

## Abstract

This report is basically concerned with the approximation of functions. The functions in question may be functions defined on a continuum typically a finite interval or functions defined only on a finite set of points. The first instance arises, for example, in the context of special functions (elementary or transcendental) that one wishes to evaluate as a part of a subroutine. Since any such evaluation must be reduced to a finite number of arithmetic operations, we must ultimately approximate the function by means of a polynomial or a rational function. The second instance is frequently encountered in the physical sciences when measurements are taken of a certain physical quantity as a function of some other physical quantity (such as time). In either case one wants to approximate the given function “as well as possible” in terms of other simpler functions

A fundamental mathematical technique is to approximate something complicated by something simple, or at least less complicated, in the hope that the simple can capture some of the essential information in the complicated. This is the core

idea of approximation with Taylor polynomials, a tool that has been central to mathematics since the calculus was first discovered. The wide-spread use of computers has made the idea of approximation even more important. Computers are basically good at doing very simple operations many times over. Effective use of computers therefore means that a problem must be broken up into (possibly very many) simple sub-problems. The result may provide only an approximation to the original problem, but this does not matter as long as the approximation is sufficiently good. The idea of approximation is often useful when it comes to studying functions. Most mathematical functions only exist in quite abstract mathematical terms and cannot be expressed as combinations of the elementary functions we know from school. In spite of this, virtually all functions of practical interest can be approximated arbitrarily well by simple functions like polynomials, trigonometric or exponential functions. Polynomials in particular are very appealing for use on a computer since the value of a polynomial at a point can be computed by utilising simple operations like addition and

multiplication that computers can perform extremely quickly