Abstract:

All rings considered in this paper will be associative with an identity element, and all modules will be right unitary R-modules. Let R be a ring and M an R-module. N is a submodule of M or M is an extension of N Denoted by $N \leq M$. A submodule N of M is essential (or large) in M if for every nonzero submodule $K \leq M$, we have $N \cap K \neq 0$. A relative complement for A in M is any submodule B of M which is maximal with respect to the property $A \cap B = 0$. N is said to be closed submodule of M if N has no proper essential extension in M. As a generalization of closed submodules, Goodearl introduced closed submodules in [3] with properties, remarks and examples of this concept.

Closed submodules have oﬀ ered rich topics of research, especially in the last 20 years, due to their important role played in ring and module theory and relative homological algebra. In parallel, several generalizations of closed submodules have been considered. For instance, neat submodules, coneat submodules and S-closed submodules are some of these generalizations. The purpose of research is to study closed submodules so many properties and examples are given.