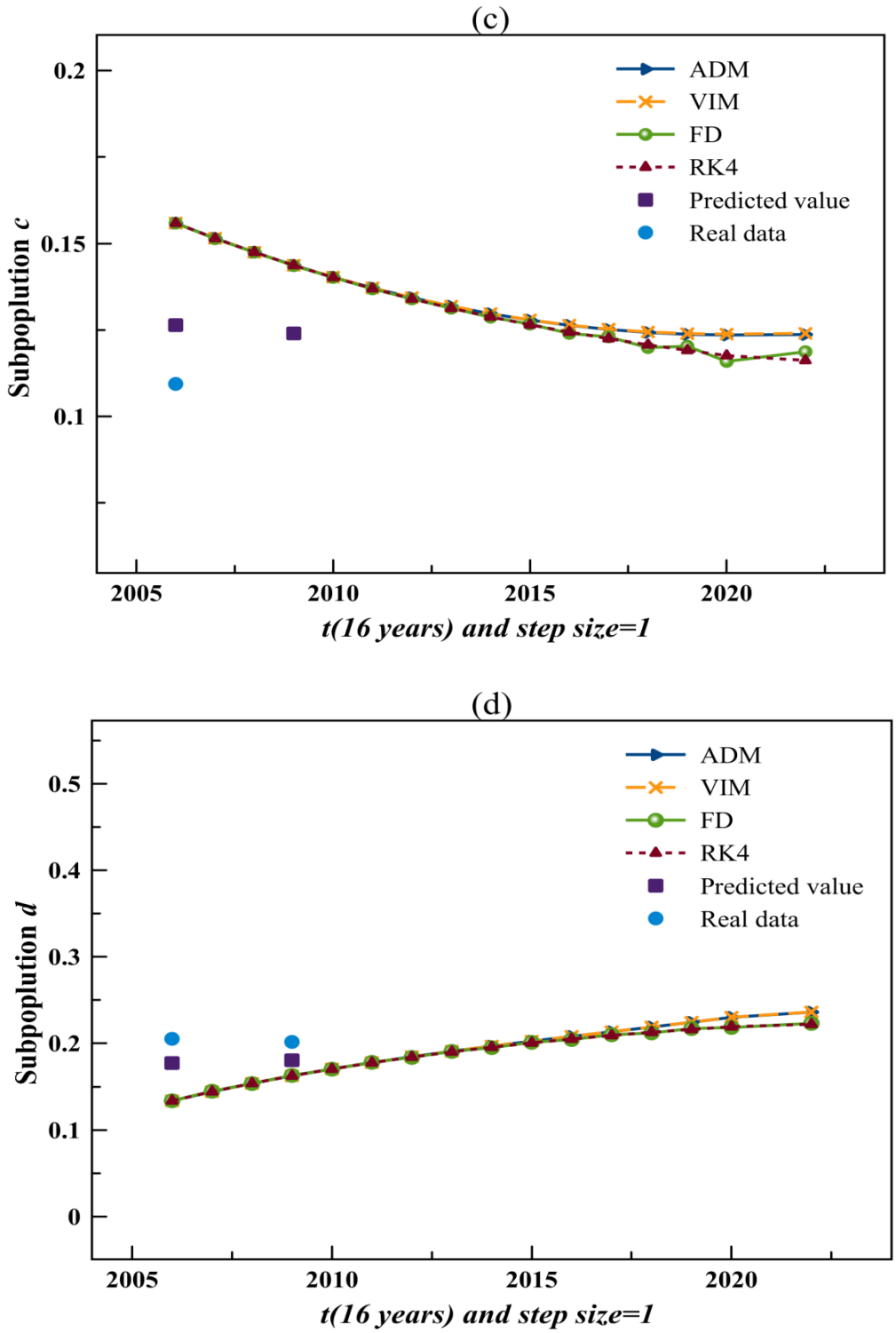

Figure 3.1 (c, d): Variation of approximate and numerical solutions by using ADM, VIM, FD and RK4 around predicted values [33] of (c) $c(t)$ and (d) $d(t)$ from 2006 to 2022 years when h=1

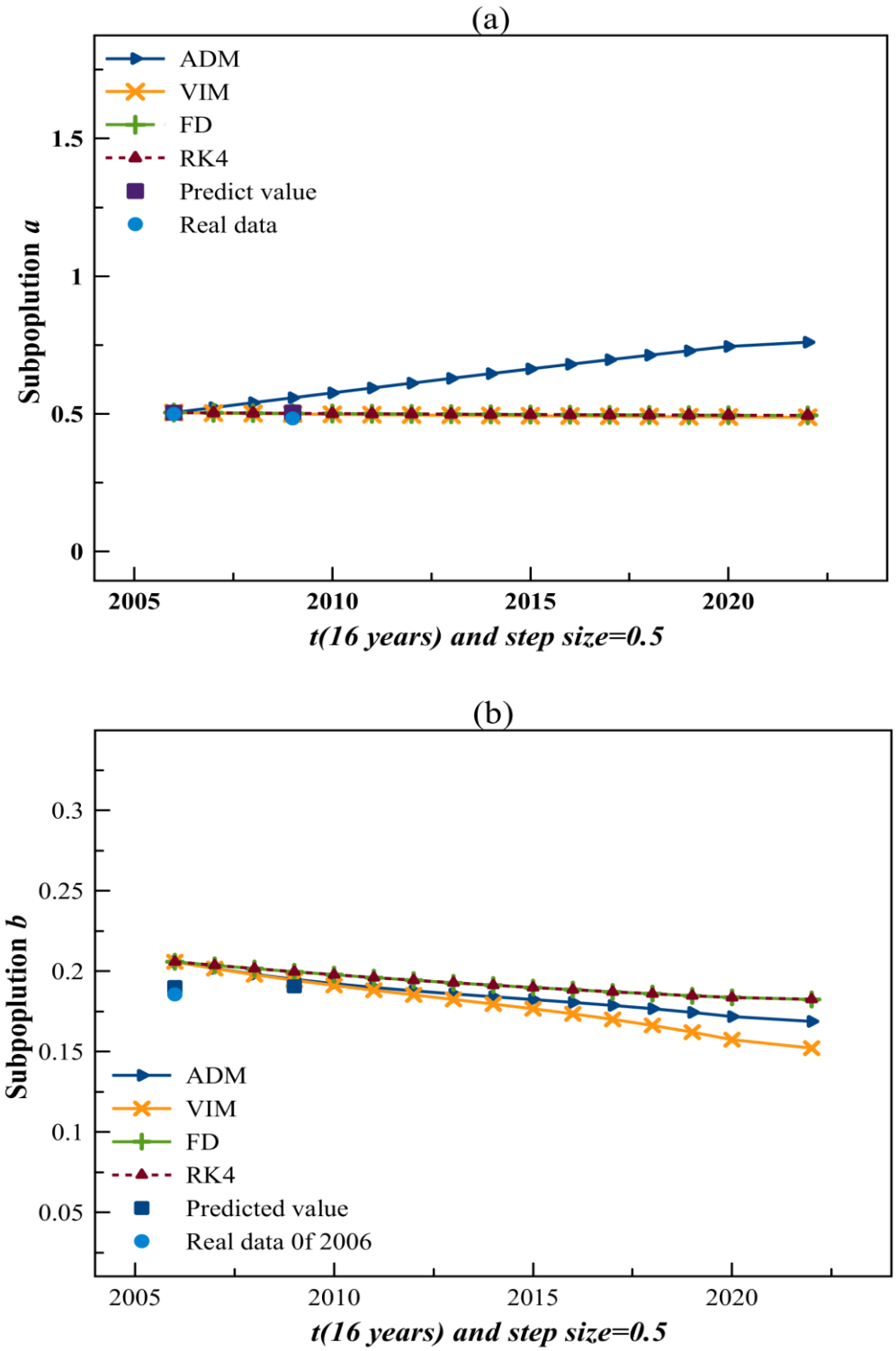

Figure 3.2 (a, b): Variation of approximate and numerical solutions by using ADM, VIM, FD and RK4 around predicted values [33] of (a) $a(t)$ and (b) $b(t)$ from 2006 to 2022 years when h=0.5

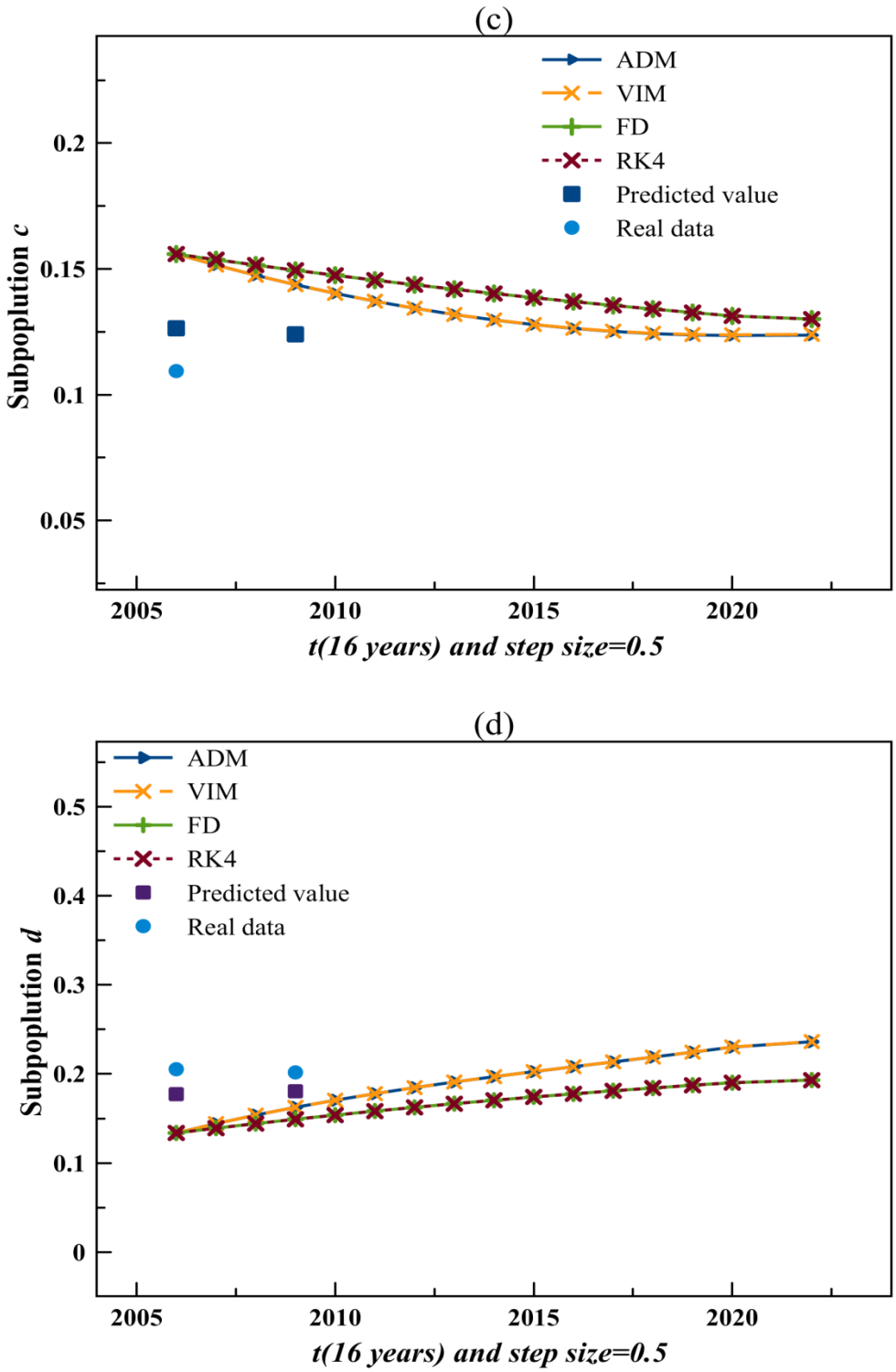

Figure 3.2 (c, d): Variation of approximate and numerical solutions by using ADM, VIM, FD and RK4 around predicted values [33] of (c) $c(t)$ and (d) $d(t)$ from 2006 to 2022 years when h=0.5

3.6 Results Analysis:

In the current study, the behavior of the bad social habit of the nonlinear epidemic model is analyzed through sixteen years under study from 2006 to 2022. In our work, some reliable approximate methods are used for solving a nonlinear system of epidemic models for ordinary differential equations of the first order. There is a convergence in the results of the analytic methods which are ADM and VIM and numerical methods which are FD and RK4 that examined in the nonlinear case. The analytic ADM and VIM with numerical FD and RK4 methods help to analyze the effects of the bad social habit of smoking habit model. The results obtained showed that subpopulation $a(t)$ of non-smokers stay stable along sixteen years except with ADM curve. While subpopulation $b(t)$ of normal-smoke and subpopulation $c(t)$ of excessive smokers are gradually declining until 2022. At least the subpopulation $d(t)$ of ex-smokers is a rising to 2022 that refer to increase the smoking habit in this region. The most predicted values [33] around the ADM, VIM, FD and RK4 curves that mean to the reliability of the obtained results.

Other analytical methods can solve such system under study like homotopy perturbation method, homotopy analysis method and semi analytical iterative method Temimi and Ansari. On the other hands, there are other numerical methods such as the iteration methods can solve the system under study.

CHAPTER 4:
Numerical Simulation Methods

4.1 Introduction:

In this chapter, two numerical simulation methods are used which are Mean Monte Carlo finite difference (MMCFD) that is applied for compression and a modified Mean Monte Carlo Runge-Kutta (MMCRK) that is created at the first time in our study. These methods are used to solve nonlinear IVP systems of ODEs representing the two social epidemic models about alcohol consumption and smoking habit. The results of these methods are called as numerical simulation solutions. Since the previous results for the real epidemic models under study are available. Therefore, the comparison between the numerical simulation results with the predicted values is discussed.

The importance of this work comes from it can expect the behavior of the population at some next years because the randomness in the numerical simulation methods of MMCFD and MMCRK that come from simulation technique for the parameters of the model under study.

4.2 Mean Monte Carlo Runge-Kutta (MMCRK) Method:

The numerical simulation method that merges between Monte Carlo simulation process (MC) and Runge-Kutta numerical iteration method (RK) is called the Mean Monte Carlo Runge-Kutta (MMCRK). MMCRK is a modified numerical simulation process that differs the MMCFD that the new one use RK numerical method instead of FD numerical method. The RK iteration numerical method is more accurate than FD since RK4 is of order 4 while FD is of order 2 for the central form. Therefore, MMCRK may be given numerical simulation results

more accurate than MMCFD mostly. In the present study, RK4 is used as a numerical iteration method. This mixed method MMCRK simulates the parameters of a model firstly by MC technique, when these parameters as random variables that distribute uniformly on (a, b) such that $a = q - p$ and $b = q + p$, when p is predicted value from [73] and $q \in R^{+}$. Then the system is solved numerically m times using RK4 with the first simulated estimation parameters. The m numerical simulated results have been gotten. The last numerical iteration result has been selected which is RK4 of order m iteration result (RK4_m) that is called the final solution. This process is returned with the second simulated estimation parameters, and so on until the last number of simulations. Finally, the mean of the n-time simulations for the final solutions is considered the Mean Monte Carlo Runge-Kutta (MMCRK). Is approximate method used to solve the nonlinear system of ordinary differential equations numerically, with more details, see Figure 4.1.

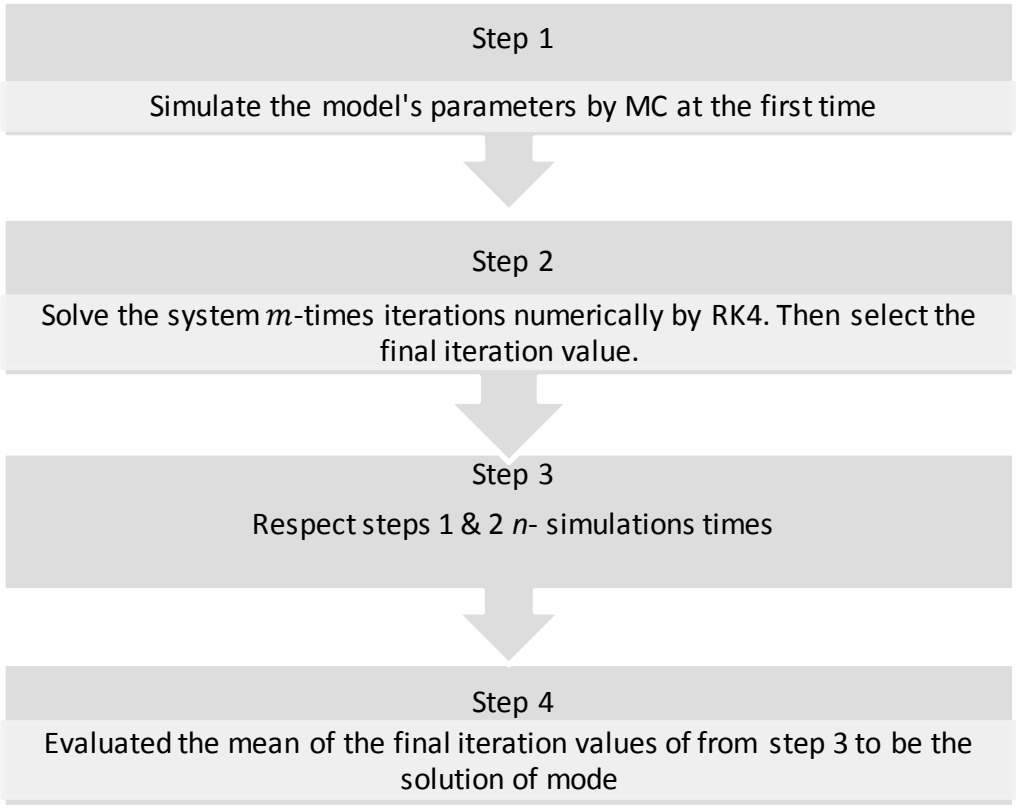

Figure 4.1: The steps of MMCRK process

4.3 The Mathematical Models:

Two social epidemic models which are alcohol consumption and smoking habit are solved in this chapter to verify the modified numerical simulation method MMCRK, then to compare with the numerical simulation method MMCFD.

4.3.1 Alcohol Consumption Model:

The model of alcohol consumption that is mentioned in Chapter 1 is considered. In this section, this model under study has been solved by numerical simulation methods MMCFD and MMCRK. For compression purpose, it is solved by analytic and numerical methods.

4.3.1.1 Results and Discussion:

The numerical simulation solutions for the nonlinear alcohol consumption model are discussed and analyzed in this section where the results are listed in Table 4.1. The predicted values of variables $a(t)$, $m(t)$ and $r(t)$ for alcohol consumption model, [73] had been given. Therefore, a comparison has been made between the numerical simulation solutions of MMCFD method and MMCRK method in the interval (0,10) from 1997 to 2007. For the purpose of comparison, the difference measure error of $a(t)$, $m(t)$ and $r(t)$ between predicted value from 1997 to 2007 and the results of ADM, VIM, FD, RK4, MMCFD and MMCRK methods are shown numerically in Table 4.2, where the difference measure error $|E_p|$ in this study is the difference between the approximate solutions and the predicted value or the difference between the numerical simulation solutions and the predicted value.

Let m be the number of iteration (number of years), n be the number of simulation of MC process and h is real step size, then for validity purpose from 1997 to 2007, let us not that:

- The smallest error of $a(t)$ is (0.119825) when h=1, n=100 and m=10 of MMCRK.
- For $m(t)$, the smallest error is (0.106700) when h=1, n=100 and m=10 of MMCRK method.
- The smallest error of $r(t)$ is (0.003081) when h=0.25, n=100 and m=10 of MMCRK method.

For the above results, we note that the MMCRK method has the smallest difference errors, therefore considered the best method.

Table 4.1: Solutions for the alcohol consumption model from 1997 to 2007

Model Variables														
$r(t)$		$m(t)$			$a(t)$			Predicted Values [73]	ADM	VIM	Step Size h (year)	FD	RK4	Present MMCFD Results		Present MMCRK Results	
														100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
0.25	0.5	1	0.25	0.5	1	0.25	0.5	1									
0.063228	0.063225	0.063215	0.706401	0.706372	0.706254	0.230370	0.230402	0.230529									
0.061377	0.061383	0.061395	0.706346	0.706346	0.706347	0.230389	0.230388	0.230388									
0.062968	0.062876	0.062693	0.703078	0.700743	0.696079	0.233954	0.236381	0.241228									
0.063087	0.062993	0.062806	0.703481	0.701146	0.696482	0.233432	0.235861	0.240712									
0.060081	0.060112	0.060168	0.691997	0.690549	0.687700	0.237175	0.238791	0.242174									
0.060160	0.060192	0.060247	0.692219	0.690804	0.687959	0.236553	0.238160	0.241526									
0.057		0.051282		0.585481	0.362			0.361013									
0.581		0.051281		0.585663	0.363056												

Table 4.2: Difference measure error, $|E_p|$ is between ADM, VIM, FD, RK4, MMCFD and MMCRK results and the predicted values [73] from 1997 to 2007

Model Variables	ADM	VIM	Step Size h (year)	FD	RK4	Present MMCFD Results		Present MMCRK Results	
						100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
$a(t)$	0.000987	0.001056	1	0.139826	0.131612	0.120772	0.121288	0.119825	0.120473
			0.5	0.143209	0.131611	0.125619	0.126139	0.123208	0.123839
			0.25	0.144825	0.131611	0.128046	0.128568	0.124825	0.125447
$m(t)$	0.004481	0.004663	1	0.106700	0.125346	0.115078	0.115482	0.106700	0.106959
			0.5	0.109549	0.125346	0.119743	0.120146	0.109549	0.109804
			0.25	0.110997	0.125345	0.122078	0.122481	0.110997	0.111219
$r(t)$	0.005718	0.005719	1	0.003168	0.004395	0.005693	0.005806	0.003168	0.003247
			0.5	0.003112	0.004383	0.005876	0.005993	0.003112	0.003192
			0.25	0.003081	0.004377	0.005968	0.006087	0.003081	0.003160

Table 4.3: Expectation solutions for the alcohol consumption model from 1997 to 2027

Model Variables		ADM	VIM	Step Size h (year)	FD	RK4	Present MMCFD Results		Present MMCRK Results	
							100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
$r(t)$	0.041659	0.592328	0.593809	1	0.807342	0.808154	0.805598	0.805545	0.770368	0.769931
	0.041658			0.5	0.808196	0.808153	0.805972	0.805878	0.770578	0.770053
				0.25	0.808410	0.808151	0.806331	0.806222	0.770654	0.770166
$m(t)$										
$a(t)$		0.359047	0.364532	1	0.127421	0.126276	0.120103	0.120132	0.120417	0.120292
				0.5	0.126511	0.126277	0.120018	0.120129	0.120621	0.120505
				0.25	0.126283	0.126276	0.1200012	0.1200097	0.120716	0.120604
$r(t)$	0.041659	0.065306	0.059811	1	0.065236	0.059841	0.065099	0.065322	0.056836	0.056944
	0.041658			0.5	0.065292	0.059821	0.065009	0.065233	0.056631	0.056740
				0.25	0.065306	0.059811	0.064977	0.065200	0.056529	0.056639

The Table 4.3 for future solutions for alcohol consumption model, shows us that the value of MMCRK method is near to the predicted value than MMCFD method, when h=0.25 (real step size) and n=100 (number of simulations) for non-drink alcohol people $a(t)$. While $m(t)$ of non-risk-drink alcohol people the value of MMCRK is near to the predicted h=0.5 and n=1000. The value of MMCRK when h=1 and n=1000 of risk-drink alcohol people. By notice, Table 4.4, seeing that the values of the mean within the interval.

Table 4.4: Prediction intervals (5th percentile, 95th percentile) for MMCFD and MMCRK solutions

MMCFD from 1997 to 2027 ($t \leq 30$)			
Subpopulation	**(100 repetitions)**	**Mean**	**(1000 repetitions)**
$a(t)$	(0.095907, 0.161453)	0.129303	(0.097544, 0.167752)
$m(t)$	(0.776219, 0.835442)	0.805598	(0.770562, 0.834359)
$r(t)$	(0.061126, 0.069548)	0.065098	(0.061161, 0.069172)
MMCRK from 1997 to 2027 ($t \leq 30$)			
Subpopulation	**(100 repetitions)**	**Mean**	**(1000 repetitions)**
$a(t)$	(0.090362, 0.149554)	0.120417	(0.091593, 0.155077)
$m(t)$	(0.745822, 0.793217)	0.770369	(0.740725, 0.793143)
$r(t)$	(0.053550, 0.060442)	0.056836	(0.053247, 0.060224)

Table 4.5: Results of MSE for MMCFD and MMCRK from 1997 to 2007

Model Variables	**Step Size, h (year)**	**Present MMCFD Results**		**Present MMCRK Results**	
		100 repetitions	**1000 repetitions**	**100 repetitions**	**1000 repetitions**
$a(t)$	1	0.015120	0.014727	0.014066	0.014018
	0.5	0.016346	0.015932	0.014630	0.014607
	0.25	0.016978	0.016554	0.014199	0.014178
$m(t)$	1	0.013749	0.013399	0.011825	0.011542
	0.5	0.014876	0.014507	0.012457	0.012170
	0.25	0.015457	0.015079	0.012784	0.012489
$r(t)$	1	0.000033	0.000032	0.000031	0.0000026
	0.5	0.000035	0.000034	0.000029	0.0000025
	0.25	0.000037	0.000035	0.000027	0.0000023

The Table 4.5 obtain the results of mean square error of the a new numerical simulation method MMCRK and compare it with MMCFD by using MSE then can be noted, let m be the number of iterations (number of years), n be the number of simulations of MC process and h is real step size, then for validity purpose from 1997 to 2007, let us not that:

- The smallest error of $a(t)$ is (0.014018) when h=1, n=1000 and m=10 of MMCRK.
- For $m(t)$, the smallest error is (0.011542) when h=1, n=1000 and m=10 of MMCRK method.
- The smallest error of $r(t)$ is (0.0000023) when h=0.25, n= 1000 and m=10 of MMCRK method.

From the above results, we noted that the MMCRK method has the smallest mean square errors, therefore considered the best method.

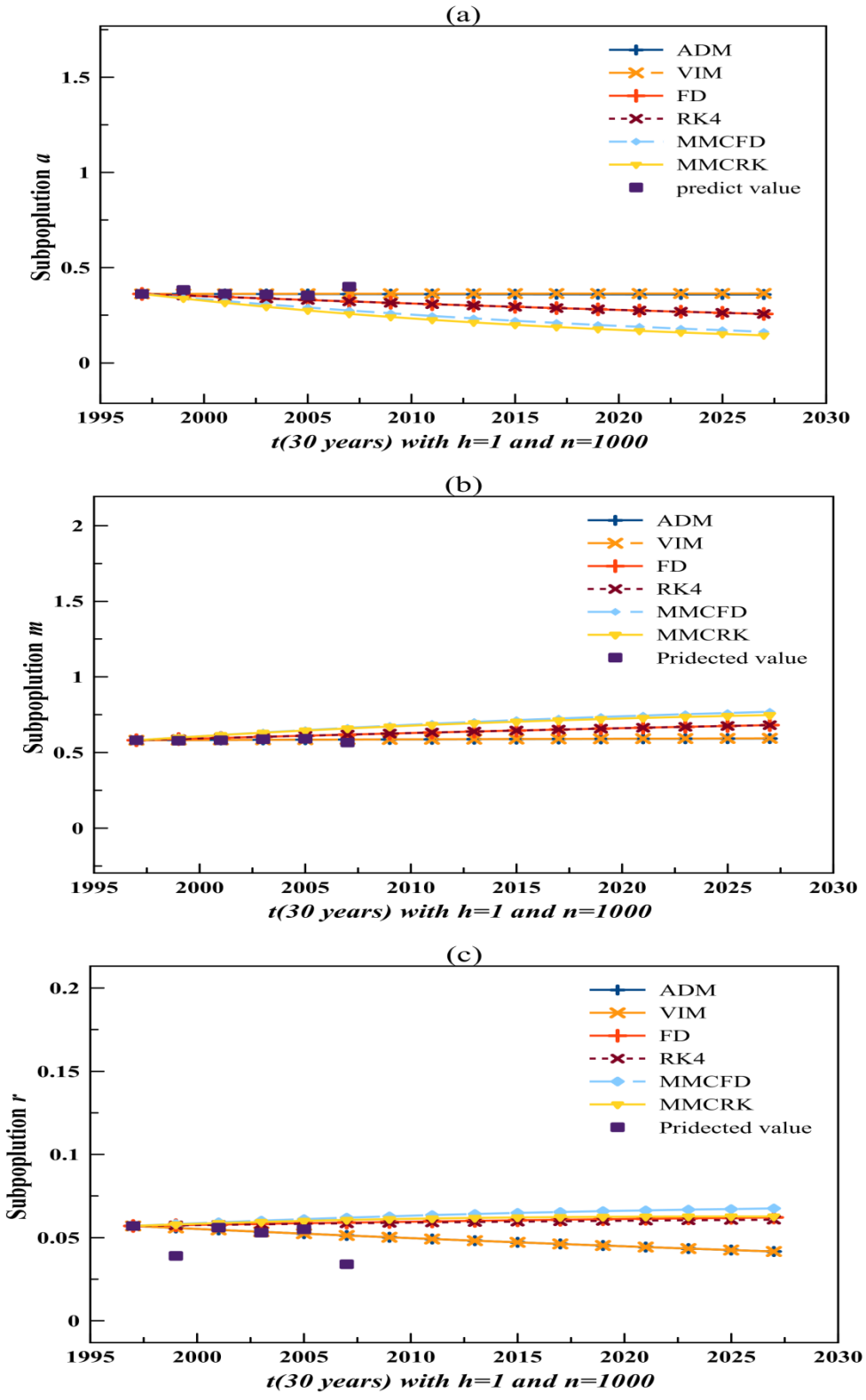

Figure 4.2: Variation of approximate and numerical solutions by using ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [73] when real step size (h=1) and simulations (n=1000) of (c) $a(t)$, (b) $m(t)$ and (c) $r(t)$ from 1997 to 2027 years

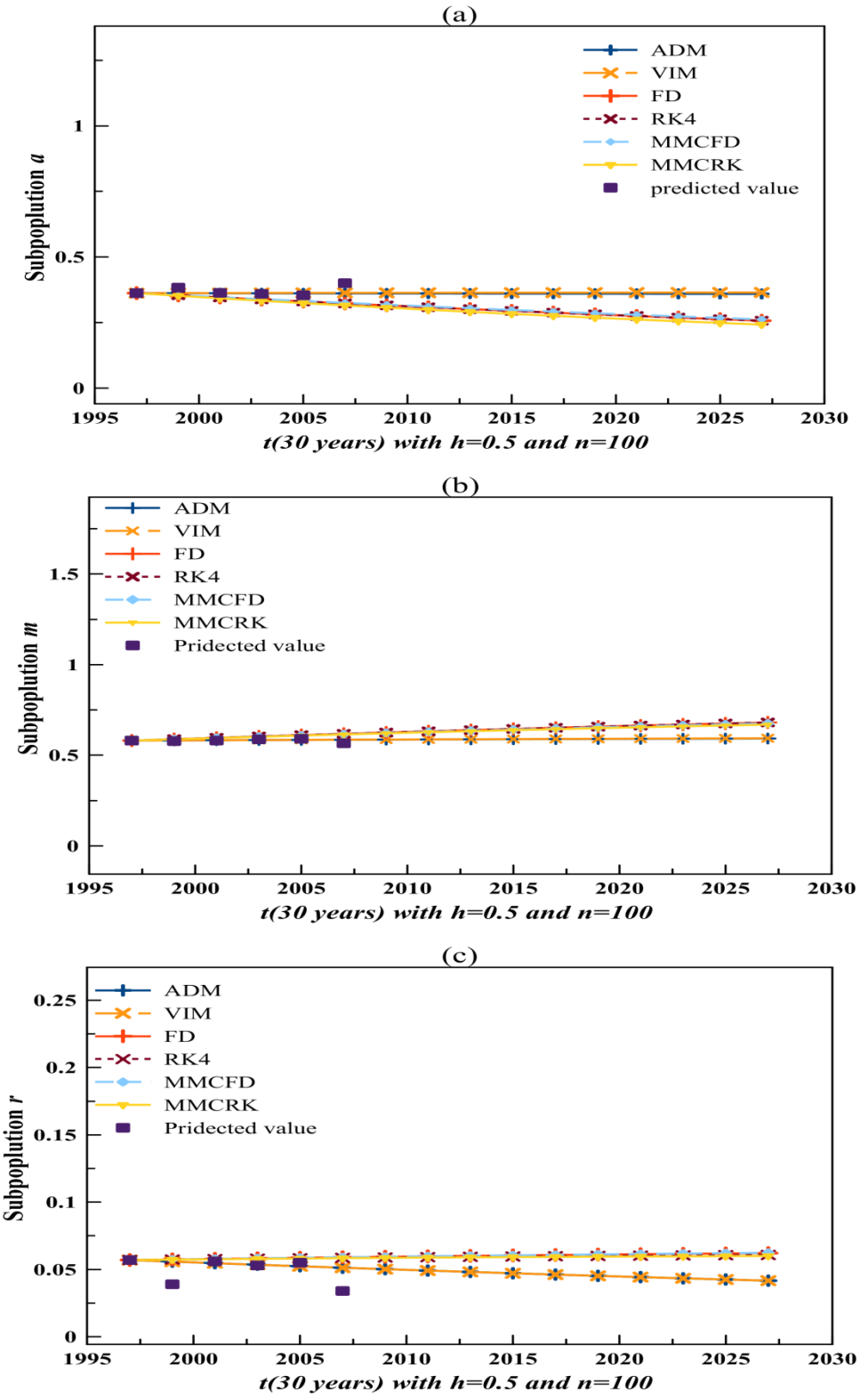

Figure 4.3: Variation of approximate and numerical simulation solutions by using ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [73] when real step size (h=0.5) and simulations (n=100) of (a) $a(t)$, (b) $m(t)$ and (c) $r(t)$ from 1997 to 2027 years

In Figure 4.2, we note the behavior of the alcohol consumption habit from 1997 to 2027 with m=30 (number of years), n=1000 (number of simulations) and h=1 (real step size). Figure 4.2 (a) the curves of numerical simulation methods MMCFD and MMCRK are near of predicted values from 1997 to 2005 and decreasing gradually until 2027. In Figure 4.2 (b), the curves of MMCFD and MMCRK around predicted values into 2007 then the curves are increasing step by step until 2027. While Figure 4.2 (c) shows the curves of MMCFD and MMCRK results converge of the predicted value in 1999 until 2005.

The Figure 4.3 describes the behavior of alcohol consumption habit from 1997 to 2027 with m=30 which is number of iterations (number of years), h=0.5 (step size) and n=100 be the number of simulations. Figure 4.3 (a) of $a(t)$ shows the MMCFD and MMCRK curves obtained results near to some predicted values in 2001 until 2005.While Figure 4.3 (b) of $m(t)$ shows the predicted values around both MMCFD and MMCRK curves. Regarding to Figure 4.3 (c) of $r(t)$, both MMCFD and MMCRK curves for results converge to the predicted values in 1999 until 2005.

In Figure 4.3 (a) that related to non-drink alcohol people $a(t)$, the MMCFD curve decreasing from 1997 to 2027. More other, there exists a variation between the curves such that the MMCFD and MMCRK curves are higher level than the curve of other methods. On the other hand, both MMCFD and MMCRK curves of non-risk-drink alcohol people $m(t)$ have higher that appears during the thirty years from 1997 until 20027 in Figure 4.3 (b). Figure 4.3 (c) illustrates the decrease in the risk-drink alcohol people $r(t)$ from 1997 to 2027 years under study for both MMCFD and MMCRK curves.

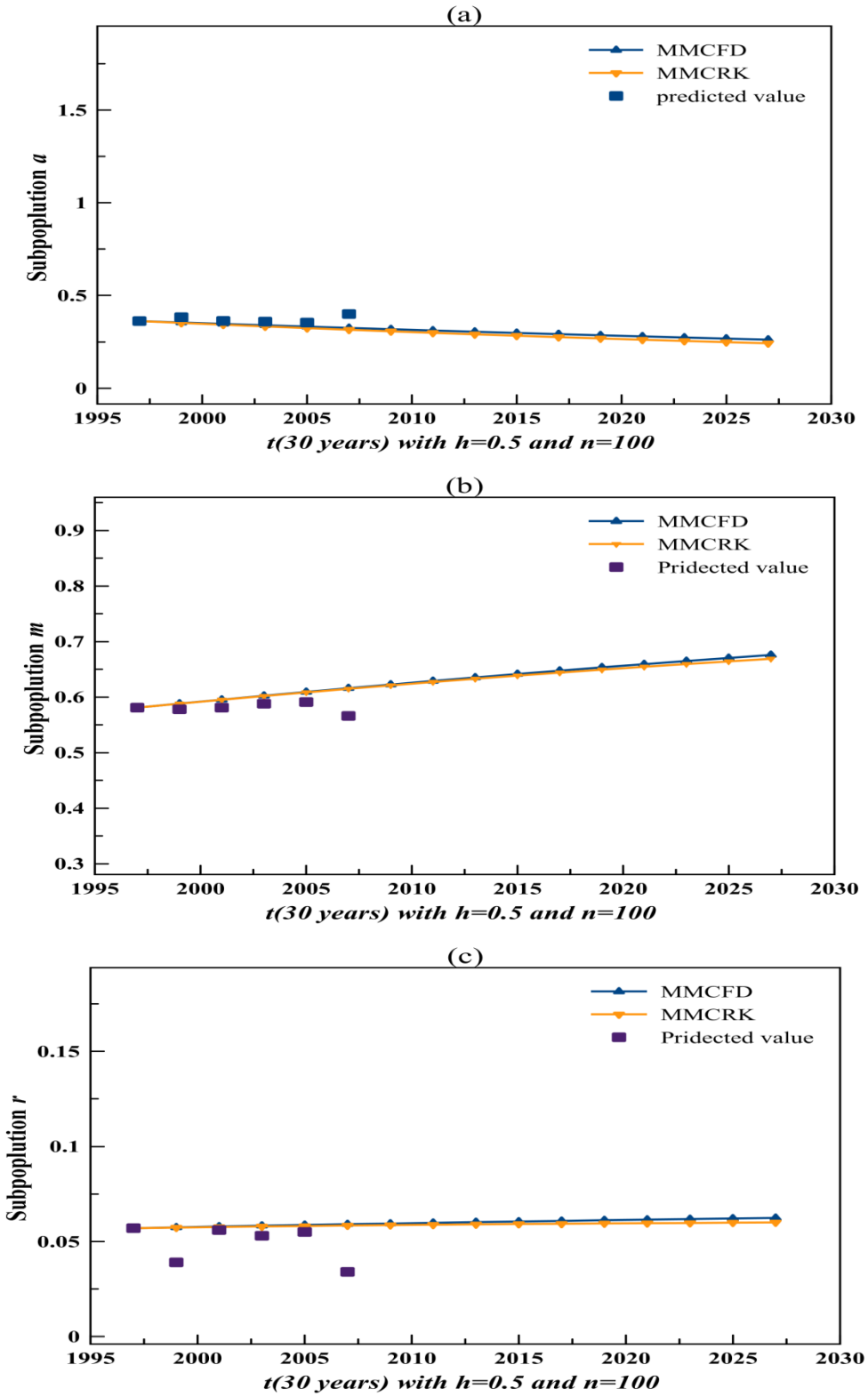

Figure 4.4 Variation of numerical simulation solutions by using MMCFD and MMCRK around predicted values [73], when real step size (h=0.5) and simulations (n=100) of (a) $a(t)$, (b) $m(t)$ and (c) $r(t)$, from 1997 to 2027 years

The Figure 4.4 is to compare between MMCFD and a new proposed method MMCRK when real step size h=0.5, m=30 (number of years) and n=100 (number of simulations), then in Figure 4.4 (a), the curve of results of MMCRK is more approach of the predicted values then starts to decline until 2027. Also Figure 4.4 (b) the curve of MMCRK is nearer to the predicted values than MMCFD method then starts increasing until 2027. The curve of a new modified MMCRK of Figure 4.4 (c) is near than MMCFD method of the predicted values with small decreasing to 2027.

Generally, for the interval (0,30) the percentage of non-drink alcohol people $a(t)$ and the risk-drink alcohol people $r(t)$ are almost decrease, but there is an increase with the non-risk-drink alcohol people $m(t)$. The results are calculated by Matlab 2013 software, the figures are drawn by the Magic Plot program.

4.3.1.2 Results Analysis:

In the current study, the convergence of the results for the numerical simulation methods which are MMCFD and the new proposed MMCRK are examined in the nonlinear case. These methods are consider from reliable methods for solving a system of ordinary differential equations. In our work, they are used for solving a system of nonlinear ordinary differential equations. The behavior of bad social habit which is alcohol consumption in Spain is analyzed, through thirteen years from 1997 to 2027 under study. The modified MMCRK method helps to analyze the effects of the bad social habit of alcohol consumption. The obtained results are shown that there is increasing in alcohol consumption with non-risk-drink consumers and declining the risk-drink consumers during

the thirty years under study. For the number of the non-drink consumers has a decrease with the MMCRK method. The most predicted values [73] around the MMCRK curves. That means MMCRK can expect that the increase may be happening in the future for alcohol consumption habit in Spain.

4.3.2 Smoking Habit Model:

The model of smoking habit model that is mentioned in Chapter 3 is considered In this section, this model under study has been solved by the new numerical simulation method MMCRK. Then MMCRK is compared with the analytic methods ADM and VIM, and with the numerical methods FD and RK4, as well as with the numerical simulation method MMCFD [55]. For comparison purpose, the difference measure error and the mean square error.

4.3.2.1 Results and Discussion:

Approximate and numerical solutions for nonlinear smoking habit model in Spain are analyzed and discussed in this section where they are listed in Table 4.6 a and Table 4.6 b. The predicted values of variables $a(t)$, $b(t)$, $c(t)$ and $d(t)$ for smoking habit evaluation model, [33] had been given. The exact solution is not available in the current model. Therefore, a comparison is done between the predicted values or the real data that available in some years with the numerical simulation solutions for MMCFD and MMCRK in the interval (0,3) from 2006 to 2009. For the purpose of comparison, the difference measure error for $a(t)$, $b(t)$, $c(t)$ and $d(t)$ between predicted value from 2006 to 2009 and ADM, VIM, FD, RK4, MMCFD, MMCRK methods are shown numerically in

Table 4.8 a and Table 4.8 b , where the difference measure error $|E_p|$ in this study is the difference between the approximate solutions and the predicted value or the difference between the numerical solutions and the predicted value. Some points can be noted: let n be a number of simulations, m be a number of iterations which is number of years and h be a step size.

- The smallest error for $a(t)$ is (0.01431403) when h=1, n=1000 and m=16 of MMCRK method.
- For $b(t)$, the smallest error is (0.04631671) when h=0.25, n=1000 m=16 of MMCRK method.
- For $c(t)$, the smallest error is (0.00742477) when h=1, n=100 and m=16 of MMCRK method.
- For $d(t)$, the smallest error is (0.03825620) when h=1, n=100 and m=16 of MMCRK method.

  For the above results, we note that the MMCRK method has the smallest difference measure errors. Therefore, it is considered the best method.

Table 4.6 a: Approximate solutions of the smoking habit model from 2006 to 2009

Model Variables		Real Data [33]		Predicted Values [33]		ADM	VIM (3 iter.)	Step Size h (year)		FD (3 iter.)		RK4 (3 iter.)		MMCFD		MMCRK	
$b(t)$		in 2006	in 2009	in 2006	in 2009									100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
0.1856		0.4997															
–		0.4835															
0.1902		0.5041															
0.1906		0.5049															
0.19500158		0.55832470															
0.19437918		0.49939633															
0.25	0.5	1	0.25	0.5	1												
0.19579540	0.19580836	0.19561906	0.49879116	0.49879849	0.49868229												
0.19435139	0.19438163	0.19406428	0.49942288	0.49943026	0.49931556												
0.19648255	0.19724219	0.19881534	0.49986469	0.50023156	0.50099586												
0.19516595	0.19601513	0.19778229	0.49978984	0.50016225	0.50093837												
0.19579540	0.19580836	0.19561906	0.49879116	0.49879849	0.49868229												
0.19435139	0.19438163	0.19406428	0.49942288	0.49943026	0.49931556												

Table 4.6 b: Approximate solution of the smoking habit model from 2006 to 2009

$d(t)$			$c(t)$			Model Variables	
0.2053			0.1094			**in 2006**	**Real Data [33]**
0.2017			_			**in 2009**	
0.1773			0.1264			**in 2006**	**Predicted Values [33]**
0.1805			0.1240			**in 2009**	
0.16250281			0.14372090			**ADM**	
0.16250304			0.14372145			**VIM (3 iter.)**	
0.25	0.5	1	0.25	0.5	1	**Step Size h (year)**	
0.19579540	0.19580836	0.19561906	0.14331860	0.14333171	0.14310983	**FD (3 iter.)**	
0.19435139	0.19438163	0.19406428	0.14381608	0.14381286	0.14370169	**RK4 (3 iter.)**	
0.19648255	0.19724219	0.19881534	0.14422752	0.14517334	0.14713032	**100 repetitions**	**MMCFD**
0.19516595	0.19601513	0.19778229	0.14472517	0.14565114	0.14755629	**1000 repetitions**	
0.19579540	0.19580836	0.19561906	0.14331860	0.14333171	0.14310983	**100 repetitions**	**MMCRK**
0.19435139	0.19438163	0.19406428	0.14381608	0.14381286	0.14370169	**1000 repetitions**	

Table 4.7 a: The expected of approximate solutions and numerical simulation results the smoking habit model from 2006 to 2022

Model Variables	ADM	VIM (16 iter.)	Step Size, h (year)	FD (16 iter.)	RK4 (16 iter.)	Present MMCFD Results		Present MMCRK Results	
						1000 repetitions	100 repetitions	1000 repetitions	100 repetitions
$a(t)$	0.77521396	0.48643685	1	0.49049314	0.48747909	0.49051075	0.49039192	0.49038596	0.49025842
			0.5	0.49031746	0.48728619	0.49032471	0.49023571	0.49023588	0.49025841
			0.25	0.49027319	0.48723681	0.49027743	0.49016189	0.49016524	0.49025841
$b(t)$	0.16532272	0.14627349	1	0.17438741	0.17524276	0.17608199	0.17420935	0.17109599	0.17011593
			0.5	0.17128695	0.17363810	0.17175225	0.17371569	0.17057094	0.17011592
			0.25	0.17041579	0.17316280	0.17051241	0.17347673	0.17031671	0.17011591

Table 4.7 b: The expected of approximate solutions and numerical simulation results the smoking habit model from 2006 to 2022

Model Variables	ADM	VIM (16 iter.)	Step Size, h (year)	FD (16 iter.)	RK4 (16 iter.)	Present MMCFD Results		Present MMCRK Results	
						1000 repetitions	100 repetitions	1000 repetitions	100 repetitions
$c(t)$	0.12413672	0.12457661	1	0.1159163	0.11433901	0.11036054	0.11657523	0.1648078	0.11499751
			0.5	0.11404419	0.11499877	0.11385917	0.11597224	0.11584439	0.11499750
			0.25	0.11475214	0.11522423	0.11488335	0.11568072	0.11553643	0.11499750
$d(t)$	0.24252661	0.24427130	1	0.22352783	0.22293914	0.22304672	0.21875620	0.22181206	0.22462814
			0.5	0.22435139	0.22407693	0.22406387	0.22000255	0.22310206	0.22462817
			0.25	0.22455887	0.22437616	0.22432680	0.22060353	0.22372391	0.22462817

Table 4.6 a and Table 4.6 b for h=1, 0.5, 0.25 and m=3, is the number of iterations is to compare between real and predicted values (2006-2009) with approximate solutions at the same time under study.

Table 4.7 a and Table 4.7 b contain the future solution for smoking habit from 2006 to 2022. The value of a new MMCRK for $a(t)$ can be noted is near of the predicted when h=1 (real step size) and n=1000 (number of simulations). While the value of MMCRK for $b(t)$ is an approach of the predicted value when h=1 and n=100. For $c(t)$ the value of MMCRK is near of predicted value when h=1 and n=100. Finally, the value of MMCRK is near of predicted value when h=1 and n=100.

Table 4.8 a: Difference measure error $|E_p|$ between ADM, VIM, FD, RK4, MMCFD and MMCRK solutions and the predicted values [33] from 2006 to 2009

Model Variables										
$b(t)$			$a(t)$			ADM in 2009	VIM (16 iter.) in 2009	Step Size h year	RK4 (16 iter.) in 2009	FD (16 iter.) in 2009	Present MMCFD Results		Present MMCRK Results	
											100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
0.00440158			0.0534247				0.00550367	1	0.01464158	0.01440686	0.01742091	0.01438925	0.01450807	0.01431403
								0.5	0.01464158	0.01458253	0.01761381	0.01457528	0.01466429	0.01456411
								0.25	0.01464158	0.01462680	0.01766319	0.01462257	0.01473811	0.01461476
							0.00377918	1	0.29965841	0.29989313	0.05124276	0.05208199	0.05020934	0.04709599
								0.5	0.29965841	0.29971746	0.04963810	0.04775224	0.04971568	0.04657093
								0.25	0.29965841	0.29967319	0.04916280	0.04651241	0.04947672	0.04631671

Table 4.8 b: Difference measure error $|E_p|$ between ADM, VIM, FD, RK4, MMCFD and MMCRK solutions and the predicted values [33] from 2006 to 2009

Model Variables										
$d(t)$			$c(t)$			ADM in 2009	VIM (16 iter.) in 2009	Step Size h year	RK4 (16 iter.) in 2009	FD (16 iter.) in 2009	Present MMCFD Results		Present MMCRK Results	
											100 repetitions	1000 repetitions	100 repetitions	1000 repetitions
0.25	0.5	1	0.25	0.5	1	0.0197209	0.01972145							
0.30975841	0.30975841	0.30975841	0.36625841	0.36625841	0.36625841									
0.30977319	0.30981746	0.30999313	0.36627319	0.36631746	0.36649313									
0.04387615	0.04357693	0.04243913	0.00877577	0.00900123	0.00966098									
0.04382680	0.04356387	0.04254671	0.00911664	0.01014083	0.01363945									
0.04010353	0.03950255	0.03825620	0.00831927	0.00802775	0.00742477									
0.04322390	0.04260205	0.04131205	0.00846356	0.00815560	0.00751921									

Prediction interval is a predicted region for the numerical simulation results from 1997 to 2027 in Table 4.9. By note the Table 4.9 we see that the values of mean within the interval.

Table 4.9: Prediction intervals (5th percentile, 95th percentile) for MMCFD and MMCRK solutions

MMCFD from 1997 to 2022 ($t \leq 16$)			
Subpopulation	**(100 repetitions)**	**Mean**	**(1000 repetitions)**
$a(t)$	(0.46517544, 0.51525916)	0.48747909	(0.46784111, 0.51429054)
$b(t)$	(0.12727289, 0.22106877)	0.17524276	(0.13021315, 0.22237849)
$c(t)$	(0.05673551, 0.16653102)	0.11433901	(0.05846895, 0.15524955)
$d(t)$	(0.20241392, 0.24716537)	0.22293914	(0.20089752, 0.24437106)
MMCRK from 2006 to 2022 ($t \leq 16$)			
Subpopulation	**(100 repetitions)**	**Mean**	**(1000 repetitions)**
$a(t)$	(0.46495973, 0.51131111)	0.49039192	(0.46787991, 0.51307728)
$b(t)$	(0.15501609, 0.19879246)	0.17420935	(0.14936064, 0.19367405)
$c(t)$	(0.09742801, 0.13664182)	0.11657523	(0.09758071, 0.13563891)
$d(t)$	(0.19987210, 0.23872143)	0.21875620	(0.20066171, 0.24414046)

Table 4.10: Results of MSE for MMCFD and MMCRK from 2006 to 2009

Model Variables	**Step Size, h (year)**	**Present MMCFD Results**		**Present MMCRK Results**	
		100 repetitions	**1000 repetitions**	**100 repetitions**	**100 repetitions**
$a(t)$	1	0.00002917	0.00001738	0.00004396	0.00000944
	0.5	0.00004347	0.00002566	0.00006521	0.00001332
	0.25	0.00005082	0.00002974	0.00007711	0.00001541
$b(t)$	1	0.00598064	0.00584571	0.00601748	0.00585020
	0.5	0.00583757	0.00566605	0.00585286	0.00565108
	0.25	0.00575351	0.00556638	0.00577226	0.00555408
$c(t)$	1	0.00049382	0.00044614	0.00048569	0.00043818
	0.5	0.00039421	0.00034294	0.00039892	0.00034625
	0.25	0.00035902	0.00030589	0.00036061	0.00030524
$d(t)$	1	0.00075837	0.00075480	0.00070756	0.00078692
	0.5	0.00052999	0.00053100	0.00048720	0.00057448
	0.25	0.00044310	0.00044627	0.00039527	0.00048476

The Table 4.10 contains the results of mean square error of the a new numerical simulation method MMCRK and compare it with MMCFD by using MSE then can be noted, let n be a number of simulations, m be a number of iterations which is number of years and h be a step size.

- The smallest error for $a(t)$ is (0.00000944) when h=1, n=1000 and m=16 of MMCRK method.
- For $b(t)$, the smallest error is (0.00555408) when h=0.25, n=1000 m=16 of MMCRK method.
- For $c(t)$, the the smallest error is (0.00030524) when h=0.25, n=1000 and m=16 of MMCRK method.
- For $d(t)$, the the smallest error is (0.00039527) when h=0.25, n=100 and m=16 of MMCRK method.

For the above results, we note that the MMCRK method has the smallest mean square errors, therefore it is considered the best method.

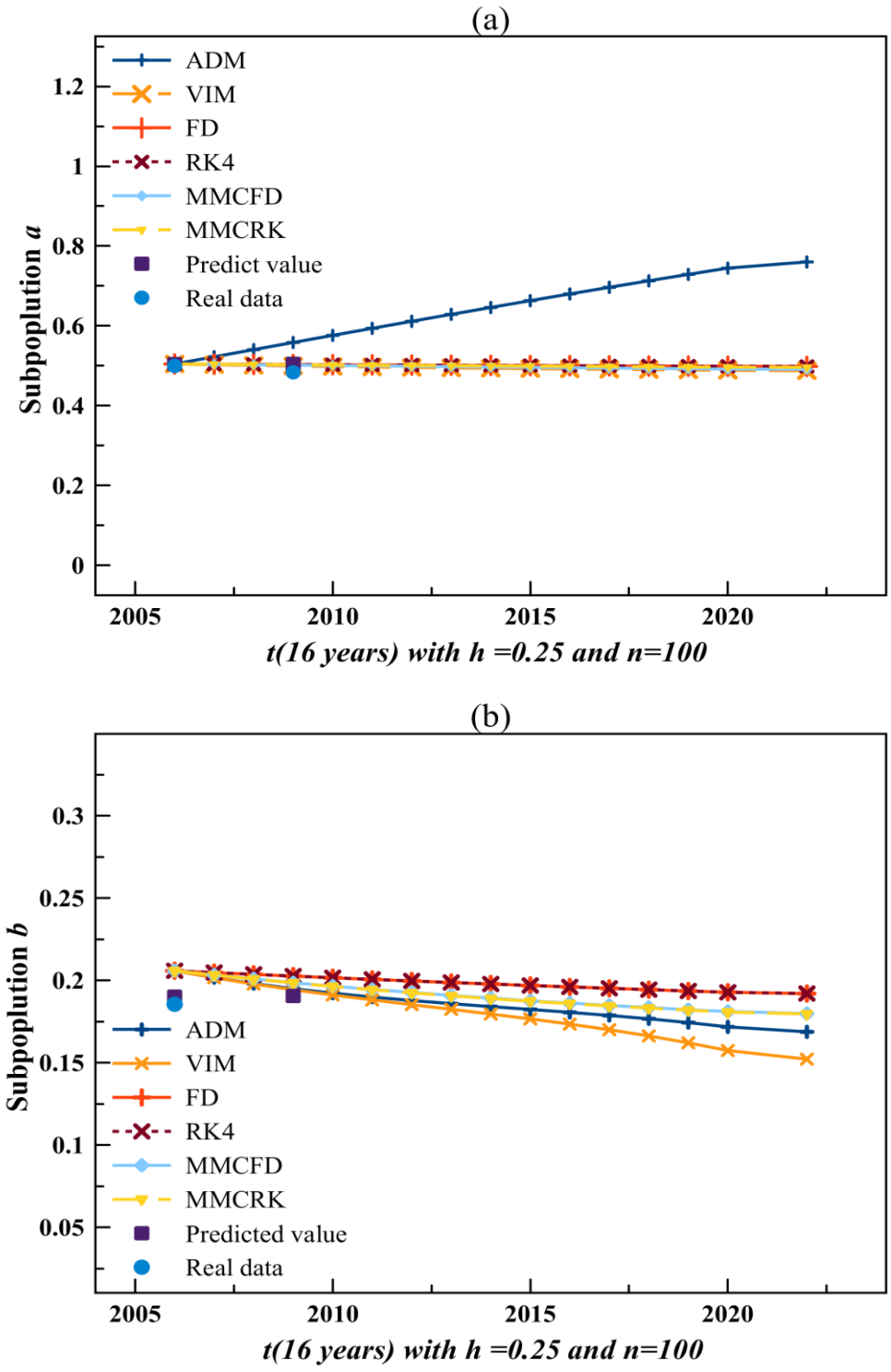

Figure 4.5 (a, b): Variation of approximate and numerical simulation solutions by ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [33] when h=0.25 (real step size) and n=100 (number of simulations) of (a) $a(t)$ and (b) $b(t)$ from 2006 to 2022 years

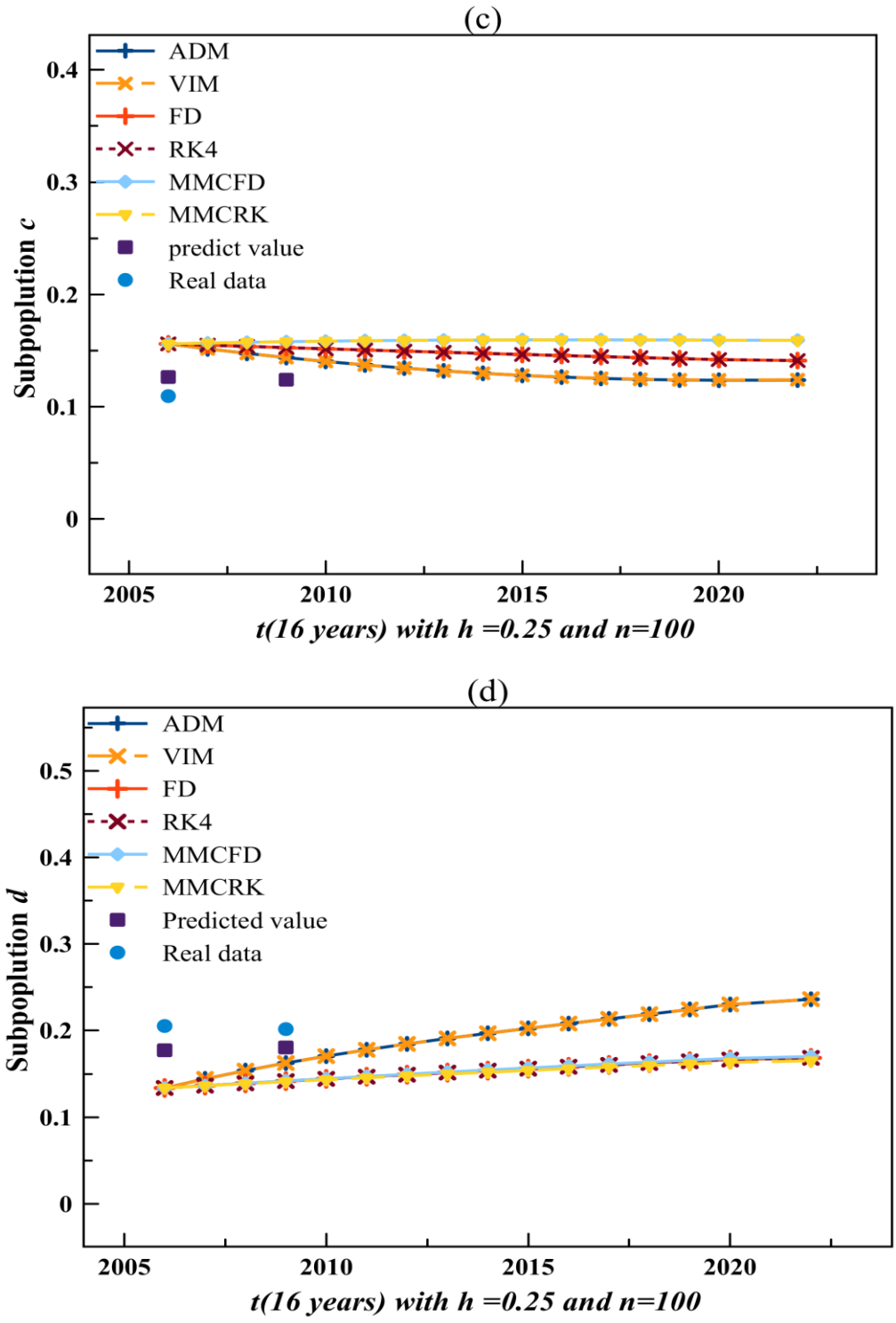

Figure 4.5 (c, d): Variation of approximate and numerical solutions by ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [33] when h=0.25 (real step size) and n=100 (number of simulations) of (c) $c(t)$ and (d) $d(t)$ from 2006 to 2022 years

Figure 4.5 (a, b) and Figure 4.5 (c, d) are describe the behavior of smoking habit from 2006 to 2022. Figure 4.5 (a) that related to non-smoke people $a(t)$, the curve of MMCRK method near to the predict value more than the curve of MMCFD method and keep on its level. This means, the people who do not smoke through 16 years to 2022 have stable case. Therefore, the behavior of curves of these methods is agree with the behavior of curves for $a(t)$ in the previous study (Figure 2, page. 249) [33]. Figure 4.5 (b) of $b(t)$ that related to normal smoke people is showing us the curves of the methods that are MMCFD, MMCRK, FD, VIM and RK4 methods are near from 2006 until 2013, then the curves of mentioned methods are gradually decreasing yearly to 2022, while with MMCRK is more decreasing from 2013 until 2022.

For Figure 4.5 (c) of $c(t)$ that related to the excessive smokers all the curves of the methods under study are decreasing, the numerical simulations for the methods (MMCRK, MMCFD) is more increasing than the other methods and agree with previous study the previous study (Figure 2, page. 249) [33].

Figure 4.5 (d) of $d(t)$, that related to ex-smokers, there is increasing from 2006 to 2022, for all the curves of the numerical simulation methods are decreasing more than the curves of the other methods from 2013 until 2022 and agree with the previous study (Figure 2, page. 249) [33].

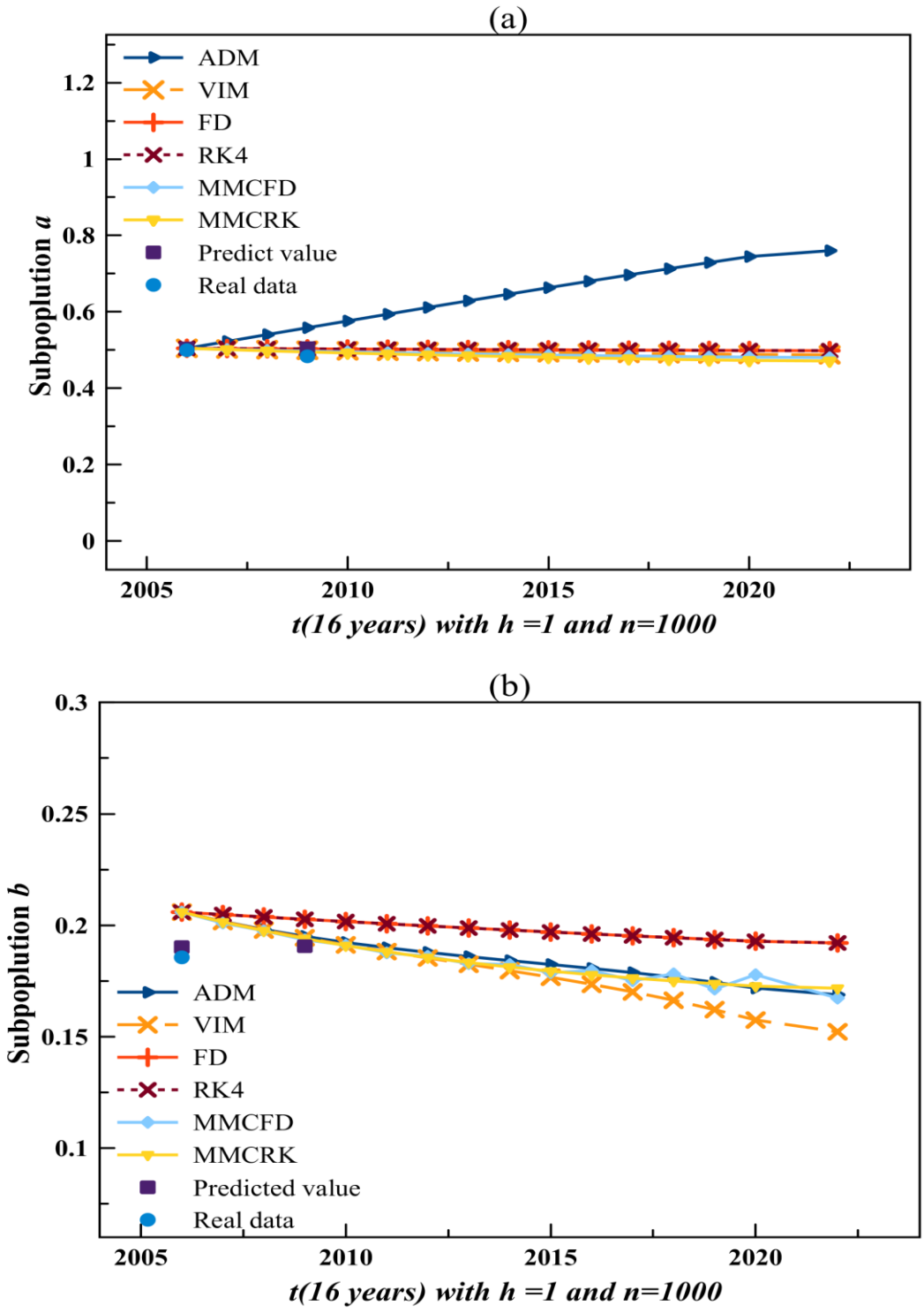

Figure 4.6 (a, b): Variation of approximate and numerical solutions by ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [33] when h=1 (real step size) and n=1000 (number of simulations) of (a) $a(t)$ and (b) $b(t)$, from 2006 to 2022 years

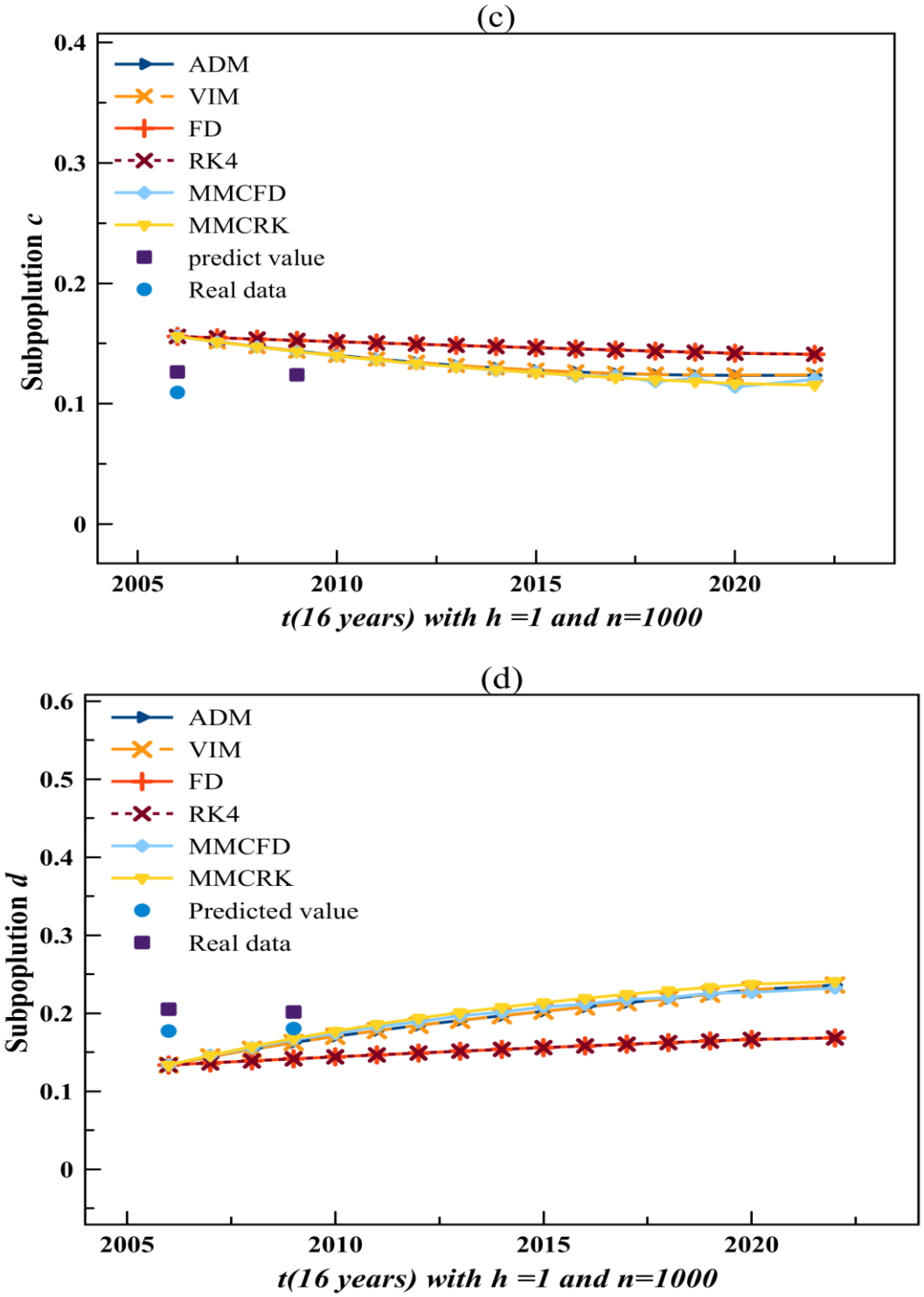

Figure 4.6 (c, d): Variation of approximate and numerical solutions by using ADM, VIM, FD, RK4, MMCFD and MMCRK around predicted values [33] when h=1 (real step size) and n=1000 (number of simulations) of (c) $c(t)$ and d $d(t)$ from 2006 to 2022 years.

Figure 4.7 (a, b) and Figure 4.7 (c, d) when h=1 (real step size), n=1000 (number of simulation) and m=16 (number of iteration) are describe the behavior of smoking habit from 2006 to 2022. Figure 4.6 (a) the curve of MMCRK method near to the predict value more than the curve of MMCFD method and keep on its level. Therefore, the behavior of curves of these methods is agree with the behavior of curves for $a(t)$ in the previous study (Figure 2, page. 249) [33]. Figure 4.6 (b) of $b(t)$ is explain us the curves of the methods that are MMCFD, MMCRK, FD, VIM and RK4 methods are near from 2006 until 2013, then the curves of mentioned methods are gradually decreasing yearly to 2022, while with MMCRK is more decreasing from 2013 until 2022.

For Figure 4.6 (c) of $c(t)$ shows us all the curves of the methods under study are decreasing, the numerical simulation methods (MMCRK, MMCFD) more increasing than the other methods and agree with previous study the previous study (Figure 2, page. 249) [33].

Figure 4.6 (d) of $d(t)$, there is increasing from 2006 to 2022, for all the curves of the numerical simulation methods are decreasing more than the curves of the other methods from 2013 until 2022 and agree with the previous study (Figure 2, page. 249) [33].

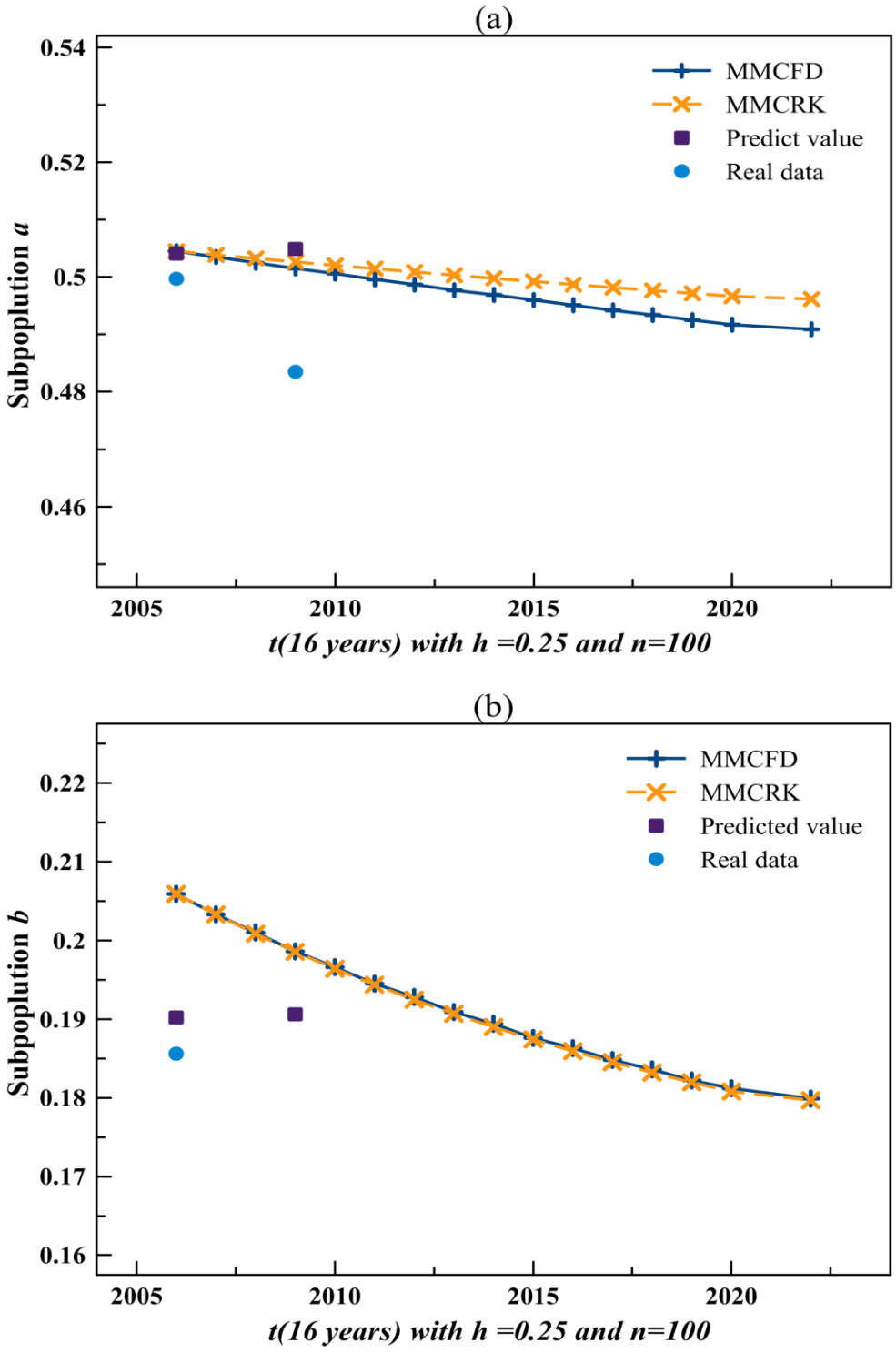

Figure 4.7 (a, b): Numerical simulation solutions using MMCFD and MMCRK around predicted values and Real data [33] when h=0.25 (real step size) and n=100 (number of simulations) of (a) $a(t)$ and (b) $b(t)$ from 2006 to 2022 years

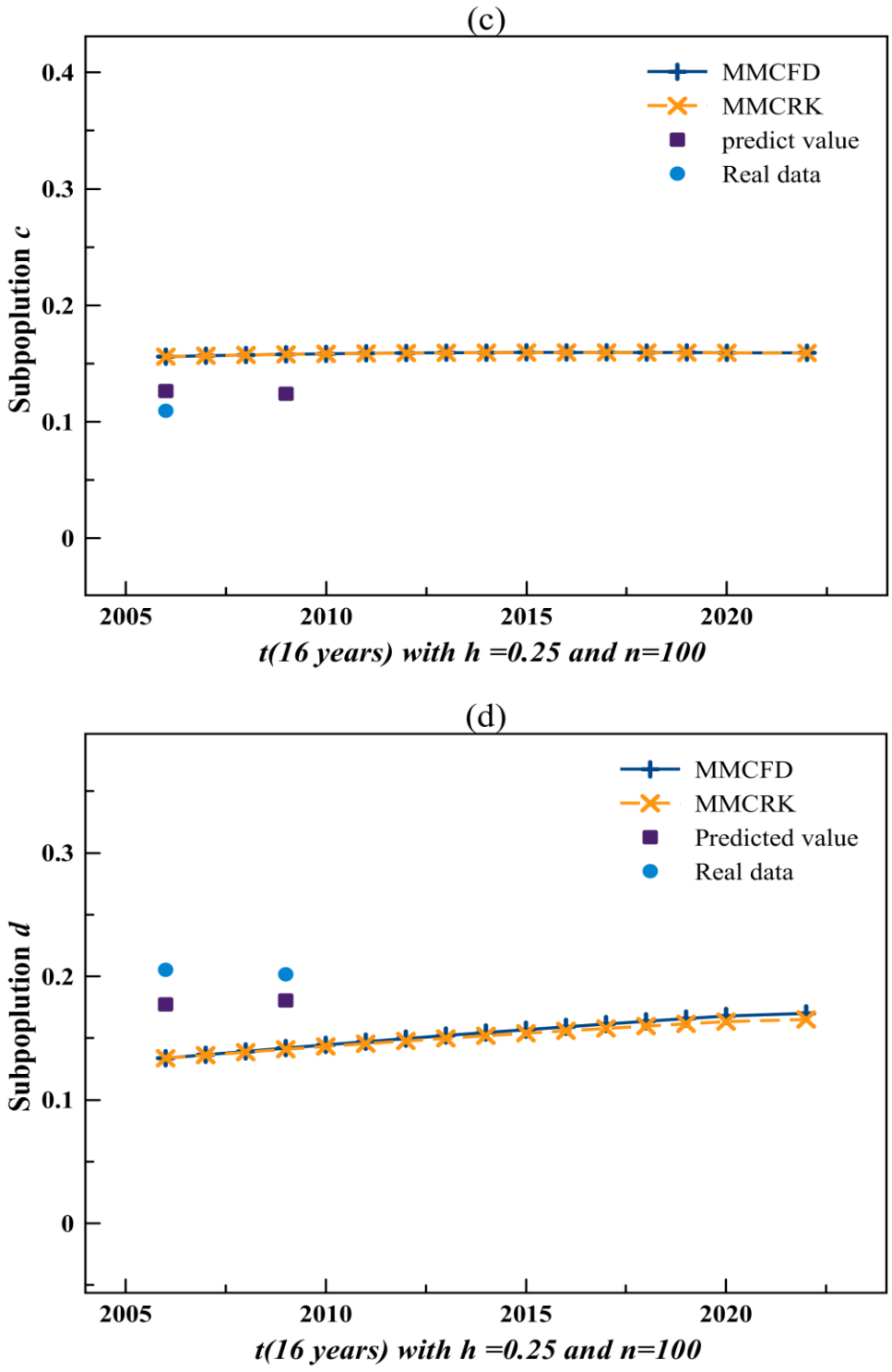

Figure 4.7 (c, d): Numerical simulation solutions by MMCFD and MMCRK around predicted values and Real data [33] when h=0.25 (real step size) and n=100 (number of simulations) of (c) $c(t)$ and (d) $d(t)$ from 2006 to 2022 years.

Figure 4.7 (a, b) and Figure 4.7 (c, d) when h=0.25, n=100 and m=16 are described approaches the curve of MMCRK of the predicted value. Figure 4.7 (a) the curve of MMCRK method near to the predict value more than the curve of MMCFD method and start decreasing gradually until 2022. Figure 4.7 (b) of $b(t)$ is explain us the curves of the MMCRK method is near from 2006 until 2013, then the curves of mentioned method is decreasing step by step yearly to 2022. For Figure 4.7 (c) of $c(t)$ the curve of MMCRK method under study is near of predicted value and starts with little increasing until 2022. Figure 4.7 (d) of $d(t)$, there is increasing from 2006 to 2022, for the curve of the numerical simulation method MMCRK and the mentioned method is near of the predicted value.

4.3.2.2 Results Analysis:

In the current study, the behavior of the bad social habit of the smoking habit of the nonlinear epidemic model is analyzed through sixteen years under study from 2006 to 2022. In our work, some reliable numerical simulation methods are used to solve a nonlinear system of epidemic models for ordinary differential equations of the first order. There is a convergence in the results of the new modified MMCRK are smaller than MMCFD errors in the nonlinear case. The numerical simulation methods help to analyze the effects of the bad social habit of smoking habit model and expect the behavior of the population in the future about this habit.

Because of the randomness in the numerical simulation methods, this feature cannot be found in the approximate (analytic, numerical) methods, therefore, the expectation of the next years for the smoking habit can be studied through using the numerical simulation methods.

The results obtained are shown the subpopulation $a(t)$ of non-smokers stay stable along sixteen years with MMCFD and MMCRK curves. While subpopulation $b(t)$ of normal-smoke it is decreasing until 2022. According subpopulation $c(t)$ of excessive smokers it is keep on the same level until 2022. Finally the subpopulation $d(t)$ of ex-smokers has a small increase to 2022 that refer to there is increase to smoking habit in this region. The most predicted values [33] around the ADM, VIM, FD, RK4, MMCFD and MCRK curves, that mean to the reliability of the obtained results.

CHAPTER 5:

CONCLUTION AND FUTUER WORKS

5.1 Conclusion:

The aim of this study is to solve a system by a special process that considers a randomized to merge with a numerical iteration method as a new process that proposed for the first time. Since the approximate methods (analytic, numerical) are inappropriate to solve such models that have constant coefficients, therefore the proposed method is more suitable to solve this type of the systems that have a random variable in their coefficients.

The importance of the proposed method has been highlighted; the proposed method can expect the behavior of a population under study the next few years in a predicted period, in order to help to analyze the behavior of some models such as epidemic models that have been applied which are represented in alcohol consumption and smoking habit. While the other analytical and numerical methods despite its efficiency, but they find only the current solution since there is no randomness in their coefficients. We do not say the proposed method is always better than the approximate methods in the area of the precision and the approach to the solutions. The proposed numerical simulation method is more appropriated to solve such systems that have their coefficients as random variables which depend on the variable time, these coefficients are treated by the simulation process.

In this thesis, a new modified numerical simulation technique for solving nonlinear epidemic models is proposed. The importance of the current study, the research objective, the problem statement which

highlighted why this study is necessary have been provided. The scope of the research and the outline of thesis have also been displayed. The important role of simulation technique has been explained for solving the epidemic models with random parameters.

In the current study that appears in **Chapter 2**, there is a convergence in the results of the used analytic methods which are AM and VIM methods are examined in the nonlinear case. In our work, they are used for solving a system of nonlinear ordinary differential equations. The behavior of bad social habit which is alcohol consumption in Spain is analyzed, based on the epidemiological model through ten years under study from 1997 to 2007. The obtained results are shown that there is increasing in alcohol consumption with the non- risk-drink consumers and declining the risk-drink consumers during the ten years under study from 1997 to 2007. For the non-drink consumers have a small increase with the VIM keeping the same level with the ADM. The most predicted values [73] are around the ADM and VIM curves.

In our work, some reliable numerical methods which are FD and RK4. The behavior of the bad social habit of the nonlinear epidemic model is analyzed through ten years under study from 1997 to 2007. The numerical FD and RK4 methods help to show the effects of the bad social habit of alcohol consumption on Spanish population during the years under study, in **Chapter 2**.

In **Chapter 3**, the behavior of the bad social habit of the nonlinear epidemic model is analyzed through sixteen years under study from 2006 to 2022. There is a convergence in the results of the analytic methods which are ADM and VIM and the numerical methods which are FD and RK4 that examined in the nonlinear case. The analytic methods ADM and VIM with the numerical methods FD and RK4 methods help to note the effects of the bad social of smoking habit. The

results obtained have been shown that the subpopulation $a(t)$ of non-smokers stay stable along three years under study from 2006 to 2009 except the ADM curve. While the subpopulation $b(t)$ of normal-smoke and the subpopulation $c(t)$ of excessive smokers are gradually declining until 2022. Finally, the subpopulation $d(t)$ of ex-smokers is arising to 2022 that refer to there is increasing in the smoking habit of this region with the ex-smokers, in spite of the law of a social smoking habit was applied. But the law can useful to reduce this habit with the other subpopulations. The most predicted values [33] are around the ADM,VIM, FD and RK4 curves that mean to the reliability of the obtained results.

In **Chapter 4**, the new numerical simulation solutions of the method MMCFD and the new numerical simulation proposed method MMCRK, for the nonlinear epidemic models have been discussed and analyzed. The two epidemic models under study, which are alcohol consumption and smoking habit are applied in this chapter on the numerical simulation methods. The mean square error and the difference measure error are used for comparison between the approximate methods or the numerical simulation solutions and the predicted value. The alcohol consumption model has the smallest mean square error when n=1000 (number of simulations), step size h=1 and iterations m=10 for subpopulations $a(t)$ and $m(t)$ of MMCRK, subpopulation $r(t)$ has smalles error when h=25, n=1000, and iterations m=10 and has the smallest difference measure error when h=1 (real step size), repetitions n=100 and iterations m=10 for subpopulation $a(t)$, $m(t)$ and h=0.25 for subpopulation $r(t)$. The obtained results are shown that there is an increasing in alcohol consumption with the non- risk-drink consumers and declining the risk-drink consumers during the thirty years from 1997 to 2027 under study with MMRK. For the non-drink consumers have a

decreasing with the MMCRK method. The most predicted values [73] around the MMCRK curves. That means MMCRK can expect that the increasing may be happening in the future for alcohol consumption habit in Spain.

The results obtained from smoking habit model are shown the subpopulation $a(t)$ of non-smokers stay stable along sixteen years from 2006 to 2022 with MMCFD and MMCRK curves. While subpopulation $b(t)$ of normal-smoke it is decreasing from 2006 to 2022. According to the subpopulation $c(t)$ of excessive smokers, it is keep on the same level from 2006 until 2022. Finally the subpopulation $d(t)$ of ex-smokers has a small increase from 2006 to 2022 that refer to there is an increase in smoking habit of $d(t)$ only in this region under study, in spite of the law of avoid the smoking habit was applied. The most predicted values [33] around the MMCFD and MMCRK curves that mean to the reliability of the obtained results.

The epidemic models under study have the smallest mean square error and the smallest difference error with MMCRK with all subpopulations. In other words, MMCRK is better than MMCFD according to the expectation the next years.

The results are calculated by the Mathematica.11 software for analytical methods ADM and VIM and MATLAB 2013 software for numerical methods FD and RK4, the figures are drawn by the Magic Plot software.

5.2 Recommendations and Future Works:

The current study deals with modification of the numerical simulation methods.We recommend the following ideas:

1. We recommend to apply the methods under study, to other epidemic models.

2. MMCRK method is suggested to solve an autonomous system of nonlinear IVP of higher order ODEs, partial DEs, fractional ODEs, and all types if they have a random variable as a coefficient in the model under research [38].

3. The advanced of RK numerical iteration methods with different orders such as RK45 and RK78 can be suggested to merge with MC simulation techniques as a new method to solve deterministic models with random parameters [58].
4. Other analytical methods can be suggested to solve such system under study like homotopy perturbation analysis method and Semi analytical iterative method Temimi and Ansari.
5. On the other hands, there are other numerical iteration methods can be suggested to solve the system under study.
6. Other kinds of simulation techniques like a Latin Hypercube Sampling, Box–Muller transform and so on, can be used to simulate the random parameters of stochastic deterministic models.
7. We recommend to change the process of the numerical simulation methods to get the optimal number of iteration and simulation that help to obtain the best expectation for the nature of subpopulations under study in the future.

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Appendixes

Appendix A

MMCFD Application of Alcohol Consumption Model

```
%FDM with MC to solve model of the nonlinear ODE system
%********************************************************
clc
clear
close all
format long
%Nonlinear system of differential Equations for Solving
%alcohol consumption in Spain%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%parameters of the model%
disp('parameters of model')
b11 =0.01;          %prportion of the birth rate in Spain.
b22 =0.00144;       %proportion of the rate at which a
risk consumer becomes a non-consumer.
b33 =0.08;          %proportion of the death rate in
Spain.
b44 =0.0284;        %proportion the transmission rate due
to social pressure to increase the alcohol consumption.
b55 =0.009;         %prportion of the augmented death rate
due alcohol consumption.
b66 =0.000110247;   %prportion of the rate at which a non-
risk consumer moves to the risk consumption
subpopulation.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Generate values from the uniform distribution on the
interval [a, b].
  k=1000;      %number of simulation
  q=10;        %number of iteration
  h=0.25;      %step size

for j=1:k
  rand('seed',k)
  b1=(b11-0.2*b11)+((b11+0.2*b11)-(b11-
0.2*b11))*rand(j);
  b2=(b22-0.2*b22)+((b22+0.2*b22)-(b22-
0.2*b22))*rand(j);
  b3=(b33-0.2*b33)+((b33+0.2*b33)-(b33-
0.2*b33))*rand(j);
  b4=(b44-0.2*b44)+((b44+0.2*b44)-(b44-
0.2*b44))*rand(j);
```

```
  b5=(b55-0.2*b55)+((b55+0.2*b55)-(b55-
0.2*b55))*rand(j);
  b6=(b66-0.2*b66)+((b66+0.2*b66)-(b66-
0.2*b66))*rand(j);
  j=j+1;
end

disp('b1 b2 b3 b4 b5 b6')
Par=[ Parameter_b1'   Parameter_b2'   Parameter_b3'
Parameter_b4'    Parameter_b5'    Parameter_b6' ]


%initial conditions%
%x0=[a0 m0 r0];
disp('initial conditions')
disp('-------------------------------------------------')
for j=1:k

%initial conditions%
%x0=[a0 m0 r0];
%disp('a0 = initial a ')
a0=0.362; a(1,j)=a0;          %initial value of a(t)
%disp('m0 = initial m ')
m0=0.581; m(1,j)=m0;          %initial value of m(t)
%disp('r0 = initial r ')
r0=0.057; r(1,j)=r0;          %initial value of r(t)
%disp('t0 = initial t ')
t0=0;                         %initial condition of t
%time per year

%Backward FD of the model%
a(2,j)=a(1,j)+h*(b1(j)+b2(j)*r(1,j)-b3(j)*a(1,j)-
b4(j)*a(1,j)*(m(1,j)+r(1,j))-a(1,j)*(b1(j)-b3(j)*a(1,j)-
b5(j)*m(1,j)-b5(j)*r(1,j)));
m(2,j)=m(1,j)+h*(b4(j)*a(1,j)*(m(1,j)+r(1,j))-
b6(j)*m(1,j)+b3(j)*a(1,j)*m(1,j)-b5(j)*a(1,j)*m(1,j)-
b1(j)*m(1,j));
r(2,j)=r(1,j)+h*(b6(j)*m(1,j)-
b2(j)*r(1,j)+b3(j)*a(1,j)*r(1,j)-b5(j)*a(1,j)*r(1,j)-
b1(j)*r(1,j));

%Centeral FD of the model%
%ai+1-ai-1/2h = b1+b2*ri-b3*ai-b4*ai(mi+ri)-ai[b1-b3*ai-
b5*mi-b5*ri]
%mi+1-mi+1/2h = b4*ai(mi+ri)-b6*mi+b3*ai*mi-b5*ai*mi-
b1*mi
%ri+1-ri-1/2h = b6*mi-b2*ri+b3*ai*ri-b5*ai*ri-b1*ri


for i=2:q/h
```

```
a(i+1,j)=a(i-1,j)+2*h*(b1(j)+b2(j)*r(i,j)-b3(j)*a(i,j)-
b4(j)*a(i,j)*(m(i,j)+r(i,j))-a(i,j)*(b1(j)-b3(j)*a(i,j)-
b5(j)*m(i,j)-b5(j)*r(i,j)));
m(i+1,j)=m(i-1,j)+2*h*(b4(j)*a(i,j)*(m(i,j)+r(i,j))-
b6(j)*m(i,j)+b3(j)*a(i,j)*m(i,j)-b5(j)*a(i,j)*m(i,j)-
b1(j)*m(i,j));
r(i+1,j)=r(i-1,j)+2*h*(b6(j)*m(i,j)-
b2(j)*r(i,j)+b3(j)*a(i,j)*r(i,j)-b5(j)*a(i,j)*r(i,j)-
b1(j)*r(i,j));
 sol=zeros(q,3);
 sol(i,j,1) = a(i,j);
 sol(i,j,2) = m(i,j);
 sol(i,j,3) = r(i,j);
 i=i+1;
   end
j=j+1;
end

%result=[a' m' r']
 result=zeros(q/h,k,3);
for i=1:q/h
  res(i,k,1) = a(i,k);
  res(i,k,2) = m(i,k);
  res(i,k,3) = r(i,k);
  i=i+1;
end
result=[ res(:,k,1)    res(:,k,2)     res(:,k,3) ]

disp('[ a m r ]')
for j=1:k
  sol=[  a(:,j)    m(:,j)    r(:,j)  ]
end

solfinal=zeros(q/h,3);
for j=1:k
    solfinal_a(j)=a(q/h,j);
    solfinal_m(j)=m(q/h,j);
    solfinal_r(j)=r(q/h,j);
    j=j+1;
end
disp('[   solfinal_a      solfinal_m      solfinal_r
]')
solfinal=[solfinal_a(:)   solfinal_m(:)   solfinal_r(:)]
fprintf('solfinal_a       solfinal_m       solfinal_r
\n')
fprintf( '%1.5 f  %1.5 f  %1.5 f  \n' , solfinal_a  ,
solfinal_m  , solfinal_r )

musolfinal_a = mean(solfinal_a);
musolfinal_m = mean(solfinal_m);
musolfinal_r = mean(solfinal_r);
```

```
disp('[ mean_a                    mean_m
mean_r ]')
disp([musolfinal_a              musolfinal_m
musolfinal_r])

%Predicted values in 2027, when t=30 years
predict_a=0.362;
predict_m=0.581;
predict_r=0.057;

%Difference error
Diff_a=abs(predict_a-musolfinal_a);
Diff_m=abs(predict_m-musolfinal_m);
Diff_r=abs(predict_r-musolfinal_r);

disp('[Diff_a      Diff_b      Diff_c     Diff_d]')
Diff_error=[Diff_a       Diff_m     Diff_r]

%Absolute relative approximate error
RE_a =abs((predict_a-musolfinal_a)/musolfinal_a);
RE_m =abs((predict_m-musolfinal_m)/musolfinal_m);
RE_r =abs((predict_r-musolfinal_r)/musolfinal_r);

disp('[RE_a    RE_m     RE_r]')
RE_error = [RE_a    RE_m    RE_r]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('[prctile_a]')
Pa=prctile(solfinal_a,[5 95])
disp('[prctile_m]')
Pm=prctile(solfinal_m,[5 95])
disp('[prctile_r]')
Pr=prctile(solfinal_r,[5 95])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('Results of MMCFD of alcohol consumption')
%Predicted values in --------, when t years
predicta=0.362;
predictm=0.581;
predictr=0.057;

MMCFDa=musolfinal_a;
MMCFDm=musolfinal_m;
MMCFDr=musolfinal_r;

disp('[Numerical Simulation for the System]')
disp('[MMCFD_a  MMCFD_m  MMCFD_r]')
disp([MMCFDa   MMCFDm   MMCFDr])
%#####################################################
%Estimate of error by mean square error
```

```
disp('MSE to measure results error')

   %summation account
   aMSEsum1=0;
   mMSEsum1=0;
   rMSEsum1=0;

 k=1;
 while k<=q
   aMSEsum1=aMSEsum1+(predicta-solfinal_a(k))^2;
   mMSEsum1=mMSEsum1+(predictm-solfinal_m(k))^2;
   rMSEsum1=rMSEsum1+(predictr-solfinal_r(k))^2;
   k=k+1;
 end

   aMSE_SOL =(1/q)*(aMSEsum1);
   mMSE_SOL =(1/q)*(mMSEsum1);
   rMSE_SOL =(1/q)*(rMSEsum1);

disp('[aMSE_SOL       mMSE_SOL         rMSE_SOL]')
MSE_SOL=[aMSE_SOL      mMSE_SOL      rMSE_SOL]
%@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

figure(1)
%Sketch the results of the last simulation
disp('[  last simulation_a                  last
simulation_m                        last simulation_r ]')
last_sim=[  a(:,k)    m(:,k)    r(:,k)   ]
t=t0:h:q; %Calculates up to t_final results
plot(t,a(:,k),'o',t,m(:,k),'x',t,r(:,k),'*')
title('MMCFD solutions of alcohol consumption model');

xlabel('30 Years');
ylabel('Subpopulations a,m,r');
legend('a','m','r')

figure(2)
t=t0:h:q;
subplot(3,1,1)
plot(t,a(:,k),'o')
title('MMCFD of a solution of alcohol consumption
model');
xlabel('30 Years ');
ylabel('a');

subplot(3,1,2)
plot(t,m(:,k),'x')
title('MMCFD of m solution of alcohol consumption
model');
xlabel('30 Years ');
ylabel('m');
```

```
subplot(3,1,3)
plot(t,r(:,k),'*')
title('MMCFD of r solution of the alcohol consumption
model');
xlabel('30 Years ');
ylabel('r');
```

Appendix B

MMCRK Application of Alcohol Consumption Model

```
%RK4 with MC to solve models of the nonlinear ODE system
%*****************************************************
clc
clear
close all
format long

%To solve a model for the evolution of alcohol
consumption in Spain%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%model%
%da_dt = b1+b2*ri-b3*ai-b4*ai(mi+ri)-ai[b1-b3*ai-b5*mi-
b5*ri]
%dm_dt = b4*ai(mi+ri)-b6*mi+b3*ai*mi-b5*ai*mi-b1*mi
%dr_dt = b6*mi-b2*ri+b3*ai*ri-b5*ai*ri-b1*ri

%initial values%
%t0=0;                  %time per year
%y0=[a0 m0 r0];         %y0 = initial condition of a
system
%a0=0.362; a(1)=a0;     %initial value of a(t)
%m0=0.581; m(1)=m0;     %initial value of m(t)
%r0=0.057; r(1)=r0;     %initial value of r(t)

%Model with RK4%
%dy/dt in form of f(t,y).it can be a function of both
variables t and y,
%da_dt =f1(t,a,m,r);
%dm_dt =f2(t,a,m,r);
%dr_dt =f3(t,a,m,r);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%parameters of the model%
%disp('parameters of model')
%a11 =0.01;          %prportion of the birth rate in Spain
%a22 =0.00144;       %proportion of the rate at which a
risk consumer becomes a non-consumer.
%a33 =0.08;          %proportion of the death rate in
Spain.
%a44 =0.0284;        %proportion the transmission rate due
to social pressure to increase the alcohol consumption.
%a55 =0.009;         %prportion of the augmented death
rate due to alcohol consumption.
%a66 =0.000110247; %prportion of the rate at which a
non-risk consumer moves to the risk consumption
subpopulation.
```

```
disp('******************Input Data*******************')
  k=100;      %number of simulation
              %q=number of iteration= numbers of years
              between 2006 & 2009
  q=10;       %number of iteration
  h=0.25;     %step size

%disp('parameters of model')
 a11=0.01;
 a22=0.00144;
 a33=0.08;
 a44=0.0284;
 a55=0.009;
 a66=0.000110247;

%Generate values from the uniform distribution on the
interval[a, b].
for j=1:k
  rand('seed',k)
  a1=(a11-0.2*a11)+((a11+0.2*a11)-(a11-
0.2*a11))*rand(j);
  a2=(a22-0.2*a22)+((a22+0.2*a22)-(a22-
0.2*a22))*rand(j);
  a3=(a33-0.2*a33)+((a33+0.2*a33)-(a33-
0.2*a33))*rand(j);
  a4=(a44-0.2*a44)+((a44+0.2*a44)-(a44-
0.2*a44))*rand(j);
  a5=(a55-0.2*a55)+((a55+0.2*a55)-(a55-
0.2*a55))*rand(j);
  a6=(a66-0.2*a66)+((a66+0.2*a66)-(a66-
0.2*a66))*rand(j);
  j=j+1;
end

disp('a1 a2 a3 a4 a5 a6')
Par=[ Parameter_a1'   Parameter_a2'   Parameter_a3'
Parameter_a4'    Parameter_a5'    Parameter_a6' ]

disp('-----------------------------------------------')
for j=1:k
  for i=1:q/h

%initial conditions %
%x0=[a0 m0 ro];
%disp('a0 = initial a ')
a0=0.362; a(1,j)=a0;          %initial value of a(t)
%disp('m0 = initial m ')
m0=0.581; m(1,j)=m0;          %initial value of m(t)
%disp('r0 = initial r ')
r0=0.057; r(1,j)=r0;          %initial value of r(t)
%disp('t0 = initial t ')
```

```
t0=0;                          %initial condition of t
%time per year

ka1=zeros(q/h,k);
ka2=zeros(q/h,k);
ka3=zeros(q/h,k);
ka4=zeros(q/h,k);

km1=zeros(q/h,k);
km2=zeros(q/h,k);
km3=zeros(q/h,k);
km4=zeros(q/h,k);

kr1=zeros(q/h,k);
kr2=zeros(q/h,k);
kr3=zeros(q/h,k);
kr4=zeros(q/h,k);

%Calculating ka1,km1,kr1
ka1(i,j) = a1(j)+a2(j)*r(i,j)-a3(j)*a(i,j)-
a4(j)*a(i,j)*(m(i,j)+r(i,j)-a(i,j)*(a1(j)-a3(j)*a(i,j)-
a5(j)*m(i,j)-a5(j)*r(i,j)));

km1(i,j) = a4(j)*a(i,j)*(m(i,j)+r(i,j))-
a6(j)*m(i,j)+a3(j)*a(i,j)*m(i,j)-a5(i)*a(i,j)*m(i,j)-
a1(j)*m(i,j);

kr1(i,j) = a6(j)*m(i,j)-
a2(j)*r(i,j)+a3(j)*a(i,j)*r(i,j)-a5(j)*a(i,j)*r(i,j)-
a1(j)*r(i,j);



%Calculating ka2,km2,kr2

ka2(i,j)=a1(j)+a2(j)*(r(i,j)+0.5*kr1(i,j))-
a3(j)*(a(i,j)+0.5*ka1(i,j))-
a4(j)*(a(i,j)+0.5*ka1(i,j))*((m(i,j)+0.5*km1(i,j)+(r(i,j
)+0.5*kr1(i,j)))-(a(i,j)+ka1(i,j))*(a1(j)-
a3(j)*(a(i,j)+ka1(i,j))-a5(j)*(m(i,j)+km1(i,j))-
a5(j)*(r(i,j)+0.5*kr1(i,j))));

km2(i,j)=a4(j)*(a(i,j)+0.5*ka1(i,j))*((m(i,j)+0.5*km1(i,
j)+r(i,j)+0.5*kr1(i,j)))-
a6(j)*(m(i,j)+0.5*km1(i,j))+a3(j)*(a(i,j)+0.5*ka1(i,j))*
(m(i,j)+0.5*km1(i,j))-
a5(j)*(a(i,j)+0.5*ka1(i,j))*(m(i,j)+0.5*km1(i,j))-
a1(j)*(m(i,j)+0.5*km1(i,j));

kr2(i,j)=a6(j)*(m(i,j)+0.5*km1(i,j))-
a2(j)*(r(i,j)+kr1(i,j))+a3(j)*(a(i,j)+0.5*ka1(i,j))*(r(i
```

```
,j)+0.5*kr1(i,j))-
a5(j)*(a(i,j)+0.5+ka1(i,j))*(r(i,j)+0.5*kr1(i,j))-
a1(j)*(r(i,j)+0.5*kr1(i,j));

%Calculating ka3,km3,kr3

ka3(i,j)=a1(j)+a2(j)*(r(i,j)+0.5*kr2(i,j))-
a3(j)*(a(i,j)+0.5*ka2(i,j))-
a4(j)*(a(i,j)+0.5*ka2(i,j))*((m(i,j)+0.5*km2(i,j)+r(i,j)
+0.5*kr2(i,j)))-(a(i,j)+ka2(i,j))*(a1(j)-
a3(j)*(a(i,j)+ka2(i,j))-a5(j)*(m(i,j)+km2(i,j))-
a5(j)*(r(i,j)+0.5*kr2(i,j)));

km3(i,j)=a4(j)*(a(i,j)+0.5*ka2(i,j))*((m(i,j)+0.5*km2(i,
j))+(r(i,j)+0.5*kr2(i,j)))-
a6(j)*(m(i,j)+0.5*km2(i,j))+a3(j)*(a(i,j)+0.5*ka2(i,j))*
(m(i,j)+0.5*km2(i,j))-
a5(j)*(a(i,j)+0.5*ka2(i,j))*(m(i,j)+0.5*km2(i,j))-
a1(j)*(m(i,j)+0.5*km2(i,j));

kr3(i,j)=a6(j)*(m(i,j)+0.5*km2(i,j))-
a2(j)*(r(i,j)+kr2(i,j))+a3(j)*(a(i,j)+0.5*ka2(i,j))*(r(i
,j)+0.5*kr2(i,j))-
a5(j)*(a(i,j)+0.5+ka2(i,j))*(r(i,j)+0.5*kr2(i,j))-
a1(j)*(r(i,j)+0.5*kr2(i,j));

%Calculating ka4,km4,kr4

ka4(i,j)=a1(j)+a2(j)*(r(i,j)+0.5*kr3(i,j))-
a3(j)*(a(i,j)+0.5*ka3(i,j))-
a4(j)*(a(i,j)+0.5*ka3(i,j))*((m(i,j)+0.5*km3(i,j))+(r(i,
j)+0.5*kr3(i,j)))-(a(i,j)+ka3(i,j))*(a1(j)-
a3(j)*(a(i,j)+ka3(i,j))-a5(j)*(m(i,j)+km3(i,j))-
a5(j)*(r(i,j)+0.5*kr3(i,j)));

km4(i,j)=a4(j)*(a(i,j)+0.5*ka3(i,j))*((m(i,j)+0.5*km3(i,
j))+(r(i,j)+0.5*kr3(i,j)))-
a6(j)*(m(i,j)+0.5*km3(i,j))+a3(j)*(a(i,j)+0.5*ka3(i,j))*
(m(i,j)+0.5*km3(i,j))-
a5(j)*(a(i,j)+0.5*ka3(i,j))*(m(i,j)+0.5*km3(i,j))-
a1(j)*(m(i,j)+0.5*km3(i,j));

kr4(i,j)=a6(j)*(m(i,j)+0.5*km3(i,j))-
a2(j)*(r(i,j)+kr3(i,j))+a3(j)*(a(i,j)+0.5*ka3(i,j))*(r(i
,j)+0.5*kr3(i,j))-
a5(j)*(a(i,j)+0.5+ka3(i,j))*(r(i,j)+0.5*kr3(i,j))-
a1(j)*(r(i,j)+0.5*kr3(i,j));

%Using 4th Order Runge-Kutta formula
```

```
a(i+1,j)=a(i,j)+(1/6)*(ka1(i,j)+2*ka2(i,j)+2*ka3(i,j)+ka
4(i,j))*h;

m(i+1,j)=m(i,j)+(1/6)*(km1(i,j)+2*km2(i,j)+2*km3(i,j)+km
4(i,j))*h;

r(i+1,j)=r(i,j)+(1/6)*(kr1(i,j)+2*kr2(i,j)+2*kr3(i,j)+kr
4(i,j))*h;

  i=i+1;
  end
j=j+1;
end

%result=[a' m' r']
 result=zeros(q/h,k,3);
for i=1:q/h
  res(i,k,1) = a(i,k);
  res(i,k,2) = m(i,k);
  res(i,k,3) = r(i,k);
 i=i+1;
end
result=[ res(:,k,1)    res(:,k,2)     res(:,k,3) ]

 disp('[ a m r ]')
for j=1:k
  sol=[  a(:,j)    m(:,j)    r(:,j)  ]
end
  solfinal=zeros(q/h,3);
  for j=1:k
    solfinal_a(j)=a(q/h,j);
    solfinal_m(j)=m(q/h,j);
    solfinal_r(j)=r(q/h,j);
    j=j+1;
  end

disp('[   solfinal_a      solfinal_m      solfinal_r
]')
solfinal=[solfinal_a(:)   solfinal_m(:)   solfinal_r(:)]
fprintf('solfinal_a       solfinal_m       solfinal_r
\n')
fprintf( '%1.5 f  %1.5 f  %1.5 f  \n' , solfinal_a  ,
solfinal_m  , solfinal_r )

musolfinal_a = mean(solfinal_a);
musolfinal_m = mean(solfinal_m);
musolfinal_r = mean(solfinal_r);

disp('[ mean_a                    mean_m
mean_r ]')
```

```
disp([musolfinal_a                musolfinal_m
musolfinal_r])

%Predicted values in 2009, when t=3 years
predict_a=0.362;
predict_m=0.581;
predict_r=0.057;

%Difference error
Diff_a=abs(predict_a-musolfinal_a);
Diff_m=abs(predict_m-musolfinal_m);
Diff_r=abs(predict_r-musolfinal_r);

disp('[Diff_a      Diff_b      Diff_c      Diff_d]')
Diff_error=[Diff_a        Diff_m      Diff_r]

%Absolute relative approximate error
AE_a =abs((predict_a-musolfinal_a)/musolfinal_a);
AE_m =abs((predict_m-musolfinal_m)/musolfinal_m);
AE_r =abs((predict_r-musolfinal_r)/musolfinal_r);

disp('[AE_a    AE_m     AE_r]')
AE_error = [AE_a    AE_m    AE_r]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('[prctile_a]')
Pa=prctile(solfinal_a,[5 95])
disp('[prctile_m]')
Pm=prctile(solfinal_m,[5 95])
disp('[prctile_r]')
Pr=prctile(solfinal_r,[5 95])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('Results of MMCRK of alcohol consumption')

%Predicted values in  --------, when t years
predicta=0.362;
predictm=0.581;
predictr=0.057;

MMCRKa=musolfinal_a;
MMCRKm=musolfinal_m;
MMCRKr=musolfinal_r;

disp('[Numerical Simulation for the System]')
disp('[MMCRK_a  MMCRK_m  MMCRK_r]')
disp([MMCRKa   MMCRKm   MMCRKr])
%#########################################################
```

```
%Estimate error by mean square error
disp('MSE to measure results error')

%summation account
   aMSEsum1=0;
   mMSEsum1=0;
   rMSEsum1=0;

 k=1;
 while k<=q
   aMSEsum1=aMSEsum1+(predicta-solfinal_a(k))^2;
   mMSEsum1=mMSEsum1+(predictm-solfinal_m(k))^2;
   rMSEsum1=rMSEsum1+(predictr-solfinal_r(k))^2;
    k=k+1;
 end

   aMSE_SOL =(1/q)*(aMSEsum1);
   mMSE_SOL =(1/q)*(mMSEsum1);
   rMSE_SOL =(1/q)*(rMSEsum1);

disp('[aMSE_SOL       mMSE_SOL         rMSE_SOL]')
MSE_SOL=[aMSE_SOL      mMSE_SOL      rMSE_SOL]
%@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

figure(1)
%Sketch the results of the last simulation
disp('[  last simulation_a                  last
simulation_m                      last simulation_r ]')
last_sim=[  a(:,k)    m(:,k)    r(:,k)   ]
t=t0:h:q;     %Calculates up to t_final results
plot(t,a(:,k),'o',t,m(:,k),'x',t,r(:,k),'*')
title('MMCRK solutions of alcohol consumption model');
xlabel('30 Years');
ylabel('Subpopulations a,m,r');
legend('a','m','r')

figure(2)
t=t0:h:q;
subplot(3,1,1)
plot(t,a(:,k),'o')
title('MMCRK of a solution of alcohol consumption
model');
xlabel('30 Years ');
ylabel('a');

subplot(3,1,2)
plot(t,m(:,k),'x')
title('MMCRK of m solution of alcohol consumption
model');
xlabel('30 Years ');
ylabel('m');
```

```
subplot(3,1,3)
plot(t,r(:,k),'*')
title('MMCRK of r solution of the alcohol consumption
model');
xlabel('30 Years ');
ylabel('r');
```

Appendix C

MMCFD Application of Smoking Habit Model

```
%FDM with MC to solve model of the nonlinear ODE system
%*******************************************************
clc
clear
close all
format long
%%%Nonlinear system of differential Equations for
Solving a model for the Evolution of smoking habit in
Spain%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%parameters of the model%%
disp('parameters of model')
b11 =0.01;          %birth rate in Spain.
b22 =0.0381;        %transmission rate due to social
pressure to adopt smoking habit.
b33 =0.0425;        %rate at which ex-smokers return to
smoking.
b44 =0.1244;        %rate at which an excessive smoker
becomes a normal smoker by decreasing the number of
cigarettes per day.
b55 =0.1175;        %rate at which normal smokers become
excessive smokers by increasing the number of cigarettes
per day.
b66 =0.0498;        %rate at which normal smokers stop
smoking.
b77 =0.0498;        %rate at which excessive smokers stop
smoking.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%Generate values from the uniform distribution on the
interval [a, b].
  k=1000;     %number of simulation
  q=3;        %number of iteration
  h=0.25;     %step size

for j=1:k
  rand('seed',k)
  b1=(b11-0.2*b11)+((b11+0.2*b11)-(b11-
0.2*b11))*rand(j);
  b2=(b22-0.2*b22)+((b22+0.2*b22)-(b22-
0.2*b22))*rand(j);
  b3=(b33-0.2*b33)+((b33+0.2*b33)-(b33-
0.2*b33))*rand(j);
  b4=(b44-0.2*b44)+((b44+0.2*b44)-(b44-
0.2*b44))*rand(j);
```

```
  b5=(b55-0.2*b55)+((b55+0.2*b55)-(b55-
0.2*b55))*rand(j);
  b6=(b66-0.2*b66)+((b66+0.2*b66)-(b66-
0.2*b66))*rand(j);
  b7=(b77-0.2*b77)+((b77+0.2*b77)-(b77-
0.2*b77))*rand(j);
  j=j+1;
end

disp('b1 b2 b3 b4 b5 b6')
Par=[ Parameter_b1'    Parameter_b2'   Parameter_b3'
Parameter_b4'     Parameter_b5'   Parameter_b6'
Parameter_b7' ]

disp('---------------------------------')
%initial conditions%
disp('initial conditions')

for j=1:k
%initial conditions %
%disp('t0 = initial t ')
t0=0;           %initial condition of t  %time per year

%disp('a0 = initial a ')
a0=0.5045;
a(1,j)=a0;      %initial value of a(t)
%disp('a0 = initial a ')
b0=0.2059;
b(1,j)=b0;      %initial value of b(t)
%disp('a0 = initial a ')
c0=0.1559;
c(1,j)=c0;      %initial value of c(t)
%disp('a0 = initial a ')
d0=0.1337;
d(1,j)=d0;      %initial value of d(t)

%Backward FD of the model%
a(2,j)=a(1,j)+h*(b1(j)*(1-a(1,j))-
b2(j)*a(1,j)*(b(1,j)+c(1,j)));
b(2,j)=b(1,j)+h*(b2(j)*a(1,j)*(b(1,j)+c(1,j))+b3(j)*d(1,
j)+b4(j)*c(1,j)-(b5(j)+b6(j)+b1(j))*b(1,j));
c(2,j)=c(1,j)+h*(b5(j)*b(1,j)-
(b4(j)+b7(j)+b1(j))*c(1,j));
d(2,j)=d(1,j)+h*(b6(j)*b(1,j)+b7(j)*c(1,j)-
(b3(j)+b1(j))*d(1,j));

%Centeral FD of the model%
%a(i+1)-a(i-1)/2h = b1*(1-a(i))-b2*a(i)*(b(i)+c(i))
%b(i+1)-b(i+1)/2h = b2*a(i)*(b(i)+c(i))+b3*d(i)+b4*c(i)-
(b5+b6+b1)*b(i)
%c(i+1)-c(i-1)/2h = b5*b(i)-(b4+b7+b1)*c(i)
```

```
%d(i+1)-d(i-1)/2h = b6*s(i)+b7*c(i)-(b3+b1)*d(i)

   for i=2:q/h
a(i+1,j)=a(i-1,j)+2*h*(b1(j)*(1-a(i,j))-
b2(j)*a(i,j)*(b(i,j)+c(i,j)));
b(i+1,j)=b(i-
1,j)+2*h*(b2(j)*a(i,j)*(b(i,j)+c(i,j))+b3(j)*d(i,j)+b4(j
)*c(i,j)-(b5(j)+b6(j)+b1(j))*b(i,j));
c(i+1,j)=c(i-1,j)+2*h*(b5(j)*b(i,j)-
(b4(j)+b7(j)+b1(j))*c(i,j));
d(i+1,j)=d(i-1,j)+2*h*(b6(j)*b(i,j)+b7(j)*c(i,j)-
(b3(j)+b1(j))*d(i,j));

   sol=zeros(q/h,4);
   sol(i,j,1) = a(i,j);
   sol(i,j,2) = b(i,j);
   sol(i,j,3) = c(i,j);
   sol(i,j,4) = d(i,j);

 i=i+1;
   end
j=j+1;
end

%result=[a'  b'  c'  d']
 result=zeros(q/h,k,4);
for i=1:q/h
  res(i,k,1) = a(i,k);
  res(i,k,2) = b(i,k);
  res(i,k,3) = c(i,k);
  res(i,k,4) = d(i,k);
  i=i+1;
end
result=[ res(:,k,1)    res(:,k,2)     res(:,k,3)
res(:,k,4)]

disp('[ a  b  c  d ]')
for j=1:k
  sol=[  a(:,j)   b(:,j)    c(:,j)     d(:,j) ]
end

  solfinal=zeros(q/h,4);
  for j=1:k
    solfinal_a(j)=a(q/h,j);
    solfinal_b(j)=b(q/h,j);
    solfinal_c(j)=c(q/h,j);
    solfinal_d(j)=d(q/h,j);
    j=j+1;
  end
```

```
disp('[   solfinal_a      solfinal_b      solfinal_c
solfinal_d]')
solfinal=[solfinal_a(:)   solfinal_b(:)   solfinal_c(:)
solfinal_d(:)]
fprintf('solfinal_a       solfinal_b       solfinal_c
solfinal_d \n')
fprintf( '%1.5 f    %1.5 f    %1.5 f    %1.5 f\n' ,
solfinal_a  , solfinal_b, solfinal_c, solfinal_d )

musolfinal_a = mean(solfinal_a);
musolfinal_b = mean(solfinal_b);
musolfinal_c = mean(solfinal_c);
musolfinal_d = mean(solfinal_d);

disp('[ mean_a                    mean_b
mean_c                mean_d ]')
disp([musolfinal_a              musolfinal_b
musolfinal_c               musolfinal_d])

%Predicted values in 2009, when t=3 years
predict_a=0.5049;
predict_b=0.1240;
predict_c=0.1240;
predict_d=0.1805;

%Difference error
Diff_a=abs(predict_a-musolfinal_a);
Diff_b=abs(predict_b-musolfinal_b);
Diff_c=abs(predict_c-musolfinal_c);
Diff_d=abs(predict_d-musolfinal_d);

disp('[Diff_a      Diff_b      Diff_c     Diff_d]')
Diff_error=[Diff_a       Diff_b     Diff_c       Diff_d]

%Absolute relative approximate error
AE_a =abs((predict_a-musolfinal_a)/musolfinal_a);
AE_b =abs((predict_b-musolfinal_b)/musolfinal_b);
AE_c =abs((predict_c-musolfinal_c)/musolfinal_c);
AE_d =abs((predict_d-musolfinal_d)/musolfinal_d);

disp('[AE_a    AE_b     AE_c     AE_d]')
AE_error = [AE_a    AE_b    AE_c    AE_d]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('[prctile_a]')
Pa=prctile(solfinal_a,[5 95])
disp('[prctile_b]')
Pb=prctile(solfinal_b,[5 95])
disp('[prctile_c]')
Pc=prctile(solfinal_c,[5 95])
disp('[prctile_d]')
```

```
Pd=prctile(solfinal_d,[5 95])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('Results of MMCFD of smoking habit')

%Predicted values in --------, when t years
predicta=0.5049;
predictb=0.1240;
predictc=0.1240;
predictd=0.1805;

MMCFDa=musolfinal_a;
MMCFDb=musolfinal_b;
MMCFDc=musolfinal_c;
MMCFDd=musolfinal_d;

disp('[Numerical Simulation for the System]')
disp('[MMCFD_a   MMCFD_b   MMCFD_c  MMCFD_d]')
disp([MMCFDa    MMCFDb    MMCFDc    MMCFDd])
%#########################################################

%Estimate error by mean square error
disp('MSE to measure results error')

%summation account
   aMSEsum1=0;
   bMSEsum1=0;
   cMSEsum1=0;
   dMSEsum1=0;

 k=1;
 while k<=q
   aMSEsum1=aMSEsum1+(predicta-solfinal_a(k))^2;
   bMSEsum1=bMSEsum1+(predictb-solfinal_b(k))^2;
   cMSEsum1=cMSEsum1+(predictc-solfinal_c(k))^2;
   dMSEsum1=dMSEsum1+(predictd-solfinal_d(k))^2;
    k=k+1;
 end

   aMSE_SOL =(1/q)*(aMSEsum1);
   bMSE_SOL =(1/q)*(bMSEsum1);
   cMSE_SOL =(1/q)*(cMSEsum1);
   dMSE_SOL =(1/q)*(dMSEsum1);

disp('[aMSE_SOL       bMSE_SOL        cMSE_SOL
dMSE_SOL]')
MSE_SOL=[aMSE_SOL      bMSE_SOL        cMSE_SOL
dMSE_SOL]
%@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
```

```
figure(1)
%Sketch the results of the last simulation
disp('[  last simulation_a                    last
simulation_b                  last simulation_c
last simulation_d  ]')
last_sim=[  a(:,k)    b(:,k)    c(:,k)      d(:,k) ]
t=t0:h:q; %Calculates up to t_final results
plot(t,a(:,k),'o',t,b(:,k),'x',t,c(:,k),'*',t,d(:,k),'+'
)
title('MMCFD solutions of smoking habit model');
xlabel('3 Years');
ylabel('Subpopulations a,b,c,d');
legend('a','b','c','d')

figure(2)
t=t0:h:q;
subplot(4,1,1)
plot(t,a(:,k),'o')
title('MMCFD of a solution of smoking habit model');
xlabel('3 Years ');
ylabel('a');

subplot(4,1,2)
plot(t,b(:,k),'x')
title('MMCFD of b solution of smoking habit model');
xlabel('3 Years ');
ylabel('b');

subplot(4,1,3)
plot(t,c(:,k),'*')
title('MMCFD of c solution of smoking habit model');
xlabel('3 Years ');
ylabel('c');

subplot(4,1,3)
plot(t,d(:,k),'+')
title('MMCFD of d solution of smoking habit model');
xlabel('3 Years ');
ylabel('d');
```

Appendix D

MMCRK Application of Smoking Habit Model

```
%RK4 with MC to solve model of the nonlinear ODE system
%*****************************************************
clear
close all
format long

%%%To solve a model for the evolution of smoking habit
in Spain%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%model%
%da_dt = a1*(1-a(t))-a3*a(t)*(b(t)+c(t));
%db_dt = a3*a(t)*(b(t)+c(t))+a2*d(t)+a4*c(t)-
(a5+a6+a1)*b(t);
%dc_dt = a5*b(t)-(a4+a7+a1)*c(t);
%dd_dt = a6*b(t)+a7*c(t)-(a2+a1)*d(t);

%initial values%
%t0=0;                  %time per year
%y0=[a0 b0 c0 d0];      %y0 = initial condition of a
system
%a0=0.5045;a(1)=n0;     %initial value of a(t)
%b0=0.2059; b(1)=s0;    %initial value of b(t)
%c0=0.1559; c(1)=c0;    %initial value of c(t)
%d0=0.1337; d(1)=e0;    %initial value of d(t)

% model with RK4 %
%dy/dt in form of f(t,y).it can be a function of both
variables t and y,
%where y is n or s or c or e.
%da_dt =f1(t,a,b,c,d);
%db_dt =f2(t,a,b,c,d);
%dc_dt =f3(t,a,b,c,d);
%dd_dt =f4(t,a,b,c,d);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('******************Input Data*********************')

 k=1000;    %number of simulation
 h=1;       %step size
 q=3;       %number of iteration

%Generate values from the uniform distribution on the
interval [a, b].
```

```
disp('parameters of model')
 a11=0.01;
 a22=0.0425;
 a33=0.0381;
 a44=0.1244;
 a55=0.1175;
 a66=0.0498;
 a77=0.0498;

for j=1:k
  rand('seed',k)
  a1=(a11-0.2*a11)+((a11+0.2*a11)-(a11-
0.2*a11))*rand(j);
  a2=(a22-0.2*a22)+((a22+0.2*a22)-(a22-
0.2*a22))*rand(j);
  a3=(a33-0.2*a33)+((a33+0.2*a33)-(a33-
0.2*a33))*rand(j);
  a4=(a44-0.2*a44)+((a44+0.2*a44)-(a44-
0.2*a44))*rand(j);
  a5=(a55-0.2*a55)+((a55+0.2*a55)-(a55-
0.2*a55))*rand(j);
  a6=(a66-0.2*a66)+((a66+0.2*a66)-(a66-
0.2*a66))*rand(j);
  a7=(a77-0.2*a77)+((a77+0.2*a77)-(a77-
0.2*a77))*rand(j);
  j=j+1;
end

%parameters=[ a1  a2  a3  a4  a5  a6  a7 ]
disp('a1  a2  a3  a4  a5  a6  a7')
par=[ par_a1    par_a2    par_a3    par_a4    par_a5
par_a6    par_a7 ];

 disp('-------------------------------------------')
for j=1:k
  for i=1:q/h
%q=number of iteration= number of years between 2006 &
2009
%disp('t0 = initial t ')
t0=0;        %initial condition of t  %time per year
%y0=[a0 b0 c0 d0];
%disp('a0 = initial a ')
a0=0.5045;
a(1,j)=a0;  %initial value of a(t)
%disp('b0 = initial b ')
b0=0.2059;
b(1,j)=b0;  %initial value of b(t)
%disp('c0 = initial c ')
c0=0.1559;
c(1,j)=c0;  %initial value of c(t)
%disp('d0 = initial d ')
```

```
d0=0.1337;
d(1,j)=d0;  %initial value of d(t)

ka1=zeros(q/h,k);
ka2=zeros(q/h,k);
ka3=zeros(q/h,k);
ka4=zeros(q/h,k);

kb1=zeros(q/h,k);
kb2=zeros(q/h,k);
kb3=zeros(q/h,k);
kb4=zeros(q/h,k);


kc1=zeros(q/h,k);
kc2=zeros(q/h,k);
kc3=zeros(q/h,k);
kc4=zeros(q/h,k);

kd1=zeros(q/h,k);
kd2=zeros(q/h,k);
kd3=zeros(q/h,k);
kd4=zeros(q/h,k);

%Calculating ka1,kb1,kc1,kd1
 %ka1= f1(t(i),a(i),b(i),c(i),d(i));
ka1(i,j)= a1(j)*(1-a(i,j))-a3(j)*a(i,j)*(b(i,j)+c(i,j));
 %kb1= f2(t(i),a(i),b(i), c(i), d(i));
 kb1(i,j)=
a3(j)*a(i,j)*(b(i,j)+c(i,j))+a2(j)*d(i,j)+a4(j)*c(i,j)-
(a5(j)+a6(j)+a1(j))*b(i,j);
 %kc1= f3(t(i),a(i),b(i), c(i), d(i));
 kc1(i,j)= a5(j)*b(i,j)-(a4(j)+a7(j)+a1(j))*c(i,j);
 %kd1 = f4(t(i),a(i),b(i), c(i), d(i));
 kd1(i,j)= a6(j)*b(i,j)+a7(j)*c(i,j)-
(a2(j)+a1(j))*d(i,j);

%Calculating ka2,kb2,kc2,kd2
 %ka2
=f1(t(i)+0.5*h,a(i)+0.5*ka1(i)*h,b(i)+0.5*kb1(i)*h,c(i)+
0.5*kc1(i)*h,d(i)+0.5*kd1(i)*h);
 ka2(i,j)= a1(j)*(1-(a(i,j)+0.5*ka1(i,j)*h))-
a3(j)*(a(i,j)+0.5*ka1(i,j)*h)*((b(i,j)+0.5*kb1(i,j)*h)+(
c(i,j)+0.5*kc1(i,j)*h));
 %kb2(i)=
f2(t(i)+0.5*h,a(i)+0.5*ka1(i)*h,b(i)+0.5*kb1(i)*h,c(i)+0
.5*kc1(i)*h,d(i)+0.5*kd1(i)*h);
 kb2(i,j) =
a3(j)*(a(i,j)+0.5*ka1(i,j)*h)*((b(i,j)+0.5*kb1(i,j)*h)+(
c(i,j)+0.5*kc1(i,j)*h))+a2(j)*(d(i,j)+0.5*kd1(i,j)*h)+a4
```

```
(j)*(c(i,j)+0.5*kc1(i,j)*h)-
(a5(j)+a6(j)+a1(j))*(b(i,j)+0.5*kb1(i,j)*h);
 %kc2=
f3(t(i)+0.5*h,a(i)+0.5*ka1(i)*h,b(i)+0.5*kb1(i)*h,c(i)+0
.5*kc1(i)*h,d(i)+0.5*kd1(i)*h) ;
 kc2(i,j) = a5(j)*(b(i,j)+0.5*kb1(i,j)*h)-
(a4(j)+a7(j)+a1(j))*(c(i,j)+0.5*kc1(i,j)*h);
 %kd2(i,j) =
a6(j)*(b(i,j)+0.5*kb1(i,j)*h)+a7(j)*(c(i,j)+0.5*kc1(i,j)
*h)-(a2(j)+a1(j))*(d(i,j)+0.5*kd1(i,j)*h);
 kd2(i,j) =
a6(j)*(b(i,j)+0.5*kb1(i,j)*h)+a7(j)*(c(i,j)+0.5*kc1(i,j)
*h)-(a2(j)+a1(j))*(d(i,j)+0.5*kd1(i,j)*h);

 % Calculating ka3,kb3,kc3,kd3
 %ka3
=f1(t(i)+0.5*h,a(i)+0.5*ka2(i)*h,b(i)+0.5*kb2(i)*h,c(i)+
0.5*kc2(i)*h,d(i)+0.5*kd2(i)*h);
 ka3(i,j) = a1(j)*(1-(a(i,j)+0.5*ka2(i,j)*h))-
a3(j)*(a(i,j)+0.5*ka2(i,j)*h)*((b(i,j)+0.5*kb2(i,j)*h)+(
c(i,j)+0.5*kc2(i,j)*h));
 %kb3=
f2(t(i)+0.5*h,a(i)+0.5*ka2(i)*h,b(i)+0.5*kb2(i)*h,c(i)+0
.5*kc2(i)*h,d(i)+0.5*kd2(i)*h);
 kb3(i,j) =
a3(j)*(a(i,j)+0.5*ka2(i,j)*h)*((b(i,j)+0.5*kb2(i,j)*h)+(
c(i,j)+0.5*kc2(i,j)*h))+a2(j)*(d(i,j)+0.5*kd2(i,j)*h)+a4
(j)*(c(i,j)+0.5*kc2(i,j)*h)-
(a5(j)+a6(j)+a1(j))*(b(i,j)+0.5*kb2(i,j)*h);
 %kc3 =
f3(t(i)+0.5*h,a(i)+0.5*ka2(i)*h,b(i)+0.5*kb2(i)*h,c(i)+0
.5*kc2(i)*h,d(i)+0.5*kd2(i)*h) ;
 kc3(i,j) = a5(j)*(b(i,j)+0.5*kb2(i,j)*h)-
(a4(j)+a7(j)+a1(j))*(c(i,j)+0.5*kc2(i,j)*h);
 %ke3 =
f4(t(i)+0.5*h,a(i)+0.5*ka2(i)*h,b(i)+0.5*kb2(i)*h,c(i)+0
.5*kc2(i)*h,d(i)+0.5*kd2(i)*h);
 kd3(i,j) =
a6(j)*(b(i,j)+0.5*kb2(i,j)*h)+a7(j)*(c(i,j)+0.5*kc2(i,j)
*h)-(a2(j)+a1(j))*(d(i,j)+0.5*kd2(i,j)*h);

 %Calculating ka4,kb4,kc4,kd4
 %ka4
=f1(t(i)+h,a(i)+ka3(i)*h,b(i)+kb3(i)*h,c(i)+kc3(i)*h,d(i
)+kd3(i)*h);
 ka4(i,j) =a1(j)*(1-(a(i,j)+ka3(i,j)*h))-
a3(j)*(a(i)+ka3(i,j)*h)*((b(i,j)+
kb3(i,j)*h)+(c(i,j)+kc3(i,j)*h));
 %kb4=
f2(t(i)+h,a(i)+ka3(i)*h,b(i)+kb3(i)*h,c(i)+kc3(i)*h,d(i)
+kd3(i)*h);
```

```
kb4(i,j)=a3(j)*(a(i,j)+ka3(i,j)*h)*((b(i,j)+kb3(i,j)*h)+
(c(i,j)+kc3(i,j)*h))+a2(j)*(d(i,j)+kd3(i,j)*h)+a4(j)*(c(
i,j)+kc3(i,j)*h)-
(a5(j)+a6(j)+a1(j))*(b(i,j)+kb3(i,j)*h);
 %kc4 =
f3(t(i)+h,a(i)+ka3(i)*h,b(i)+kb3(i)*h,c(i)+kc3(i)*h,d(i)
+kd3(i)*h);
 kc4(i,j) = a5(j)*(b(i,j)+kb3(i,j)*h)-
(a4(j)+a7(j)+a1(j))*(c(i,j)+kc3(i,j)*h);
 %kd4 =
f4(t(i)+h,a(i)+ka3(i)*h,b(i)+kb3(i)*h,c(i)+kc3(i)*h,d(i)
+kd3(i)*h);
 kd4(i,j) =
a6(j)*(b(i,j)+kb3(i,j)*h)+a7(j)*(c(i,j)+kc3(i,j)*h)-
(a2(j)+a1(j))*(d(i,j)+kd3(i,j)*h);

 %Using 4th Order Runge-Kutta formula
a(i+1,j)=a(i,j)+(1/6)*(ka1(i,j)+2*ka2(i,j)+2*ka3(i,j)+ka
4(i,j))*h;

b(i+1,j)=b(i,j)+(1/6)*(kb1(i,j)+2*kb2(i,j)+2*kb3(i,j)+kb
4(i,j))*h;

c(i+1,j)=c(i,j)+(1/6)*(kc1(i,j)+2*kc2(i,j)+2*kc3(i,j)+kc
4(i,j))*h;

d(i+1,j)=d(i,j)+(1/6)*(kd1(i,j)+2*kd2(i,j)+2*kd3(i,j)+kd
4(i,j))*h;

  i=i+1;
  end
j=j+1;
end

%result=[a'  b'  c'  d']
 result=zeros(q/h,k,4);
for i=1:q/h
  res(i,k,1) = a(i,k);
  res(i,k,2) = b(i,k);
  res(i,k,3) = c(i,k);
  res(i,k,4) = d(i,k);
 i=i+1;
end
result=[ res(:,k,1)    res(:,k,2)     res(:,k,3)
res(:,k,4)]

disp('[ a  b  c  d ]')
for j=1:k
  sol=[  a(:,j)   b(:,j)    c(:,j)     d(:,j) ]
end
```

```
  solfinal=zeros(q/h,4);
  for j=1:k
    solfinal_a(j)=a(q/h,j);
    solfinal_b(j)=b(q/h,j);
    solfinal_c(j)=c(q/h,j);
    solfinal_d(j)=d(q/h,j);
    j=j+1;
  end

disp('[   solfinal_a      solfinal_b      solfinal_c
solfinal_d]')
solfinal=[solfinal_a(:)   solfinal_b(:)   solfinal_c(:)
solfinal_d(:)]
fprintf('solfinal_a       solfinal_b       solfinal_c
solfinal_d \n')
fprintf( '%1.5 f    %1.5 f    %1.5 f    %1.5 f\n' ,
solfinal_a  , solfinal_b, solfinal_c, solfinal_d )


musolfinal_a = mean(solfinal_a);
musolfinal_b = mean(solfinal_b);
musolfinal_c = mean(solfinal_c);
musolfinal_d = mean(solfinal_d);

disp('[ mean_a                    mean_b
mean_c               mean_d ]')
disp([musolfinal_a              musolfinal_b
musolfinal_c              musolfinal_d])

%Predicted values at 2009, when t=3 years
predict_a=0.5049;
predict_b=0.1240;
predict_c=0.1240;
predict_d=0.1805;

%Difference error
Diff_a=abs(predict_a-musolfinal_a);
Diff_b=abs(predict_b-musolfinal_b);
Diff_c=abs(predict_c-musolfinal_c);
Diff_d=abs(predict_d-musolfinal_d);

disp('[Diff_a      Diff_b      Diff_c     Diff_d]')
Diff_error=[Diff_a       Diff_b     Diff_c       Diff_d]

%Absolute relative approximate error
AE_a =abs((predict_a-musolfinal_a)/musolfinal_a);
AE_b =abs((predict_b-musolfinal_b)/musolfinal_b);
AE_c =abs((predict_c-musolfinal_c)/musolfinal_c);
AE_d =abs((predict_d-musolfinal_d)/musolfinal_d);
disp('[AE_a    AE_b     AE_c     AE_d]')
AE_error = [AE_a    AE_b    AE_c    AE_d]
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('[prctile_a]')
Pa=prctile(solfinal_a,[5 95])
disp('[prctile_b]')
Pb=prctile(solfinal_b,[5 95])
disp('[prctile_c]')
Pc=prctile(solfinal_c,[5 95])
disp('[prctile_d]')
Pd=prctile(solfinal_d,[5 95])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('Results of MMCFD of smoking habit')
%Predicted values in --------, when t years
predicta=0.5049;
predictb=0.1240;
predictc=0.1240;
predictd=0.1805;

MMCRKa=musolfinal_a;
MMCRKb=musolfinal_b;
MMCRKc=musolfinal_c;
MMCRKd=musolfinal_d;

disp('[Numerical Simulation for the System]')
disp('[MMCRK_a   MMCRK_b   MMCRK_c  MMCRK_d]')
disp([MMCRKa    MMCRKb    MMCRKc    MMCRKd])
%#########################################################

%Estimate of actual error by mean square error
disp('MSE to measure results error')

   %summation account
   aMSEsum1=0;
   bMSEsum1=0;
   cMSEsum1=0;
   dMSEsum1=0;

 k=1;
 while k<=q
   aMSEsum1=aMSEsum1+(predicta-solfinal_a(k))^2;
   bMSEsum1=bMSEsum1+(predictb-solfinal_b(k))^2;
   cMSEsum1=cMSEsum1+(predictc-solfinal_c(k))^2;
   dMSEsum1=dMSEsum1+(predictd-solfinal_d(k))^2;
    k=k+1;
 end

   aMSE_SOL =(1/q)*(aMSEsum1);
   bMSE_SOL =(1/q)*(bMSEsum1);
   cMSE_SOL =(1/q)*(cMSEsum1);
   dMSE_SOL =(1/q)*(dMSEsum1);
```

```
disp('[aMSE_SOL        bMSE_SOL         cMSE_SOL
dMSE_SOL]')
MSE_SOL=[aMSE_SOL       bMSE_SOL         cMSE_SOL
dMSE_SOL]
%@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@

figure(1)
%Sketch the results of the last simulation
disp('[  last simulation_a                   last
simulation_b                  last simulation_c
last simulation_d  ]')
last_sim=[  a(:,k)    b(:,k)    c(:,k)      d(:,k) ]
t=t0:h:q; %Calculates up to t_final results
plot(t,a(:,k),'o',t,b(:,k),'x',t,c(:,k),'*',t,d(:,k),'+'
)
title('MMCFD solutions of smoking habit model');
xlabel('3 Years');
ylabel('Subpopulations a,b,c,d');
legend('a','b','c','d')

figure(2)
t=t0:h:q;
subplot(4,1,1)
plot(t,a(:,k),'o')
title('MMCFD of a solution of smoking habit model');
xlabel('3 Years ');
ylabel('a');

subplot(4,1,2)
plot(t,b(:,k),'x')
title('MMCFD of b solution of smoking habit model');
xlabel('3 Years ');
ylabel('b');


subplot(4,1,3)
plot(t,c(:,k),'*')
title('MMCFD of c solution of smoking habit model');
xlabel('3 Years ');
ylabel('c');

subplot(4,1,3)
plot(t,d(:,k),'+')
title('MMCFD of d solution of smoking habit model');
xlabel('3 Years ');
ylabel('d')
```

المستخلص

الهدف من هذه الرسالة هو حل نظام غير خطي من مسائل القيم الابتدائية للمعادلات التفاضلية الاعتيادية من الرتبة الاولى مكون من متغيرات ومعلمات متعدده التي تمثل كمتغيرات عشوائية. في هذه الدراسة نستخدم طريقة محاكاة عددية جديده تكون مناسبة اكثر لحل مثل هذه النماذج. الطريقة المقترحة هي خليط بين طريقة مونتي كارلو ذات العمليات العشوائية و طريقة رانكا كوتا العددية، تسمى هذه الطريقة (MMCRK) طبقت لحل نموذجين وبائيين هما نموذج استهلاك الكحول ونموذج عادة التدخين. نطبق اربع طرق تقريبية على النموذجن الوبائيين، اثنين منهما تحليلية هما طريقة ادومين وطريقة التغاير والطرق الاخرى عددية هما طريقة الفروقات النسبية وطريقة رانكا كوتا. متوسط مربع الخطأ و مقياس الاختلاف أستخدما لغرض المقارنة بين حلول المحاكاة العددية للطريقة المقترحة والقيم المتوقعة. تم مقارنة طريقة (MMCRK) مع طريقة (MMCFD) للمحاكاة العددية، وتم التوصل الى ان طريقة (MMCRK) اقرب الى القيم المتوقعة من الدراسات السابقة مع نموذج استهلاك الكحول و نموذج عادة التدخين.

البرامج التي استخدمت لحساب النتائج المقدمه في هذه الرسالة هما برنامج الماثماتكا اصدار ١١ وبرنامج الماتلاب اصدار ٢٠١٣، وللرسم استخدم برنامج الراسم الساحر.

جمهورية العراق

وزارة التعليم العالي

والبحث العلمي

جامعة بغداد

كلية التربية للعلوم الصرفة (ابن الهيثم)

قسم الرياضيات

تقنيه المحاكاة العددية المعدلة لحل نماذج الوباء غير الخطية

رسالة

مقدمة الى كلية التربية للعلوم الصرفة/أبن الهيثم

جامعة بغداد وهي جزء من متطلبات نيل درجة

الماجستيرفي علوم الرياضيات

من قبل

مهدي عبد الرضا سبع

بأشراف

أ.م. د. مها عبد الجبار محمد

١٤٤١ هـ. **٢٠١٩ م.**