

Abstract

Elliptic equation, any of a class of partial differential equations describing phenomena that do not change from moment to moment, as when a flow of heat or fluid takes place within a medium with no accumulations. The Laplace equation, $u_{xx} + u_{yy} = 0$, is the simplest such equation describing this condition in two dimensions. In addition to satisfying a differential equation within the region, the elliptic equation is also determined by its values (boundary values) along the boundary of the region, which represent the effect from outside the region. These conditions can be either those of a fixed temperature distribution at points of the boundary (Dirichlet problem) or those in which heat is being supplied or removed across the boundary in such a way as to maintain a constant temperature distribution throughout (Neumann problem).

We start by reviewing a few basic properties of elliptic problems. We then introduce the maximum principle, and also formulate a similar principle for the heat equation. We prove the uniqueness and stability of solutions to the Laplace equation in two ways. One approach is based on the maximum principle, and the other approach uses the method of Green's identities. The simplest solution method for the Laplace equation is the method of separation of variables. Indeed, this method is only applicable in simple domains, such as rectangles, disks, rings, etc., but these domains are often encountered in applications. Moreover, explicit solutions in simple domains provide an insight into the solution's structure in more general domains. Towards the end of the research we shall introduce Poisson's kernel formula.